SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	**Babeş-Bolyai University of Cluj-Napoca**
1.2 Faculty	**Faculty of Mathematics and Computer Science**
1.3 Department	**Doctoral School in Mathematics and Computer Science**
1.4 Field of study	**Mathematics**
1.5 Study cycle	**Doctoral studies**
1.6 Study programme	**TRAINING PROGRAM BASED ON ADVANCED ACADEMIC STUDIES**

2. Information regarding the discipline

2.1 Name of the discipline	**Elliptic problems on manifolds / Probleme eliptice pe varietăți Teaching language: English**						
2.2 Course coordinator	**Kristály Alexandru**						
2.3 Seminar coordinator	**Kristály Alexandru**						
2.4. Year of study	**1**	2.5 Semester	**1**	2.6. Type of evaluation	**E**	2.7 Type of discipline	**Optional**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	3	Of which: 3.2 course	2	3.3 seminar/laboratory	1 sem
3.4 Total hours in the curriculum	36	Of which: 3.5 course	24	3.6 seminar/laboratory	12
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					60
Additional documentation (in libraries, on electronic platforms, field documentation)					70
Preparation for seminars/labs, homework, papers, portfolios and essays					40
Tutorship					23
Evaluations					21
Other activities:					

3.7 Total individual study hours	214
3.8 Total hours per semester	250
3.9 Number of ECTS credits	10

4. Prerequisites (if necessary)

4.1. curriculum	Riemannian and Finsler geometry	
4.2. competencies	Elliptic PDEs	

5. Conditions (if necessary)

5.1. for the course	Blackboard and videoprojector
5.2. for the seminar /lab activities	Laboratory with computers; high level programming language environment; Blackboard

6. Specific competencies acquired

Professional competencies	• Recognition of the main types of mathematical problems, selection of methods and of appropriate techniques for solving them • Explaining and interpreting mathematical models • Use of mathematical reasoning in demonstrations
Transversal competencies	• Applying the rules of rigorous and efficient work, manifesting responsible attitudes compared to the scientific field for the optimal and creative valorification of own potential in specific situations with respect to the principles and norms of ethics • Efficient use of information sources, training and development resources

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	Deepening some chapters in the theory of elliptic problems on smooth varieties and applying these results in the theory of partial differential equations
7.2 Specific objective of the discipline	● Deepening the study of Riemannian and Finslerian varieties ● Applications in the theory of partial differential equations

8. Content

8.1 Course	Teaching methods	Remarks
1. Smooth manifolds (Riemannian, Finsler)	Interactive talk, modelation, proof, exemplification	
2. Sobolev inequalities on manifolds	Interactive talk, modelation, proof, exemplification	
3. Principle of symmetric criticality	Interactive talk, modelation, proof, exemplification	
4. Yamabe problem	Interactive talk, modelation, proof, exemplification	
5. Emden-Flower-type problems on compact manifolds	Interactive talk, modelation, proof, exemplification	
6. Maxwell-Schrodinger-type problems on compact manifolds	Interactive talk, modelation, proof, exemplification	
7. Moser-Trudinger-type problems on noncompact manifolds	Interactive talk, modelation, proof, exemplification	
8. Hardy-type problems on noncompact manifolds: Rubik-cube method	Interactive talk, modelation, proof, exemplification	
9. Oscillatory problems on manifolds	Interactive talk, modelation, proof, exemplification	
10. Poisson-type problems on Finsler manifolds	Interactive talk, modelation, proof, exemplification	

11. Brezis-Vazquez-type problems on Finsler manifolds	Interactive talk, modelation, proof, exemplification	
12. Open problems, new perspectives	Interactive talk, modelation, proof, exemplification	
8.2 Seminar	Teaching methods	Remarks
1. Examples of Riemannian and Finsler manifolds (gravitational slope, Finsler-Poincare model)	Proof, conversation, cooperation, individual study	
2. Weighted Sobolev inequalities, Sobolev inequalities on manifolds	Proof, conversation, cooperation, individual study	
3. Principle of symmetric criticality: examples and counterexamples	Proof, conversation, cooperation, individual study	
4. Yamabe problem (geometric form) versus critical elliptic problem	Proof, conversation, cooperation, individual study	
5. Emden-Flower problems on compact manifolds versus elliptic problems in R^n	Proof, conversation, cooperation, individual study	
6. Properties of the energy functionals in the Maxwell-Schrodinger problem	Proof, conversation, cooperation, individual study	
7. Moser-Trudinger-type problems on manifolds	Proof, conversation, cooperation, individual study	
8. Rubik-type group decomposition: multiplicity results	Proof, conversation, cooperation, individual study	
9. Oscillatory problems on manifolds: examples and counterexamples	Proof, conversation, cooperation, individual study	
10. Poisson problem: rigidity results on manifolds	Proof, conversation, cooperation, individual study	
11. Brezis-Vazquez-type problems on Minkowski spaces	Proof, conversation, cooperation, individual study	
12. Open problems	Proof, conversation, cooperation, individual study	

Bibliography

1. E. Hebey, Nonlinear analysis on manifolds: Sobolev spaces and inequalities. Courant Lecture Notes in Mathematics, 5.New York University, Courant Institute of Mathematical Sciences, New
2. York; American Mathematical Society, Providence, RI, 1999.
3. A. Kristály, V. Radulescu, Cs. Varga, Variational Principles in Mathematical Physics, Geometry, and Economics, 2010.
4. C. Farkas, A. Kristály, C. Varga, Singular Poisson equations on Finsler-Hadamard manifolds. *Calc. Var. Partial Differential Equations* 54 (2015), no. 2, 1219–1241.
5. A. Kristály, A. Szakál, Interpolation between Brezis-Vázquez and Poincaré inequalities on nonnegatively curved spaces: sharpness and rigidities. *J. Differential Equations* 266 (2019), no. 10, 6621–6646.
6. A. Kristály, New geometric aspects of Moser-Trudinger inequalities on Riemannian manifolds: the non-compact case. *J. Funct. Anal.* 276 (2019), no. 8, 2359–2396.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The present discipline emphasizes the use of notions from mathematical analysis, Riemannian and Finsler geometry with applications in two important directions:

1. understanding/mastering some notions/techniques related to the study of smooth manifolds in terms of elliptic problems
2. mastering some notions and results from applied mathematics and their use in further research

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Evaluation of knowledge and earned competencies	2 tests (the first on the 7th week, the second on 13th week)	50%
10.5 Seminar	Seminar activity	Conversation, individual work	20 %
	Presentation of a research topic	Conversation and clarity of presentation	30 %
10.6 Minimum performance standards			
Minimal/Passing grade: 5.			

Date	Signature of course coordinator	Signature of seminar coordinator
30.06.2021	Prof. dr. Kristály Alexandru	Prof. dr. Kristály Alexandru

Date of approval	Signature of the head of doctoral school
07.07.2021	Prof. dr. Gabriela Czibula