SYLLABUS

Complements of Geometry

University year 2025-2026

1. Information regarding the programme

1.1. Higher education institution	Babeș-Bolyai University
1.2. Faculty	Mathematics and Computer Science
1.3. Department	Mathematics
1.4. Field of study	Mathematics
1.5. Study cycle	Bachelor
1.6. Study programme/Qualification	Mathematics and Computer Science (English) / Mathematician
1.7. Form of education	Full-time

2. Information regarding the discipline

2.1. Name of the discipline	**Complements of Geometry**					Discipline code	**MLE0041**
2.2. Course coordinator				Lect. univ. dr. George-Cătălin Țurcaș			
2.3. Seminar coordinator				Lect. univ. dr. George-Cătălin Țurcaș			
2.4. Year of study	II	2.5. Semester	4	2.6. Type of evaluation	V	2.7. Discipline regime	Speciality Discipline

3. Total estimated time (hours/semester of didactic activities)

3.1. Hours per week	**4**	of which: 3.2 course	**2**	3.3 seminar/laboratory	**2**
3.4. Total hours in the curriculum	56	of which: 3.5 course	28	3.6 seminar/laborator	**28**
Time allotment for individual study (ID) and self-study activities (SA)					**hours**
Learning using manual, course support, bibliography, course notes (SA)					20
Additional documentation (in libraries, on electronic platforms, field documentation)					20
Preparation for seminars/labs, homework, papers, portfolios and essays					30
Tutorship					14
Evaluations					10
Other activities:					
3.7. Total individual study hours	**94**				
3.8. Total hours per semester	**150**				
3.9. Number of ECTS credits	**6**				

4. Prerequisites (if necessary)

4.1. curriculum	Completion of **Analytic Geometry (first year)** and an understanding of vector representation of lines and planes in 3D; completion of **Analysis II (second semester)** and working knowledge of differentiation and integration of vector functions.
4.2. competencies	Proficiency in differentiating and integrating vector functions; ability to represent lines and planes in three-dimensional space using vector notation.

5. Conditions (if necessary)

5.1. for the course	Blackboard, chalk, projector
5.2. for the seminar /lab activities	Blackboard, chalk, projector

6.1. Specific competencies acquired [1]

[1] One can choose either competences or learning outcomes, or both. If only one option is chosen, the row related to the other option will be deleted, and the kept one will be numbered 6.

Professional/essential competencies	• **C1.1** Identify and utilize fundamental concepts of differential geometry and applying them to analyses curves and surfaces in the three-dimensional space. • **C2.1** Apply mathematical methods to compute geometric invariants of curves and surfaces, interpreting their significance.
Transversal competencies	• **CT1** Apply rigorous mathematical methodologies and demonstrate a responsible attitude towards the scientific field, creatively solving specific problems. • **CT3** Develop effective communication skills to convey mathematical concepts and results, both orally and in writing, using appropriate mathematical language and modern presentation techniques.

6.2. Learning outcomes

Knowledge	Upon successful completion of this course, the student knows: • The fundamental concepts of differential geometry, including curvature and torsion of curves, and the first and second fundamental forms of surfaces. • The intrinsic and extrinsic properties of surfaces; • The application of differential geometry principles in related fields such as physics, engineering and computer science.
Skills	Upon successful completion of this course, the student can: • Compute geometric invariants of curves and surfaces, including length, curvature, torsion, and various types of curvature. • Distinguish between intrinsic and extrinsic invariants of curves and surfaces. • Apply mathematical methods to analyse and classify curves and surfaces based on their geometric properties. • Present advanced concepts in geometry and topology both orally and in writing.
Responsibility and autonomy:	Upon successful completion of this course, the student can work independently to: • Explore mathematical content related to curves and surfaces, drawing on ideas and tools from previous coursework to extend their understanding. • Interpret mathematical literature on differential geometry and incorporate ideas and results into their own work. • Communicate mathematical ideas with precision, clarity, and organization, both orally and in writing.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• To provide students with a comprehensive understanding of the geometry of curves and surfaces in three-dimensional space, focusing on both theoretical foundations and practical applications
7.2 Specific objective of the discipline	• Introduce and analyse parameterized curves, including concepts such as arc length, curvature, and torsion, to understand the geometry of space curves and to explore the properties of surfaces through parameterization, examining tangent spaces, fundamental forms, and curvature measures, to comprehend the intrinsic and extrinsic geometry of surfaces.

8. Content

8.1 Course	**Teaching methods**	**Remarks**

Week 1. Definition of curves; Arc-length	Lecture, demonstration, examples	
Week 2. Reparameterization; Closed curves	Lecture, demonstration, examples	
Week 3. Level curves vs parameterized curves; Definition of curvature; Curvature for plane curves	Lecture, demonstration, examples	
Week 4. Signed curvature determines the curve; Space curves	Lecture, demonstration, examples	
Week 5. Global properties of curves; The isoperimetric inequality; The four vertex theorem	Lecture, demonstration, examples	
Week 6. First partial exam	Partial exam (assessment)	
Week 7. Definition of surfaces; Smooth surfaces; Smooth maps	Lecture, demonstration, examples	
Week 8. Tangents and derivatives; Normals and orientability	Lecture, demonstration, examples	
Week 9. The first fundamental form	Lecture, demonstration, examples	
Week 10. Curvature of surfaces; Second fundamental form	Lecture, demonstration, examples	
Week 11. Gaussian, mean and principal curvatures	Lecture, demonstration, examples	
Week 12. Gauss' Theorema Eggregium	Lecture, demonstration, examples	
Week 13. The Gauss-Bonnet Theorem	Lecture, demonstration, examples	
Week 14. Second partial exam	Partial exam (assessment)	

Bibliography

1. A. Pressley, *Elementary Differential Geometry*, 2nd ed., Springer, London, 2010.

2. M. P. do Carmo, *Differential Geometry of Curves and Surfaces*, revised and updated 2nd ed., Dover, New York, 2016.

3. B. O'Neill, *Elementary Differential Geometry*, 2nd ed., Academic Press, 2006.

4. T. F. Banchoff and S. T. Lovett, *Differential Geometry of Curves and Surfaces*, 2nd ed., CRC Press, 2016.

8.2 Seminar / laboratory	**Teaching methods**	**Remarks**
Week 1. Problems with the definition of curves; Arc-length	Examples, dialogue, explanation, demonstration, problem-solving	
Week 2. Reparameterization; Closed curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 3. Level curves vs parameterized curves; Definition of curvature; Curvature for plane curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 4. Signed curvature determines the curve; Space curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 5. Global properties of curves; The isoperimetric inequality; The four vertex theorem	Examples, dialogue, explanation, demonstration, problem-solving	
Week 6. Discussion of the partial exam. More problems with curves	Examples, dialogue, explanation, demonstration, problem-solving	
Week 7. Definition of surfaces; Smooth surfaces; Smooth maps	Examples, dialogue, explanation, demonstration, problem-solving	
Week 8. Tangents and derivatives; Normals and orientability	Examples, dialogue, explanation, demonstration, problem-solving	
Week 9. The first fundamental form	Examples, dialogue, explanation, demonstration, problem-solving	
Week 10. Curvature of surfaces; Second fundamental form	Examples, dialogue, explanation, demonstration, problem-solving	
Week 11. Gaussian, mean and principal curvatures	Examples, dialogue, explanation, demonstration, problem-solving	
Week 12. Gauss' Theorema Eggregium	Examples, dialogue, explanation, demonstration, problem-solving	

Week 13. The Gauss-Bonnet Theorem	Examples, dialogue, explanation, demonstration, problem-solving	
Week 14. Discussion of the final partial exam. More problems with curves	Examples, dialogue, explanation, demonstration, problem-solving	

Bibliography

1. A. Pressley, *Elementary Differential Geometry*, 2nd ed., Springer, London, 2010.

2. M. P. do Carmo, *Differential Geometry of Curves and Surfaces*, revised and updated 2nd ed., Dover, New York, 2016.

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4. T. F. Banchoff and S. T. Lovett, *Differential Geometry of Curves and Surfaces*, 2nd ed., CRC Press, 2016.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- Differential Geometry is widely recognized as fundamental within mathematical education, integral to curricula at leading universities internationally. The importance of its role is emphasized not only due to historical contributions by renowned mathematicians such as Gauss but also through its modern applications in fields spanning physics, engineering, and computer science.
- Students exposed to differential geometry develop robust analytical and spatial reasoning skills, preparing them effectively for advanced research and diverse professional challenges.

10. Evaluation

Activity type	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Percentage of final grade
10.4 Course	Knowledge of fundamental concepts and results; problem-solving ability	Two written tests	80%
10.5 Seminar/laboratory	Problem-solving ability based on learned concepts and theorems; Ability to synthesize mathematical content and effectively communicate it through clear presentations	Homework; solving problems at the boar; individual projects and presentations	20%
10.6 Minimum standard of performance			
• Obtaining a minimum weighted average of 5 (on a scale from 1 to 10), calculated according to the evaluation percentages specified above (80% course assessments and 20% seminar activities).			

11. Labels ODD (Sustainable Development Goals)[2]

	General label for Sustainable Development

[2] Keep only the labels that, according to the *Procedure for applying ODD labels in the academic process*, suit the discipline and delete the others, including the general one for *Sustainable Development* – if not applicable. If no label describes the discipline, delete them all and write „*Not applicable*.".

								

Date:
11.04.2025

Signature of course coordinator

Lect. dr. George Țurcaș

Signature of seminar coordinator

Lect. dr. George Țurcaș

Date of approval:
25.04.2025

Signature of the head of department

Prof. dr. Andrei Mărcuș