## **SYLLABUS**

# Qualitative theory of ordinary differential equations

# University year 2025-2026

## 1. Information regarding the programme

| 1.1. Higher education institution  | Babeş-Bolyai University          |
|------------------------------------|----------------------------------|
| 1.2. Faculty                       | Mathematics and Computer Science |
| 1.3. Department                    | Mathematics                      |
| 1.4. Field of study                | Mathematics                      |
| 1.5. Study cycle                   | Master                           |
| 1.6. Study programme/Qualification | Advanced Mathematics             |
| 1.7. Form of education             | Full Attendance                  |

## 2. Information regarding the discipline

| 2.1. Name of the disc    | cipli | <b>Qualitati</b><br>ne | Qualitative theory of ordinary differential equations |                         |  |   | Discipline<br>code | MME3109     |            |
|--------------------------|-------|------------------------|---|-------------------------|--|---|--------------------|-------------|------------|
| 2.2. Course coordinator  |       |                        |   | Conf. Dr. Adriana Buică |  |   |                    |             |            |
| 2.3. Seminar coordinator |       |                        | Conf. Dr. Adriana Buică                               |                         |  |   |                    |             |            |
| 2.4. Year of study       | 1     | 2.5. Semester          | mester <b>1</b> 2.6. Type of evaluation               |                         |  | Ε | 2.7. Discip        | line regime | Compulsory |

#### 3. Total estimated time (hours/semester of didactic activities)

| 3.1. Hours per week   | 3           | of which: 3.2 course      | 2         | 3.3 seminar/laboratory | 1  |  |
|---|-------------|---------------------------|-----------|------------------------|----|--|
| 3.4. Total hours in the curriculum                                      | 42          | of which: 3.5 course      | 28        | 3.6 seminar/laborator  | 14 |  |
| Time allotment for individual study (ID) and self-study activities (SA) |             |                           |           |                        |    |  |
| Learning using manual, course support,                                  | bibliograp  | hy, course notes (SA)     |           |                        | 47 |  |
| Additional documentation (in libraries, o                               | on electroi | nic platforms, field docu | imentatio | n)                     | 30 |  |
| Preparation for seminars/labs, homework, papers, portfolios and essays  |             |                           |           |                        |    |  |
| Tutorship   |             |                           |           |                        |    |  |
| Evaluations   |             |                           |           |                        |    |  |
| Other activities:   |             |                           |           |                        |    |  |
| 3.7. Total individual study hours   133                                 |             |                           |           |                        |    |  |
| <b>3.8. Total hours per semester</b> 175                                |             |                           |           |                        |    |  |
| 3.9. Number of ECTS credits 7   |             |                           |           |                        |    |  |

#### 4. Prerequisites (if necessary)

| 4.1. curriculum   | Mathematical Analysis; Differential Equations   |  |
|-------------------|---|--|
| 4.2. competencies | Logical thinking, as well mathematical notions and properties from the above mentioned fields |  |

#### 5. Conditions (if necessary)

| 5.1. for the course                  | Classroom with blackboard |
|--------------------------------------|---------------------------|
| 5.2. for the seminar /lab activities | Classroom with blackboard |

### 6. Specific competencies acquired <sup>1</sup>

<sup>&</sup>lt;sup>1</sup> One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

| Professional/essential<br>competencies | Ability to understand and manipulate advanced concepts, results and theories in the fields of mathematics.   |
|--|--|
| Transversal<br>competencies            | <ul> <li>Ability to inform themselves, to work independently or in a team in order to realize studies and to solve complex problems.</li> <li>Ability for continuous self-perfecting and study.</li> </ul> |

# 7. Objectives of the discipline (outcome of the acquired competencies)

| 7.1 General objective of the discipline  | • To present the fundamental results and methods for the study of the qualitative behaviour of the solutions of linear and nonlinear differential equations.   |
|--|--|
| 7.2 Specific objective of the discipline | <ul> <li>Full understanding of the dynamic of linear autonomous systems.</li> <li>Understand the fundamental results for nonlinear systems</li> <li>Ability to study the stability of equilibrium and periodic solutions for nonlinear systems.</li> </ul> |

#### 8. Content

| 8.1 Course   | Teaching methods             | Remarks |
|--|------------------------------|---------|
| 1.Linear differential systems. Fundamental theory.   | Exposition, proofs, examples |         |
| 2.Linear differential systems with constant coefficients.<br>The exponential matrix for a diagonalizable matrix.                     | Exposition, proofs, examples |         |
| 3.Linear differential systems with constant coefficients.<br>The exponential matrix for a deffective matrix.                         | Exposition, proofs, examples |         |
| 4.The asymptotic behaviour of the solutions of linear systems with constant coefficients. Stable, unstable, center manifolds.        | Exposition, proofs, examples |         |
| 5.The fundamental theorems for nonlinear systems: the existence and uniqueness theorem   | Exposition, proofs, examples |         |
| 6.The fundamental theorems for nonlinear systems: maximal interval of existence  | Exposition, proofs, examples |         |
| 7.The fundamental theorems for nonlinear systems:<br>continuity and differentiability with respect to<br>parameters and initial data | Exposition, proofs, examples |         |
| 8.Stability of equilibria of nonlinear autonomous systems by linearization.  | Exposition, proofs, examples |         |
| 9.Stability of equilibria of nonlinear autonomous systems by the Lyapunov functions method   | Exposition, proofs, examples |         |
| 10.Stability of nonautonomous linear differential systems  | Exposition, proofs, examples |         |
| 11.Stability of periodic linear differential systems.<br>Floquet theory I  | Exposition, proofs, examples |         |

| 12.Stability of periodic linear differential systems.<br>Floquet theory II | Exposition, proofs, examples |  |
|--|------------------------------|--|
| 13.Periodic solutions of linear periodic systems                           | Exposition, proofs, examples |  |
| 14.Stability of periodic solutions of periodic nonlinear systems           | Exposition, proofs, examples |  |
|  |                              |  |

Bibliography

- 1. A. Buică, Periodic solutions for nonlinear systems, Cluj University Press, 2006.
- 2. A. Buică, Lecture Notes on Qualitative theory of differential equations uploaded in Teams.
- 3. C. Chicone, Ordinary differential equations with applications, Springer, 2006.
- 4. E.A. Coddington, N. Levinson, Theory of ordinary differential equations, 1959.
- 5. P. Hartman, Ordinary differential equations, SIAM, 2002.
- 6. L. Perko, Differential equations and dynamical systems, Springer, 2001.
- 7. M. Viana, J.M Espinar, Differential equations: a dynamical systems approach to theory and practice, American Mathematical Society, 2021.

| 8.2 Seminar   | Teaching methods                  | Remarks |
|---|-----------------------------------|---------|
| 1.Exercises and problems related to the fundamental           | Examples, dialogue, explanations, |         |
| theorems for linear differential systems.                     | proofs, critical thinking         |         |
| 2.Exercises to recognize a diagonalizable matrix over R       |                                   |         |
| or over C, the computation of their exponential and the       | Examples, dialogue, explanations, |         |
| general solution of the corresponding linear differential     | critical thinking                 |         |
| system.   |                                   |         |
| 3Exercises to recognize a deffective matrix, the              | Examples dialogue explanations    |         |
| computation of their exponential and the general              | proofs critical thinking          |         |
| solution of the corresponding linear differential system.     | proofs, critical tilliking        |         |
| 4 Stable unstable and center manifolds on examples            | Examples, dialogue, explanations, |         |
| +.stable, unstable and center mannolus on examples.           | critical thinking                 |         |
| 5.The existence and uniqueness theorem on examples            | Examples, dialogue, explanations, |         |
| and its important consequences.                               | proofs, critical thinking         |         |
| 6.The maximal interval of existence for the solutions of      | Examples, dialogue, explanations, |         |
| various equations, including the pendulum equation.           | critical thinking                 |         |
| 7.Continuity and differentiability with respect to the        | Examples, dialogue, explanations, |         |
| initial data and parameters on examples.                      | proofs, critical thinking         |         |
| 8.The stability by linearization of the equilibria of various | Examples, dialogue, explanations, |         |
| differential autonomous systems.                              | critical thinking                 |         |
| 9. The stability by the direct method of Lyapunov of the      | Examples, dialogue, explanations, |         |
| equilibria of various differential autonomous systems.        | proofs, critical thinking         |         |
| 10.The stability of linear nonautononous systems.             | Examples, dialogue, explanations, |         |
| Exrcises to understand how to apply the main results.         | critical thinking                 |         |
| 11 Applications of the Floquet theory                         | Examples, dialogue, explanations, |         |
|   | proofs, critical thinking         |         |
| 12.The stability of some periodic linear differential         | Examples, dialogue, explanations, |         |
| equations, including the Hill equation.                       | critical thinking                 |         |
| 13.The existence of periodic solutions of some periodic       | Examples, dialogue, explanations, |         |
| differential equations, including the Hill equation.          | proofs, critical thinking         |         |
| 14.Discussion on the stability of periodic solutions of       | Examples, dialogue, explanations, |         |
| autonomous systems.   | proofs, critical thinking         |         |
| Bibliography  |                                   |         |

1. A. Buică, Periodic solutions for nonlinear systems, Cluj University Press, 2006.

- 2. A. Buică, Lecture Notes on Qualitative theory of differential equations uploaded in Teams.
- 3. C. Chicone, Ordinary differential equations with applications, Springer, 2006.
- 4. E.A. Coddington, N. Levinson, Theory of ordinary differential equations, 1959.
- 5. P. Hartman, Ordinary differential equations, SIAM, 2002.
- 6. L. Perko, Differential equations and dynamical systems, Springer, 2001.
- 7. M. Viana, J.M Espinar, Differential equations: a dynamical systems approach to theory and practice, American Mathematical Society, 2021.

# 9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- The content of this discipline is synchronized with the curriculum of most of the important universities from our country and from abroad where the applied mathematics plays an important role.
- The content is a strong background for further doctoral studies.

#### 10. Evaluation

| Activity type                        | 10.1 Evaluation criteria  | 10.2 Evaluation methods       | 10.3 Percentage of final grade |  |  |  |
|--------------------------------------|---|-------------------------------|--------------------------------|--|--|--|
| 10.4 Course                          | Knowledge of the notions<br>and their properties by<br>examples or<br>counterexamples. Ability to<br>prove the main theoretical<br>results. | Written exam                  | 50%                            |  |  |  |
|                                      | Development of a specific<br>subject by reading the<br>bibliography.  | Report with oral presentation | 20%                            |  |  |  |
|                                      | Solving problems skills   | Evaluation of the homeworks   | 30%                            |  |  |  |
| 10.5 Seminar/laboratory              | Active participation in the classroom   |                               |                                |  |  |  |
| 10.6 Minimum standard of performance |   |                               |                                |  |  |  |
| The minimum passing grade is 5.      |   |                               |                                |  |  |  |

#### 11. Labels ODD (Sustainable Development Goals)<sup>2</sup>



Date of approval: 25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș

<sup>&</sup>lt;sup>2</sup> Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write *"Not applicable."*.