SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	**Babeş Bolyai University**
1.2 Faculty	**Faculty of Mathematics and Computer Science**
1.3 Department	**Department of Mathematics**
1.4 Field of study	**Mathematics**
1.5 Study cycle	**Master**
1.6 Study programme / Qualification	**Advanced Mathematics**

2. Information regarding the discipline

2.1 Name of the discipline	**MME3158 Calculus of Variations on Manifolds**						
2.2 Course coordinator	prof. dr. Alexandru Kristály						
2.3 Seminar coordinator	prof. dr. Alexandru Kristály						
2.4. Year of study	**1**	2.5 Semester	**1**	2.6. Type of evaluation	**E**	2.7 Type of discipline	**Optional**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	3	Of which: 3.2 course	2	3.3 seminar/l~~aboratory~~	1
3.4 Total hours in the curriculum	42	Of which: 3.5 course	28	3.6 seminar/~~laboratory~~	14
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					28
Additional documentation (in libraries, on electronic platforms, field documentation)					28
Preparation for seminars/labs, homework, papers, portfolios and essays					49
Tutorship					14
Evaluations					14
Other activities:					

3.7 Total individual study hours	133
3.8 Total hours per semester	175
3.9 Number of ECTS credits	7

4. Prerequisites (if necessary)

4.1. curriculum	- deep knowledge of bachelor level analysis, especially of the following subjects: - calculus - differential geometry
4.2. competencies	- ability to operate with abstract concepts in analysis - ability to do logical deductions - ability to solve mathematics problems bases on aquired notions

5. Conditions (if necessary)

5.1. for the course	• blackboard, projector
5.2. for the seminar /lab activities	• blackboard, projector

6. Specific competencies acquired

Professional competencies	• C1.1 Identifying the notions, describing the theories and using the specific language. • C2.3 Applying the adequate analytical theoretical methods to a given problem.
Transversal competencies	• CT1. Applying some rules of precise and efficient work, showing a responsible attitude regarding the scientific domain and teaching training for an optimal and creative development of the personal potential in specific situations, respecting the deontological norms.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Advanced knowledge in calculus of variations. Ability to solve more difficult problems
7.2 Specific objective of the discipline	• students will operate with fundamental concepts of calculus of variations • students will aquire knowlegde regarding the calculus of variations • students solve problems, theoretical and practical, using instruments of from calculus of variations on curved spaces.

8. Content

8.1 Course	Teaching methods	Remarks
1. Introduction in Calculus of Variations: examples	Explanation, dialogue, examples, proofs	
2. Isoperimetric problems	Explanation, dialogue, examples, proofs	
3. Brunn-Minkowski inequalities: an optimal mass transport approach	Explanation, dialogue, examples, proofs	
4. Euler-Lagrange equation	Explanation, dialogue, examples, proofs	
5. Brachystochron problem (classical and the tunnel problem)	Explanation, dialogue, examples, proofs	
6. Weber-type problems: Torricelli points	Explanation, dialogue, examples, proofs	
7. Weber-type problems on non-euclidean spaces: influence of gravity	Explanation, dialogue, examples, proofs	
8. Busemann-type inequalities: Thales theorem and applications in curved spaces	Explanation, dialogue, examples, proofs	
9. Minimization arguments: compact case	Explanation, dialogue, examples, proofs	
10. Minimization arguments: non-compact case	Explanation, dialogue, examples, proofs	

11. Variational principles (Ekeland, Borwein-Preiss, Ricceri)	Explanation, dialogue, examples, proofs	
12. Critical points	Explanation, dialogue, examples, proofs	
13. Minimax theorems	Explanation, dialogue, examples, proofs	
14. Applications in elliptic PDEs (existence results)	Explanation, dialogue, examples, proofs	

Bibliography

1. Kristály A., Radulescu V., Varga Cs., *Variational Principles in Mathematical Physics, Geometry, and Economics*, Cambridge University Press, Enciclopedia of Mathematics and its Applications. No 136, 2010.
2. Costea N., Kristály A, Varga C., *Variational and monotonicity methods in nonsmooth analysis*. Frontiers in Mathematics. *Birkhäuser/Springer, Cham*, 2021.

8.2 Seminar / laboratory	Teaching methods	Remarks
15. Introduction in Calculus of Variations: examples	dialogue, examples, proofs	
16. Isoperimetric problems	dialogue, examples, proofs	
17. Brunn-Minkowski inequalities: an optimal mass transport approach	dialogue, examples, proofs	
18. Euler-Lagrange equation	dialogue, examples, proofs	
19. Brachystochron problem (classical and the tunnel problem)	dialogue, examples, proofs	
20. Weber-type problems: Torricelli points	dialogue, examples, proofs	
21. Weber-type problems on non-euclidean spaces: influence of gravity	dialogue, examples, proofs	
22. Busemann-type inequalities: Thales theorem and applications in curved spaces	dialogue, examples, proofs	
23. Minimization arguments: compact case	dialogue, examples, proofs	
24. Minimization arguments: non-compact case	dialogue, examples, proofs	
25. Variational principles (Ekeland, Borwein-Preiss, Ricceri)	dialogue, examples, proofs	
26. Critical points	dialogue, examples, proofs	
27. Minimax theorems	dialogue, examples, proofs	
28. Applications in elliptic PDEs (existence results)	dialogue, examples, proofs	

Bibliography

3. Struwe M., *Variational Methods*, Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems, Fourth edition. A Series of Modern Surveys in Mathematics [Results in Mathematics and Related Areas. 3rd Series. A Series of Modern Surveys in Mathematics], 34. Springer-Verlag, Berlin, 2008.
4. Kristály A, Mezei I, Szilak K, *Elliptic differential inclusions on non-compact Riemannian manifolds*. Nonlinear Anal. Real World Appl. *69 (2023),* Paper No. 103740, 20 pp.
5. Farkas C, Kristály A, Mester A, *Compact Sobolev embeddings on non-compact manifolds via orbit expansions of isometry groups*. *Calc. Var. Partial Differential Equations* 60 (2021), no. 4, Paper No. 128, 31 pp.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- Such a course exists in the curricula of all major universities in Romania and abroad;
- Elements from calculus of variations are fundamental mathematical tools and have multiple applications in geometry, optimization, physics, etc.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	- know the basic principles of the field; - apply the new concepts	- written exam	75%
10.5 Seminar/lab activities	- problem solving	- homeworks	25%
10.6 Minimum performance standards			
➢ to aquire 5 points to pass the exam			

Date	Signature of course coordinator	Signature of seminar coordinator
14.05.2023	Prof.dr. Alexandru Kristály	Prof.dr. Alexandru Kristály

Date of approval	Signature of the head of department
....	Prof. dr. Andrei Mărcuș