SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	Babeş Bolyai University
1.2 Faculty	Faculty of Mathematics and Computer Science
1.3 Department	Department of Mathematics
1.4 Field of study	Mathematics
1.5 Study cycle	Master
1.6 Study programme / Qualification	Advanced Mathematics

2. Information regarding the discipline

2.1 Name of the discipline			Vector Optimization				
2.2 Course coordinator			Tiberiu Trif				
2.3 Seminar coordinator			Tiberiu Trif				
2.4. Year of study	2	2.5 Semester	3	2.6. Type of evaluation	VP	2.7 Type of discipline	Optional

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	3	Of which: 3.2 course	2	3.3 seminar	1
3.4 Total hours in the curriculum	42	Of which: 3.5 course	28	3.6 seminar	14
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					30
Additional documentation (in libraries, on electronic platforms, field documentation)					30
Preparation for seminars/labs, homework, papers, portfolios and essays					30
Tutorship					14
Evaluations					29
Other activities:					-

3.7 Total individual study hours	133
3.8 Total hours per semester	175
3.9 Number of ECTS credits	7

4. Prerequisites (if necessary)

4.1. curriculum	• Mathematical analysis 1 (Analysis on R); • Mathematical analysis 2 (Differential Calculus on R^n).
4.2. competencies	Ability to use abstract notions, theoretical results and practical methods of Mathematical Analysis.

5. Conditions (if necessary)

5.1. for the course	Lecture room equipped with a beamer
5.2. for the seminar /lab activities	Standard room

6. Specific competencies acquired

Professional competencies	Ability to use appropriate mathematical methods and implementable algorithms for solving practical vector optimization problems.
Transversal competencies	To apply rigorous and efficient work rules, by adopting a responsible attitude towards the scientific and didactic activities. To develop the own creative potential in specific areas, following the professional ethical norms and principles.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	Students should acquire knowledge about vector (multicriteria) optimization.
7.2 Specific objective of the discipline	Students will study several classes of practical vector optimization problems.

8. Content

8.1 Course	Teaching methods	Remarks
1. Preorder relations; maximal elements of a set with respect to a preference relation; formulation of general optimization problems. Linear prorder relations (compatible with the vector addition and multiplication of vectors by scalars).	Direct instruction, mathematical proof, exemplification	
2. Cones; characterizations of (convex, pointed, generating, totally-generating) cones; the relationship between linear preorder relations and convex cones. Topological properties of convex cones: (relative) solid and closed convex cones; the polar cone of a set; polyhedral cones.	Direct instruction, mathematical proof, exemplification	
3. Concepts of efficiency in vector optimization; efficient points and weakly efficient points w.r.t. a convex cone; efficient solutions and weakly efficient solutions of vector optimization problems.	Direct instruction, mathematical proof, exemplification	
4. Monotone and strictly monotone scalar functions (w.r.t. a preorder relation) and the their extremum points; examples of linear/nonlinear monotone functions; conical sections of a set; the existence of efficient/weakly efficient points.	Direct instruction, mathematical proof, exemplification	
5. Sufficient conditions for efficiency and weak efficiency. Cone-convex sets; necessary conditions for weak-efficiency. Proper efficient points.	Direct instruction, mathematical proof, exemplification	
6. Cone-convex vector-valued functions, their characterizations by means of the epigraph and the polar cone; the cone-convexity of the images of convex sets by cone-convex functions.	Direct instruction, mathematical proof, exemplification	
7. Explicitly cone-quasiconvex functions and lexicographic quasiconvex vector-valued functions, their characterization and some of important properties; the relationship between explicit cone-convexity and lexicographic quasiconvexity.	Direct instruction, mathematical proof, exemplification	

8. Scalarization methods for vector optimization problems: the weighting method (for convex objective functions); the parametric method (for quasiconvex/, explicitly quasiconvex/ explicitly quasiaffine objective functions).	Direct instruction, mathematical proof, exemplification	
9. The geometric and topological structure of the boundary of a closed radiant set (the homeomorphism of Bonnisseau-Cornet).	Direct instruction, mathematical proof, exemplification	
10. Simply shaded and completely shaded sets (w.r.t. a convex cone) and their characterizations. The connectedness /contractibility of the sets of efficient points.	Direct instruction, mathematical proof, exemplification	
11. The role of Helly's Theorem in reducing the number of criteria involved in vector optimization with convex/quasiconvex objective functions.	Direct instruction, mathematical proof, exemplification	
12. Pareto reducible vector optimization problems involving explicitly / lexicographic quasiconvex objective functions.	Direct instruction, mathematical proof, exemplification	
13. Approximate efficient / weakly efficient solutions and their role in numerical methods.	Direct instruction, mathematical proof, exemplification	
14. Efficient sequences and their relationship with the minimizing sequences of certain scalarization functions.	Direct instruction, mathematical proof, exemplification	

Bibliography

1. BRECKNER, B.E., POPOVICI, N.: Convexity and Optimization. An Introduction, EFES, Cluj-Napoca, 2006.
2. EHRGOT, M.: Multicriteria Optimization. Springer, Berlin Heidelberg New York, 2005.
3. GOEPFERT, A., RIAHI, H., TAMMER, C., ZALINESCU, C.: Variational Methods in Partially Ordered Spaces. Springer-Verlag, New York, 2003.
4. JAHN, J.: Vector Optimization. Theory, Applications, and Extensions. Springer, Berlin, 2004.
5. LUC, D.T.: Theory of Vector Optimization. Springer Verlag, Berlin, 1989.
6. POPOVICI, N.: Optimizare vectoriala, Casa Cartii de Stiinta, Cluj-Napoca, 2005.

8.2 Seminar	Teaching methods	Remarks
1. Geometric interpretation of the preference relations induced by the objective functions of some practical optimization problems (Fermat-Weber-type location problems, resource allocation problems, etc.)	Problem-based instruction, debate, mathematical proofs	
2. Particular classes of convex cones in the *n*-dimensional Euclidean space (polyhedral cones, the lexicographic cone, Phelps-type cones).	Problem-based instruction, debate, mathematical proofs	
3. Exercises involving the concepts of: polar cone, basis of a convex cone, the (relative) interior, and the facial structure of a convex cone.	Problem-based instruction, debate, mathematical proofs	
4. Finding the efficient / weakly efficient solutions of certain vector optimization problems by a geometric approach.	Problem-based instruction, debate, mathematical proofs	
5. Exercises concerning the (strict) monotony of certain scalar functions.	Problem-based instruction, debate, mathematical proofs	
6. Identifying the (weakly) efficient solutions of some concrete vector optimization problems in R^2 by means of the necessary and sufficient conditions of (weakly) efficiency.	Problem-based instruction, debate, mathematical proofs	
7. Geometric representations of the direct images of	Problem-based	

convex/polyhedral sets by certain cone-convex functions and their (weakly) efifcient points.	instruction, debate, mathematical proofs	
8. Geometric representation of the level sets of certain cone-quasiconvex vector-valued functions.	Problem-based instruction, debate, mathematical proofs	
9. Exercises concerning explicitly quasiconvex functions (in particular, lexicographic convex functions and linear-fractional functions).	Problem-based instruction, debate, mathematical proofs	
10. Bicriteria optimization problems solved by a geometrical approach.	Problem-based instruction, debate, mathematical proofs	
11. Linear vector optimization problems solved by the weighting scalarization method.	Problem-based instruction, debate, mathematical proofs	
12. Nonlinear vector optimization problems solved by the weighting scalarization method.	Problem-based instruction, debate, mathematical proofs	
13. Linear vector optimization problems solved by the parametric method.	Problem-based instruction, debate, mathematical proofs	
14. Nonlinear vector optimization problems solved by the parametric method.	Problem-based instruction, debate, mathematical proofs	

Bibliography

1. ALZORBA, S., GUNTHER, C., POPOVICI, N., TAMMER, C.: A new algorithm for solving planar multiobjective location problems involving the Manhattan norm, European Journal of Operational Research, Vol. 258 (1) 2017, pp. 35-46.
2. EHRGOT, M.: Multicriteria Optimization. Springer, Berlin Heidelberg New York, 2005.
3. POPOVICI, N.: Pareto reducible multicriteria optimization problems, Optimization, Vol. 54 (2005), pp. 253-263.
4. SAWARAGI, Y., NAKAYAMA, H., TANINO, T.: Theory of Multiobjective Optimization. Academic Press, New York, 1985.
5. YU, P.L.: Multiple criteria decision making: concepts, techniques and extensions. Plenum Press, New York - London, 1985.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The course ensures a solid theoretical background, according to national and international standards

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	- Knowledge of theoretical concepts and capacity to rigorously prove the main theorems; - Ability to solve practical exercises and theoretical problems	Written exam	75%
10.5 Seminar/lab activities	- Attendance and active class participation	Continuous evaluation	25%
10.6 Minimum performance standards			
The final grade should be greater than or equal to 5.			

Date	Signature of course coordinator	Signature of seminar coordinator
29.4.2023	Tiberiu Trif	Tiberiu Trif

Date of approval	Signature of the head of department
29.4.2023	