SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	**Babeş-Bolyai University of Cluj-Napoca**
1.2 Faculty	**Faculty of Mathematics and Computer Science**
1.3 Department	**Doctoral School in Mathematics and Computer Science**
1.4 Field of study	**Mathematics**
1.5 Study cycle	**Doctoral studies**
1.6 Study programme	**TRAINING PROGRAM BASED ON ADVANCED ACADEMIC STUDIES**

2. Information regarding the discipline

2.1 Name of the discipline	**Optimal mass transportation with applications / Teoria transportului optimal cu aplicații Teaching language: English**						
2.2 Course coordinator	**Kristály Alexandru**						
2.3 Seminar coordinator	**Kristály Alexandru**						
2.4. Year of study	**1**	2.5 Semester	**1**	2.6. Type of evaluation	**E**	2.7 Type of discipline	**Optional**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	3	Of which: 3.2 course	2	3.3 seminar/laboratory	1 sem
3.4 Total hours in the curriculum	36	Of which: 3.5 course	24	3.6 seminar/laboratory	12
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					60
Additional documentation (in libraries, on electronic platforms, field documentation)					70
Preparation for seminars/labs, homework, papers, portfolios and essays					40
Tutorship					23
Evaluations					21
Other activities:					

3.7 Total individual study hours	214
3.8 Total hours per semester	250
3.9 Number of ECTS credits	10

4. Prerequisites (if necessary)

4.1. curriculum	Riemannian and Finsler geometry, measure theory	
4.2. competencies	Elliptic PDEs, Heisenberg groups	

5. Conditions (if necessary)

5.1. for the course	Blackboard and videoprojector

5.2. for the seminar /lab activities	Laboratory with computers; high level programming language environment; Blackboard

6. Specific competencies acquired

Professional competencies	• Recognition of the main types of mathematical problems, selection of methods and of appropriate techniques for solving them • Explaining and interpreting mathematical models • Use of mathematical reasoning in demonstrations
Transversal competencies	• Applying the rules of rigorous and efficient work, manifesting responsible attitudes compared to the scientific field for the optimal and creative valorification of own potential in specific situations with respect to the principles and norms of ethics • Efficient use of information sources, training and development resources

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	Deepening some chapters in the theory of optimal mass transportation and applying these results in the geometry of metric measure spaces
7.2 Specific objective of the discipline	● Deepening the study of geometric inequalities on smooth/nosmooth spaces ● Applications of the optimal mass transport in the theory of partial differential equations

8. Content

8.1 Course	Teaching methods	Remarks
1. Kantorovich duality	Interactive talk, modelation, proof, exemplification	
2. c-concave functions	Interactive talk, modelation, proof, exemplification	
3. Monge-Ampere equation	Interactive talk, modelation, proof, exemplification	
4. Jacobi fields and Ricci curvature on Riemannian manifolds	Interactive talk, modelation, proof, exemplification	
5. Interpolation and displacement convexity	Interactive talk, modelation, proof, exemplification	
6. Curvature-dimension condition CD(K,N) in the sense of Lott-Sturm-Villani	Interactive talk, modelation, proof, exemplification	
7. Sharp functional inequalities via optimal mass transport	Interactive talk, modelation, proof, exemplification	
8. Geometric inequalities in the CD(K,N) sense (Brunn-Minowski, Borell-Brascamp-Lieb, Prekopa-Leindler)	Interactive talk, modelation, proof, exemplification	
9. Equality cases in geometric inequalities	Interactive talk, modelation, proof, exemplification	

10. Jacobian-type inequalities on sub-Riemannian manifolds	Interactive talk, modelation, proof, exemplification	
11. Geometric inequalities on sub-Riemannian manifolds (Carnot and Heisenberg groups)	Interactive talk, modelation, proof, exemplification	
12. Open problems in the theory of optimal mass transport		
8.2 Seminar	Teaching methods	Remarks
1. Cyclical monotonicity (Rockafellar theorem), exercises	Proof, conversation, cooperation, individual study	
2. Wasserstein distance	Proof, conversation, cooperation, individual study	
3. Transportation costs, exercises	Proof, conversation, cooperation, individual study	
4. Distortion coefficients in the sense of Lott-Sturm-Villani	Proof, conversation, cooperation, individual study	
5. Concavity of det^1/n	Proof, conversation, cooperation, individual study	
6. Co-area formula and layer cake representation	Proof, conversation, cooperation, individual study	
7. Symmetrisation (Polya-Szego-type inequality)	Proof, conversation, cooperation, individual study	
8. Isoperimetric inequality in R^N via optimal mass transportation	Proof, conversation, cooperation, individual study	
9. Rigidity results	Proof, conversation, cooperation, individual study	
10. Isoperimetric inequalities on curved spaces, examples and counterexamples	Proof, conversation, cooperation, individual study	
11. Geodesics and distortions on the Heisenberg groups	Proof, conversation, cooperation, individual study	
12. Open problems	Proof, conversation, cooperation, individual study	

Bibliography

1. C. Villani, Optimal transport. Old and new. Fundamental Principles of Mathematical Sciences, 338. Springer-Verlag, Berlin, 2009.
2. C. Villani, Topics in optimal transportation. Graduate Studies in Mathematics, 58. American Mathematical Society, Providence, RI, 2003.
3. Z. Balogh, A. Kristály, Equality in Borell-Brascamp-Lieb inequalities on curved spaces. *Adv. Math.* 339 (2018), 453–494.
4. Z. Balogh, A. Kristály, K. Sipos, Geometric inequalities on Heisenberg groups. *Calc. Var. Partial Differential Equations* 57 (2018), no. 2, Art. 61, 41 pp.
5. A. Kristály, V. Radulescu, Cs. Varga, Variational Principles in Mathematical Physics, Geometry, and Economics, Cambridge University Press, Cambridge, 2010.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The present discipline emphasizes the use of notions from mathematical analysis, Riemannian and Finsler geometry with applications in two important directions:

1. understanding/mastering some notions/techniques related to the theory of optimal mass transportation and geometric/functional inequalities

2. mastering some notions and results from applied mathematics and their use in further research

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Evaluation of knowledge and earned competencies	2 tests (the first on the 7th week, the second on 13th week)	50%
10.5 Seminar	Seminar activity	Conversation, individual work	20 %
	Presentation of a research topic	Conversation and clarity of presentation	30 %
10.6 Minimum performance standards			
Minimal/Passing grade: 5.			

Date | Signature of course coordinator | Signature of seminar coordinator

30.06.2021 | Prof. dr. Kristály Alexandru | Prof. dr. Kristály Alexandru

Date of approval | Signature of the head of doctoral school

07.07.2021 | Prof. dr. Gabriela Czibula