SYLLABUS

1.1 Higher education		
institution	Babeş-Bolyai University Cluj-Napoca	
1.2 Faculty	Faculty of Mathematics and Computer Science	
1.3 Department	Department of Mathematics	
1.4 Field of study	Mathematics	
1.5 Study cycle	Master	
1.6 Study programme /	Advanced Methometics	
Qualification	Advanced Mathematics	

1. Information regarding the programme

2. Information regarding the discipline

2.1 Name of the di	iscipli	ne (en)	(en) Integral Equations with Applications				
(ro)			Ecuații integrale cu aplicații				
2.2 Course coordin	nator		Prof. Sanda Micula, PhD. Habil.				
2.3 Seminar coord	inator		Prof. Sanda Micula, PhD. Habil.				
2.4. Year of study	2	2.5 Semester	3	2.6. Type of evaluation	E	2.7 Type of discipline	DS/Optional
2.8 Code of the discipline		MME3160		•	·		

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	3	Of which:	3.2 course	2	3.3 seminar/laboratory	1
3.4 Total hours in the curriculum	42	Of which:	3.5 course	28	3.6 seminar/laboratory	14
Time allotment:						hours
Learning using manual, course support, bibliography, course notes					40	
Additional documentation (in libraries, on electronic platforms, field documentation)					30	
Preparation for seminars/labs, homework, papers, portfolios and essays					40	
Tutorship					14	
Evaluations					9	
Other activities:						
3.7 Total individual study hours 133					<u> </u>	
3.8 Total hours per semester 175						
3.9 Number of ECTS credits		7				

4. Prerequisites (if necessary)

4.1. curriculum	Mathematical Analysis, Numerical Analysis
4.2. competencies	Knowledge of basic notions of operator theory
	Average programming skills

5. Conditions (if necessary)

5.1. for the course	Classroom with large blackboard and video projector
5.2. for the seminar /lab	Classroom with large blackboard and video projector/computers with
activities	Matlab

6. Specific competencies acquired

Professional competencies	 Ability to understand and manipulate concepts, results and advanced mathematical theories. Ability to model and analyze from the mathematical point of view real processes from other sciences, economics, and engineering. Ability to use the scientific language and to write scientific reports and papers.
Transversal competencies	 Ability to inform themselves, to work independently or in a team in order to realize studies and to solve complex problems. Ability for continuous self-perfecting and study. Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics and Applied Mathematics.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	 Acquire knowledge of the general theory of integral equations, with focus on applications. Gain the ability to apply concepts and results from integral equations theory to specific problems.
7.2 Specific objective of the discipline	 Understand and be able to use main concepts and results from general integral equations theory. Be able to analyze the solvability of specific integral equations arising in applications. Understand, use and be able to derive numerical methods for the approximate solution of integral equations arising in applications from various fields.

8. Content

8.1 Course	Teaching methods	Remarks
1. Introduction . Basic concepts. History of integral equations. Classifications and examples.	 Interactive exposure Explanation Conversation Description 	
2. Types of integral equations with exact solutions.	 Interactive exposure Explanation Conversation Description 	
3. Relationship between initial value/boundary value problems and integral equations.	 Interactive exposure Explanation Conversation Description 	
4. Volterra integral equations . The method of successive approximations. Laplace transforms. Adomian decomposition.	 Interactive exposure Explanation Conversation Description 	

5. Series solution. Volterra integral equations of	Interactive exposure
the first kind. Integral equations of the convolution	Explanation
type. Abel integral equation.	Conversation
type. Moer integral equation.	Description
6. Fredholm integral equations. The method of	Interactive exposure
successive approximations, Neumann series.	• Explanation
Adomian decomposition. Compact integral	Conversation
operators. Properties. The Fredholm alternative	Description
theorem.	1
7. Homogeneous Fredholm equations. Fredholm	Interactive exposure
integral equations of the first kind.	Explanation
	Conversation
	Description
8. Numerical methods. Degenerate kernel	Interactive exposure
methods. Taylor series approximation.	Explanation
Interpolatory degenerate kernel approximation.	Conversation
1 7 5 11	Description
9. Projection methods, collocation and Galerkin	Interactive exposure
methods. Iterated collocation and Galerkin	Explanation
methods. Error analysis.	Conversation
-	Description
10. Nyström methods. Product integration	Interactive exposure
methods. Error analysis. Discrete collocation and	Explanation
discrete Galerkin methods.	Conversation
	Description
11. Applications . Volterra's population model.	• Interactive exposure
Diffraction problems, Fresnel integrals.	Explanation
	Conversation
	Description
12. Applications to potential theory. The Thomas-	• Interactive exposure
Fermi equation.	Explanation
	Conversation
	Description
13. Applications to ocean waves. Green's function	Interactive exposure
method for waves. Seismic response of dams.	Explanation
	Conversation
	Description
14. Heat transfer and heat radiation.	Interactive exposure
	• Explanation
	Conversation
Bibliography	Description

Bibliography

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.

2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.

3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.

4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.

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2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.

3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.

4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.

6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- Courses with similar content exist in the studying program of major universities in Romania and abroad, for Mathematics and Applied Mathematics students at the Master's level;
- The knowledge and skills acquired in this course give students a foundation for launching a career in scientific research;
- The analysis and modeling abilities acquired in this course are useful in any career path students may

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	 acquire the basic principles and notions in Integral Equations theory; apply correctly various course concepts and methods 	Written exam	70%
10.5 Seminar/lab activities	 understand and be able to use theory and results in problems and applications; apply numerical procedures and algorithms to solve practical and real- life problems 	 active participation in discussing and solving problems throughout the semester individual presentation of solutions 	30%
10.6 Minimum performance	e standards		
A grade of 5 or above (on a seminar evaluation)	a scale from 1 to 10) on <u>each</u>	of the activities mentioned abo	ove (written exam,

Date	Signature of course coordinator	Signature of seminar coordinator
17.05.2023	Prof. Sanda Micula, PhD. Habil.	Prof. Sanda Micula, PhD. Habil.

Date of approval

Signature of the head of department

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