SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	**Babeş-Bolyai University of Cluj-Napoca**
1.2 Faculty	**Faculty of Mathematics and Computer Science**
1.3 Department	**Doctoral School in Mathematics and Computer Science**
1.4 Field of study	**Mathematics**
1.5 Study cycle	**Doctoral studies**
1.6 Study programme / Qualification	**TRAINING PROGRAM BASED ON ADVANCED ACADEMIC STUDIES**

2. Information regarding the discipline

2.1 Name of the discipline	**Special Topics in Complex Analysis (Capitole speciale de analiză complexă)**						
2.2 Course coordinator	**Professor PhD Mirela KOHR**						
2.3 Seminar coordinator	**Professor PhD Mirela KOHR**						
2.4. Year of study	**1**	2.5 Semester	**1**	2.6. Type of evaluation	**E**	2.7 Type of discipline	**DS/Optional**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	**3**	Of which: 3.2 course	**2**	3.3 seminar	**1 sem**
3.4 Total hours in the curriculum	**36**	Of which: 3.5 course	**24**	3.6 seminar	**12**

Time allotment:	hours
Learning using manual, course support, bibliography, course notes	40
Additional documentation (in libraries, on electronic platforms, field documentation)	40
Preparation for seminars/labs, homework, papers, portfolios and essays	40
Tutorship	50
Evaluations	44
Other activities:	-

3.7 Total individual study hours	214
3.8 Total hours per semester	250
3.9 Number of ECTS credits	10

4. Prerequisites (if necessary)

4.1. curriculum	• Complex analysis; Real functions; Partial differential equations; Differential and integral calculus in $\mathbf{R}^n$
4.2. competencies	• The are useful logical thinking and mathematical notions and results from the above mentioned fields

5. Conditions (if necessary)

5.1. for the course	• Classroom with blackboard/video projector
5.2. for the seminar /lab activities	• Classroom with blackboard/video projector

6. Specific competencies acquired

Professional competencies	• Ability to understand and manipulate concepts, individual results and advanced mathematical theories. • Ability to use scientific language and to write scientific reports and papers.
Transversal competencies	• Ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics, Applied Mathematics, or in other fields that use concepts in complex analysis. • Ability for continuous self-perfecting and study.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	· Knowledge, understanding and use of main concepts and results of complex analysis. · Knowledge, understanding and use of methods of complex analysis in the study of special problems in pure and applied mathemnatics. · Ability to use and apply concepts and fundamental results of advanced mathematics in the study of specific problems of complex analysis.
7.2 Specific objective of the discipline	• Acquiring basic and advanced knowledge in complex analysis. • Knowledge, understanding and use of advanced topics in mathematics in the study of special problems in complex analysis. • Ability of the PhD student involvement in scientific research.

8. Content

8.1 **Course**	Teaching methods	Remarks
Part I		
1. Analytic branches. Index (winding number). General properties. The Cauchy integral formulas. Applications.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Cauchy's theorem related to zeros and poles of meromorphic functions. The argument principle. Applications.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

3. Rouché's theorem. Open mapping theorem and Hurwitz's theorem. Applications.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
4. The Fréchet space H(Ω). Families of holomorphic functions. Montel and Vitali's theorems. Extremal problems on compact subsets of H(Ω).	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Conformal mappings. The automorphisms of the unit disc and the upper half-plane. The automorphisms of the complex plane.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. The Riemann mapping theorem. Extension to the boundary.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. Univalent functions. General properties. The family S. The hyperbolic metric on the unit disc. Necessary and sufficient conditions for univalence on the unit disc.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. Harmonic mappings on open subsets of **C**. Subharmonic mappings. General results. Applications.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. Conformal mappings of annuli.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
Part II		
10. Introduction in the theory of functions of several complex variables. Holomorphic functions in $\mathbf{C}^n$. The generalized Cauchy-Riemann equations. Integral representation of holomorphic functions on the polyidsc. Sequences and series of holomorphic functions in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Sets of uniqueness for the holomorphic functions in $\mathbf{C}^n$. The Montel and Vitali theorems. Holomorphic mappings. Biholomorphic mappings in $\mathbf{C}^n$. Fatou-Bieberbach domains. Poincaré's theorem.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Cartan's uniqueness theorems. The automorphisms of the Euclidean unit ball and the unit polydisc in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.

2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.

3. G. Kohr, P.T. Mocanu, *Special Topics of Complex Analysis*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).

4. P. Hamburg, P.T. Mocanu, N. Negoescu, *Mathmatical Analysis* (*Complex Functions*), Editura Didactică şi Pedagogică, Bucureşti, 1982 (in Romanian).

5. C.A. Berenstein, R. Gay, *Complex Variables: An Introduction*, Springer-Verlag New York Inc., 1991.

6. Conway, J.B., *Functions of One Complex Variable,* vol. I, Graduate Texts in Mathematics, Springer

Verlag, New York, 1978 (Second Edition).

7. K. Güerlebeck, K. Habetha, W. Spröβig, *Holomorphic Functions in the Plane and n-Dimensional Space,* Birkhäuser, Basel-Boston-Berlin, 2008.

8. R.C. Gunning, *Introduction to Holomorphic Functions of Several Variables*, vol.I. *Function Theory*, Wadsworth & Brooks/Cole, Monterey, CA, 1990.

9. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.

10. R. Narasimhan, *Several Complex Variables*, The University of Chicago Press, Chicago, 1971.

11. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables* Springer-Verlag, New York, 1986.

12. W. Rudin, *Function Theory in the Unit Ball of* $\mathbf{C}^n$, Springer-Verlag, New York, 1980.

8.2 **Seminar**	Teaching methods	Remarks
Part I		
1. Applications of residues to the computation of some real integrals.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week
2. Applications of the argument principle and Rouché’s Theorem.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week
3. Examples of compact families of holomorphic functions. Extremal problems on compact subsets of H(Ω).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week
4. Sufficient conditions of univalence for holomorphic functions of one complex variable. Examples of univalent functions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week
5. The family S. Properties and examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
6. .Necessary and sufficient conditions of univalence for holomorphic functions on the unit disc.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
7. Applications of the Riemann mapping theorem. Conformal mappings of special simply connected domains in **C** (I).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week
8. Applications of the Riemann mapping theorem. Conformal mappings of special simply connected domains in **C** (II).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week

9. Harmonic mappings. Subharmonic mappings. Examples and applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	
Part II		
10. Applications of the Cauchy integral representations on the unit polydisc in $\mathbf{C}^n$. Applications of the maximum modulus theorem and the Schwarz Lemma for holomorphic functions of several complex variables.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week
11. Pluriharmonic functions and plurisubharmonic functions. Examples	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week
12. Sufficient conditions of univalence for holomorphic mappings on the unit ball in $\mathbf{C}^n$. Examples of locally biholomorphic mappings. Examples of univalent mappings. Examples of biholomorphic automorphisms of the n-dimensional complex space $\mathbf{C}^n$. Fatou-Bieberbach domains and Runge domains in $\mathbf{C}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to PhD students.	1 hour/week

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.
3. G. Kohr, P.T. Mocanu, *Special Topics of Complex Analysis*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).
4. P. Hamburg, P.T. Mocanu, N. Negoescu, *Mathmatical Analysis* (*Complex Functions*), Editura Didactică şi Pedagogică, Bucureşti, 1982 (in Romanian).
5. Bulboacă, T., Joshi, S.B., Goswami, P., *Complex Analysis. Theory and Applications*, de Gruyter, Berlin, Boston, 2019.
6. Conway, J.B., *Functions of One Complex Variable,* vol. I, Graduate Texts in Mathematics, Springer Verlag, New York, 1978 (Second Edition).
7. K. Güerlebeck, K. Habetha, W. Spröβig, *Holomorphic Functions in the Plane and n-Dimensional Space,* Birkhäuser, Basel-Boston-Berlin, 2008.
8. R.C. Gunning, *Introduction to Holomorphic Functions of Several Variables*, vol.I. *Function Theory*, Wadsworth & Brooks/Cole, Monterey, CA, 1990.
9. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.
10. R. Narasimhan, *Several Complex Variables*, The University of Chicago Press, Chicago, 1971.
11. R. Narasimhan, Y. Nievergelt, *Complex Analysis in One Variable*, Birkhäser, 2001.
12. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables* Springer-Verlag, New York, 1986.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The content of this discipline is in accordance with the curricula of the PhD programme of all important universities from our country and abroad, where the advanced mathematics plays an essential role. This discipline is useful in specifical PhD research activities, in preparing future researchers in pure and applied mathematics, and for those who use advanced mathematics in other areas of science.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Knowledge of concepts and basic results	Written exam.	60%
	Ability to justify by proofs theoretical results		
10.5 Seminar/lab activities	Ability to apply concepts and results acquired in the course at mathematical modeling and analysis of current research problems in pure and applied mathematics.	Evaluation the activity of PhD students during the semester.	40%
10.6 Minimum performance standards			
➢ Ability to apply concepts and results acquired in the course to study special problems in complex analysis.			

Date	Signature of course coordinator	Signature of seminar coordinator
30.06.2021	Professor PhD Mirela KOHR	Professor PhD Mirela KOHR

Date of approval in the Doctoral School Council

07.07.2021

Signature of the Director of the Doctoral School in Mathematics and Computer Science

Professor PhD Gabriela Czibula