

## SYLLABUS

### 1. Information regarding the programme

1.1 Higher education institution	Babeş-Bolyai University Cluj-Napoca
1.2 Faculty	Faculty of Mathematics and Computer Science
1.3 Department	Department of Mathematics
1.4 Field of study	Mathematics
1.5 Study cycle	Bachelor of Science
1.6 Study programme / Qualification	Mathematics and Computer Science

### 2. Information regarding the discipline

2.1 Name of the discipline	Real Analysis						
2.2 Course coordinator	Conf. dr. Adriana Nicolae						
2.3 Seminar coordinator	Conf. dr. Adriana Nicolae						
2.4. Year of study	2	2.5 Semester	4	2.6. Type of evaluation	C	2.7 Type of discipline	Compulsory

### 3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	4	Of which: 3.2 course	2	3.3 seminar/laboratory	2
3.4 Total hours in the curriculum	56	Of which: 3.5 course	28	3.6 seminar/laboratory	28
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					25
Additional documentation (in libraries, on electronic platforms, field documentation)					10
Preparation for seminars/labs, homework, papers, portfolios and essays					20
Tutorship					4
Evaluations					10
Other activities					-
3.7 Total individual study hours	69				
3.8 Total hours per semester	125				
3.9 Number of ECTS credits	5				

### 4. Prerequisites (if necessary)

4.1. curriculum	<ul style="list-style-type: none"> <li>Calculus 1, 2</li> </ul>
4.2. competencies	<ul style="list-style-type: none"> <li>Analytic thinking</li> </ul>

### 5. Conditions (if necessary)

5.1. for the course	<ul style="list-style-type: none"> <li>Lecture hall equipped with blackboard</li> </ul>
5.2. for the seminar /lab activities	<ul style="list-style-type: none"> <li>Classroom equipped with blackboard</li> </ul>

### 6. Specific competencies acquired

<b>Professional competencies</b>	<ul style="list-style-type: none"> <li>C1.1 Identification of notions, description of theories and use of specific language.</li> <li>C1.4 Recognition of main classes/types of mathematical problems and of appropriate techniques for solving them.</li> <li>C5.2 Use of mathematical arguments to prove mathematical results.</li> </ul>
<b>Transversal competencies</b>	<ul style="list-style-type: none"> <li>CT1 Application of efficient and rigorous working rules by adopting responsible attitudes towards the scientific and didactic fields for the development of the own creative potential respecting professional and ethical principles.</li> </ul>

## 7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	<ul style="list-style-type: none"> <li>To acquire fundamental knowledge about general measure theory and integration and to apply it in solving problems.</li> </ul>
7.2 Specific objective of the discipline	<ul style="list-style-type: none"> <li>To acquire knowledge about elements of general measure theory and integration (e.g., <math>\sigma</math>-algebras, measures, outer measures, Lebesgue measure, integration of measurable functions, limit theorems, normed spaces, Hilbert spaces, <math>L^p</math> spaces).</li> </ul>

## 8. Content

8.1 Course	Teaching methods	Remarks
1. Introduction: the problem of measure. Measurable spaces and measure spaces	Lecture, discussion, didactical demonstration, problematisation	
2. The Lebesgue exterior measure	Lecture, discussion, didactical demonstration, problematisation	
3. The Lebesgue measure	Lecture, discussion, didactical demonstration, problematisation	
4. Properties of the Lebesgue measure	Lecture, discussion, didactical demonstration, problematisation	
5. Measurable functions	Lecture, discussion, didactical demonstration, problematisation	
6. Approximation of measurable functions	Lecture, discussion, didactical demonstration, problematisation	
7. Integration of measurable functions (I)	Lecture, discussion, didactical demonstration, problematisation	
8. Integration of measurable functions (II)	Lecture, discussion, didactical demonstration, problematisation	
9. Limit theorems and applications (I)	Lecture, discussion, didactical demonstration, problematisation	
10. Limit theorems and applications (II). The relation between the Riemann and Lebesgue integrals.	Lecture, discussion, didactical demonstration, problematisation	
11. Lebesgue's Differentiation Theorem	Lecture, discussion, didactical demonstration, problematisation	
12. Types of convergence. Normed spaces and Hilbert spaces	Lecture, discussion, didactical demonstration, problematisation	
13. $L^p$ spaces (I)	Lecture, discussion, didactical demonstration, problematisation	
14. $L^p$ spaces (II)	Lecture, discussion, didactical demonstration, problematisation	

### Bibliography

- V. Anisiu, Topologie și teoria măsurii, Universitatea "Babeș-Bolyai", Cluj-Napoca, 1993.
- J.J. Benedetto, W. Czaja, Integration and modern analysis, Birkhäuser, Boston, MA, 2009.
- D.L. Cohn, Measure theory, 2<sup>nd</sup> ed., Birkhäuser/Springer, New York, 2013.
- G.B. Folland, Real analysis. Modern techniques and their applications, 2<sup>nd</sup> ed., John Wiley & Sons, Inc., New York, 1999.
- F. Jones, Lebesgue integration on Euclidean space, Jones and Bartlett Publishers, Boston, MA, 1993.
- H.L. Royden, P.M. Fitzpatrick, Real analysis, 4th ed., Pearson, 2010.
- W. Rudin, Real and complex analysis, 3<sup>rd</sup> ed., McGraw-Hill Book Co., New York, 1987.
- E. Stein, R. Shakarchi, Real analysis. Measure theory, integration, and Hilbert spaces, Princeton University Press, Princeton, NJ, 2005.
- D.W. Stroock, A concise introduction to the theory of integration, 2<sup>nd</sup> ed., Birkhäuser Boston, Inc.,

Boston, MA, 1994.

10. T. Tao, An introduction to measure theory, American Mathematical Society, Providence, RI, 2011.

8.2 Seminar	Teaching methods	Remarks
1. Introduction: the problem of measure. Measurable spaces and measure spaces	Discussion, problem solving, didactical demonstration	
2. The Lebesgue exterior measure	Discussion, problem solving, didactical demonstration	
3. The Lebesgue measure	Discussion, problem solving, didactical demonstration	
4. Properties of the Lebesgue measure	Discussion, problem solving, didactical demonstration	
5. Measurable functions	Discussion, problem solving, didactical demonstration	
6. Approximation of measurable functions	Discussion, problem solving, didactical demonstration	
7. Integration of measurable functions (I)	Discussion, problem solving, didactical demonstration	
8. Integration of measurable functions (II)	Discussion, problem solving, didactical demonstration	
9. Limit theorems and applications (I)	Discussion, problem solving, didactical demonstration	
10. Limit theorems and applications (II). The relation between the Riemann and Lebesgue integrals.	Discussion, problem solving, didactical demonstration	
11. Lebesgue's Differentiation Theorem	Discussion, problem solving, didactical demonstration	
12. Types of convergence. Normed spaces and Hilbert spaces	Discussion, problem solving, didactical demonstration	
13. $L^p$ spaces (I)	Discussion, problem solving, didactical demonstration	
14. $L^p$ spaces (II)	Discussion, problem solving, didactical demonstration	
Bibliography (in addition to the books mentioned before which also contain exercises)		
1. R.L. Schilling, Measures, integrals and martingales, Cambridge University Press, New York, 2005.		
2. W.J. Kaczor, M.T. Nowak, Problems in Mathematical Analysis III. Integration, American Mathematical Society, Providence, RI, 2003.		
3. A. Torchinsky, Problems in real and functional analysis, American Mathematical Society, Providence, RI, 2015.		

**9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program**

The course ensures a solid theoretical background, according to national and international standards. This discipline is useful in preparing future teachers and researchers in mathematics, but is also addressed to those who use various modern mathematical methods and techniques in other areas.

**10. Evaluation**

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade
10.4 Course	- Knowledge of basic notions, examples and results - Ability to prove	- Test, exam - Lecture and seminar activity	- Test: 35% - Exam: 65% - Lecture and seminar

	theoretical results		activity: bonus max. 5%
10.5 Seminar/lab activities	- Problem solving using concepts and results acquired during the lecture classes		
<b>10.6 Minimum performance standards</b>			
<ul style="list-style-type: none"> <li>- The accumulation of at least 10 attendances at the seminar.</li> <li>- Both the exam grade and the final grade should be at least 5. The bonus points are only awarded in this case.</li> </ul>			

Date  
30.04.2024

Signature of course coordinator  
Conf. dr. Adriana Nicolae

Signature of seminar coordinator  
Conf. dr. Adriana Nicolae

Date of approval

Signature of the head of department  
Prof. dr. Andrei Mărcuş