SYLLABUS

Mathematical Methods in Fluid Mechanics

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeş-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Master
1.6. Study programme/Qualification	Advanced Mathematics
1.7. Form of education	Full time

2. Information regarding the discipline

2.1. Name of the discipline	**Mathematical Methods in Fluid Mechanics**				Discipline code	**MME3104**	
2.2. Course coordinator					**Professor PhD Mirela KOHR**		
2.3. Seminar coordinator					**Professor PhD Mirela KOHR**		
2.4. Year of study	**1**	2.5. Semester	**1**	2.6. Type of evaluation	**C**	2.7. Discipline regime	**Compulsory/DF**

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**3**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**1 sem**
3.4. Total hours in the curriculum	**42**	of which: 3.5 course	**28**	3.6 seminar/laborator	**14**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					38
Additional documentation (in libraries, on electronic platforms, field documentation)					38
Preparation for seminars/labs, homework, papers, portfolios and essays					38
Tutorship					10
Evaluations					9
Other activities:					-
3.7. Total individual study hours	**133**				
3.8. Total hours per semester	**175**				
3.9. Number of ECTS credits	**7**				

4. Prerequisites (if necessary)

4.1. curriculum	In-depth knowledge of the following disciplines: • Theoretical Mechanics; • Partial Differential Equations; • Real Analysis; • Numerical Analysis.
4.2. competencies	There are useful logical thinking and mathematical notions and results from the above-mentioned fields

5. Conditions (if necessary)

5.1. for the course	Classroom with blackboard, video projector
5.2. for the seminar /lab activities	Classroom with blackboard, video projector

6.1. Specific competencies acquired [1]

Professional/essential competencies	• Ability to understand, handle and communicate concepts, fundamental and advanced theories in the field of mathematics. • Ability to model and analyze from the mathematical point of view real processes from other sciences, Physics, Medicine, and Engineering. • Ability to understand scientific papers in the field of mathematics, to formulate new problems and to initiate new mathematical research, preparing reports and scientific papers. • Ability to analyze in a pertinent way the results obtained comparing with various alternative approaches. • Use of advanced skills to develop and manage mathematical projects of research nature, applying a wide range of quantitative and qualitative methods. • Ability to use acquired knowledge in pursuing a doctoral program in Pure Mathematics, Applied Mathematics, or in other fields that use mathematical models. • Ability for continuous self-perfecting and study.
Transversal competencies	• Ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • Application of organized and efficient work rules, a responsible attitude towards the didactic-scientific field, to bring creative value to own potential respecting professional ethics principles. • Ability to adopt and integrate in different environments from education and research. • Ability to adapt to the requirements of a dynamical society and to communicate efficiently in an international language.

6.2. Learning outcomes

Knowledge	• The master student knows, understand and use concepts, individual results and advanced mathematical theories. • The master student has the ability to develop and use efficient research skills. • The master student has the ability to model and analyze from the mathematical point of view real processes and phenomena from other sciences, physics, medicine, and engineering.
Skills	• The master student has the ability to identify and state significant problems which can be the basis for subsequent research. • The master student has the ability to use scientific language and to perform research in Mathematics.
Responsibility and autonomy:	• The master student has the ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • The master student has the ability of critical investigation of specific literature. • The master student has the ability to use international data bases of academic research.

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Knowledge, understanding and use of main concepts and results of Fluid Mechanics. • Ability to use and apply concepts and fundamental results of the theory of partial differential equations, topology, real analysis in the study of specific problems of fluid mechanics. • Knowledge, understanding and use of advanced mathematical methods in the study of special linear or non-linear problems in Fluid Mechanics.
7.2 Specific objective of the discipline	• Acquiring basic and advanced knowledge in Fluid Mechanics. • Ability to apply and use mathematical models to describe and analyze problems concerning viscous fluid flows. • Knowledge, understanding and use of main concepts and results in the mathematical theory of viscous incompressible flows at low Reynolds numbers. • Knowledge, understanding and use of advanced topics in mathematics in the study of special boundary value problems in fluid mechanics: variational techniques, layer potential theoretical methods, fixed point techniques. • Ability student involvement in scientific research.

8. Content

8.1 Course	**Teaching methods**	**Remarks**
1. Introduction in the theory of Sobolev spaces (I): The fundamental spaces of the theory of distributions. Distributions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Introduction in the theory of Sobolev spaces (II): Sobolev spaces on $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. The dual of a Sobolev space. The Sobolev continuous embedding theorem and the Rellich - Kondrachov compact embedding theorem.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Kinematics of fluids: fluid, configuration, motion. Velocity and acceleration fields. Spatial description of the motion of a fluid.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
4. Fluid Dynamics: Principle of mass conservation. The continuity equation.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Fluid Dynamics: The Cauchy stress tensor. The Cauchy equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. The constitutive equation of ideal fluid. The Euler equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. The mathematical model of viscous Newtonian fluid: The constitutive equation and the Navier-Stokes equations. Special forms of the Navier-Stokes equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. Uniqueness results for the Dirichlet and Neumann problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$. Variational approach for the weak solution of the Stokes problem in a bounded Lipschitz domain with Dirichlet boundary condition.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The method of fundamental solutions in fluid mechanics: The Oseen-Burgers tensor and the fundamental pressure vector for the Stokes system in $\mathbf{R}^n$ (n=2, 3).	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. The layer potential theory for the Stokes system (I): Bounded and compact operators, Fredholm operators on Banach spaces. The Fredholm alternative.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

11. The layer potential theory for the Stokes system (II): Single- and double layer potentials for the Stokes system. Boundedness, compactness, and Fredholm properties in Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Applications of the Stokes layer potential methods: Well-posedness results in Sobolev spaces for boundary value problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$. Existence and uniqueness in Sobolev spaces for the Dirichlet problem for the Navier-Stokes system in bounded Lipschitz domains in $\mathbf{R}^3$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
13. Applications of Stokes layer potential methods and variational techniques: Well-posedness results in weighted Sobolev spaces for the exterior Dirichlet problem for the Stokes system in $\mathbf{R}^n$. Existence results in weighted Sobolev spaces for the exterior Dirichlet problem for the Navier-Stokes system in $\mathbf{R}^3$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
14. Layer potentials, boundary and transmission problems for the Stokes and Navier-Stokes systems with variable coefficients in Lipschitz domains: Variational and layer potential approach. Well-posedness results in Sobolev spaces. Applications to viscous flow problems in the presence of interfaces and in porous media. Numerical results. Research directions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.
2. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian).
3. Kohr, M., *Special Topics of Mechanics*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).
4. Kohr, M., *Mathematical Methods in Fluid Mechanics*, Lecture Notes, 2024/2025.
5. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000.
6. Boyer, F., Fabrie, P., *Mathematical Tools for the Study of the Incompressible Navier-Stokes Equations and Related Models*, Springer, New York, 2013.
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8. Adams, R. Fournier, J., *Sobolev Spaces*, 2nd Edition, Pure and Applied Mathematics, vol. 140, Elsevier/Academic Press, Amsterdam, 2003.
9. Agranovich, M.S., *Sobolev Spaces, Their Generalizations, and Elliptic Problems in Smooth and Lipschitz Domains,* Springer, Heidelberg, 2015.
10. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 1th Edition 2008, 2nd Edition 2021.
11. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, France, 344 (2012): viii+241 pp.
12. Temam, R., *Navier-Stokes Equations. Theory and Numerical Analysis*, AMS Chelsea Edition, 2001.
13. Sayas, F-J., Brown, T.S., Hassell, M.E., *Variational Techniques for Elliptic Partial Differential Equations: Theoretical Tools and Advanced Applications*, CRC Press, Boca Raton, FL, 2019.
14. Rieutord, M., *Fluid Dynamics. An Introduction*, Springer Cham Heidelberg, 2015.

8.2 Seminar	Teaching methods	Remarks
1. Introduction in the theory of Sobolev spaces (I): The fundamental spaces of the theory of distributions. Distributions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
2. Introduction in the theory of Sobolev spaces (II): Sobolev spaces over $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. Trace theorems.	Applications of course concepts. Description of arguments and proofs for	1 hour/week

	solving problems. Homework assignments. Direct answers to the questions of students.	
3. Differential operators. Material derivatives. The Euler theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
4. Second order Cartesian tensors in $\mathbf{R}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
5. Properties of the Cauchy stress tensor: Cauchy's fundamental theorem, and the symmetry property.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
6. The mathematical model of incompressible fluid.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
7. The Killing theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
8. Variational approach for the weak solution of the Stokes problem in a bounded Lipschitz domain with homogeneous Dirichlet boundary condition.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
9. The exterior Dirichlet problem for the Stokes system in Lipschitz domains in $\mathbf{R}^n$ (n=2,3). Well-posedness results and applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
10. The method of fundamental solutions in fluid mechanics (I): Layer potential representations for the Stokes flow.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
11. The method of fundamental solutions in fluid mechanics (II): Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
12. Well-posedness results in Sobolev spaces for boundary value problems for the Stokes and Navier-Stokes systems in bounded Lipschitz domains in $\mathbf{R}^3$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework	1 hour/week

	assignments. Direct answers to the questions of students.	
13. Exterior Dirichlet problems for the Stokes and Navier-Stokes systems in $\mathbf{R}^3$, with data in weighted Sobolev spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week
14. Stokes and Navier-Stokes systems with variable coefficients in Lipschitz domains: Variational and layer potential approach. Applications to flow problems in porous media. Numerical results. Research directions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to the questions of students.	1 hour/week

Bibliography

1. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.
2. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian).
3. Kohr, M., *Special Topics of Mechanics*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).
4. Kohr, M., *Mathematical Methods in Fluid Mechanics*, Seminar Notes, 2024/2025.
5. Kohr, M., Mikhailov, S.E., Wendland, W.L., *Non-homogeneous Dirichlet-transmission problems for the anisotropic Stokes and Navier-Stokes systems in Lipschitz domains with transversal interfaces*, Calculus of Variations and Partial Differential Equations, 61:198 (2022), 47 pp.
6. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000.
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8. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 1th Edition 2008, 2nd Edition 2021.
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13. McLean, W., *Strongly Elliptic Systems and Boundary Integral Equations*, Cambridge University Press, Cambridge, UK, 2000.
14. Wloka, J. T. , Rowley, B., Lawruk, B., *Boundary Value Problems for Elliptic Systems*, Cambridge University Press, Cambridge, 1995.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- The content of this discipline is in accordance with the curricula of the most important universities in Romania and abroad, where the advanced mathematics plays an essential role.
- This discipline is useful in specific PhD research activities, in preparing future researchers in pure and applied mathematics, and for those who use mathematical models and advanced methods of study in other areas.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of concepts and basic results.	Colloquium	60%
	Ability to justify by proofs theoretical results.		
10.5 Seminar	Ability to apply concepts and results acquired in the course in mathematical modeling and analysis of problems in Fluid Mechanics.	Evaluation of reports and homework during the semester, and active participation in the seminar activity. A midterm written test.	15% 25%
There are valid the official rules of the faculty concerning the attendance of students at teaching activities.			
10.6 Minimum standard of performance			
• The final grade should be at least 5 (from a scale of 1 to 10).			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development							
								9 INDUSTRY, INNOVATION AND INFRASTRUCTURE

Date:
11.04.2025

Signature of course coordinator

Prof.PhD. Mirela KOHR

Signature of seminar coordinator

Prof.PhD. Mirela KOHR

Date of approval:
25.04.2025

Signature of the head of department

Prof.PhD. Andrei MĂRCUŞ

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.".