SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	Babeş-Bolyai University Cluj-Napoca
1.2 Faculty	Faculty of Mathematics and Computer Science
1.3 Department	Department of Mathematics
1.4 Field of study	Mathematics
1.5 Study cycle	Bachelor
1.6 Study programme / Qualification	Mathematics and Computer Science

2. Information regarding the discipline

2.1 Name of the discipline (en) (ro)			Functional Analysis (Analiză funcțională)				
2.2 Course coordinator			Conf. dr. Brigitte Breckner				
2.3 Seminar coordinator			Conf. dr. Brigitte Breckner				
2.4. Year of study	3	2.5 Semester	5	2.6. Type of evaluation	E	2.7 Type of discipline	O
2.8 Code of the discipline		MLE0004					

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	4	Of which: 3.2 course	2	3.3 seminar/ laboratory	2
3.4 Total hours in the curriculum	56	Of which: 3.5 course	28	3.6 seminar/ laboratory	28
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					45
Additional documentation (in libraries, on electronic platforms, field documentation)					10
Preparation for seminars/labs, homework, papers, portfolios and essays					20
Tutorship					15
Evaluations					4
Other activities:					-

3.7 Total individual study hours	94
3.8 Total hours per semester	150

3.9 Number of ECTS credits	6

4. Prerequisites (if necessary)

4.1. curriculum	• linear algebra; general topology; mathematical analysis
4.2. competencies	• abstract and logical thinking

5. Conditions (if necessary)

5.1. for the course	
5.2. for the seminar /lab activities	

6. Specific competencies acquired

Professional competencies	C1.1 To identify the appropriate notions, to describe the speficic topic and to use an appropriate language. C1.3 To apply correctly basic methods and principles in order to solve mathematical problems.
Transversal competencies	CT1 To apply efficient and rigorous working rules, to manifest responsible attitudes towards the scientific and didactic fields, respecting the professional and ethical principles.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	Presentation of the basic concepts and fundamental results of Functional Analysis
7.2 Specific objective of the discipline	To become familiar with the abstract thinking and the problematization specific to Functional Analysis

8. Content

8.1 Course	Teaching methods	Remarks
1. Basics of linear spaces (definition, linear subspaces, linear hull, basis, linear functionals, sublinear functionals)	Lecture with mathematical proofs, problematization, discussion	

2. Basics of linear spaces (the extension lemma of Helly, the Hahn–Banach theorem for real linear spaces)	Lecture with mathematical proofs, problematization, discussion	
3. Basics of linear spaces (the relation between complex linear functionals and real linear functionals, the Hahn–Banach theorem for complex linear spaces, the Bohnenblust-Sobczyk-Suhomlinov theorem)	Lecture with mathematical proofs, problematization, discussion	
4. Linear topological spaces (definition, basic properties)	Lecture with mathematical proofs, problematization, discussion	
5. Locally convex spaces (preliminaries: the topology generated by a family of functions; the topology of locally convex spaces, seminormed spaces)	Lecture with mathematical proofs, problematization, discussion	
6. Normed spaces (equivalence of two norms, the equivalence of norms on finite dimensional linear spaces, bounded sequences)	Lecture with mathematical proofs, problematization, discussion	
7. Normed spaces (the Riesz lemma, characterizations of finite dimensional normed spaces, summable families of vectors in a normed space)	Lecture with mathematical proofs, problematization, discussion	
8. Inner product spaces (definition of the inner product, properties of the inner product, characterizations of the norms generated by inner products, the definition of inner product spaces and of Hilbert spaces)	Lecture with mathematical proofs, problematization, discussion	
9. Inner product spaces (the continuity of the inner product, orthogonality, properties of the orthogonal complement, best approximation points, chracterizations of best approximation points)	Lecture with mathematical proofs, problematization, discussion	
10. Inner product spaces (the orthogonal decomposition of an inner product space, the orthogonal decomposition of a Hilbert space, orthonormal families, orthonormal bases, characterizations of orthonormal bases in inner product spaces and in Hilbert spaces, Fourier coefficients and Fourier expansions)	Lecture with mathematical proofs, problematization, discussion	

11. Continuous linear operators (characterizations of the continuity of linear operators between locally convex spaces, characterizations of the continuity of linear operators between normed spaces, the normed space of linear continuous operators between normed spaces)	Lecture with mathematical proofs, problematization, discussion	
12. Continuous linear operators between normed spaces (pointwise convergence of sequences of continuous linear operators, principle of condensation of singularities, the uniform boundedness principle)	Lecture with mathematical proofs, problematization, discussion	
13. Continuous linear operators between normed spaces (the Banach-Steinhaus theorem, the completeness of the normed space of continuous linear operators, the convergence of quadrature formulas)	Lecture with mathematical proofs, problematization, discussion	
14. Continuous linear operators between normed spaces (the open mapping theorem, the bounded inverse theorem, the closed graph theorem)	Lecture with mathematical proofs, problematization, discussion	

Bibliography

Bibliografie

1. BRECKNER W. W.: Analiză funcțională, Presa Universitară Clujeană, Cluj-Napoca, 2009.

2. BREZIS H.: Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.

3. CONWAY J. B.: A Course in Functional Analysis. Second Edition, Springer-Verlag, New-York –Berlin – Heidelberg, 1999.

4. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage, B. G. Teubner, Stuttgart, 1992.

5. KANTOROVICI L.V., AKILOV G. P.: Analiză funcţională. Editura Ştiinţifică şi Enciclopedică, Bucureşti, 1986.

6. MUNTEAN I.: Analiză funcţională, Universitatea "Babeş-Bolyai", Cluj-Napoca, 1993.

7. PRECUPANU T.: Analiză funcţională pe spaţii liniare normate, Editura Universităţii "Alexandru Ioan Cuza", Iaşi, 2005.

8.2 Seminar / laboratory	Teaching methods	Remarks
1. Basics of linear spaces	Problematization, discussion, team work	
2. Basics of linear spaces	Problematization, discussion, team work	
3. Review on topological notions and results used in functional analysis	Problematization, discussion, team work	

4. Linear topological spaces. The inequalities of Young, Hölder and Minkowski	Problematization, discussion, team work	
5. Locally convex spaces. Normed spaces (the $\|\cdot\|_p$ norm on the linear space $\mathbf{K}^m$, the supremum norm on the linear space B(T, **K**), the norm on the linear space l_p)	Problematization, discussion, team work	
6. Banach spaces (properties). Examples of Banach spaces: finite dimensional normed spaces; the spaces B(T, **K**), CB(T, **K**), C(T, **K**)	Problematization, discussion, team work	
7. Examples of Banach spaces: the spaces C^1[a,b], l_∞, c, c_0, l_p	Problematization, discussion, team work	
8. Inner product spaces	Problematization, discussion, team work	
9. The Chebyshev approximation problem. Best approximation points. Inner product spaces	Problematization, discussion, team work	
10. The orthogonal decomposition of Hilbert spaces	Problematization, discussion, team work	
11. Continuous linear functionals on normed spaces (characterization of the continuity of linear functionals). Computing the norm of continuous linear operators between normed spaces	Problematization, discussion, team work	
12. Continuous linear functionals on normed spaces (the extension theorems of Hahn). The uniform and the pointwise convergence of continuous linear operators between normed spaces	Problematization, discussion, team work	
13. The general form of continuous linear functionals on Hilbert spaces	Problematization, discussion, team work	
14. The general form of continuous linear functionals on the spaces l_p	Problematization, discussion, team work	

Bibliography

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2. HEUSER H.: Funktionalanalysis. Theorie und Anwendung, 3. Auflage. B. G. Teubner, Stuttgart, 1992.

3. POPA E.: Culegere de probleme de analiză funcţională, Editura Didactică şi Pedagogică, Bucureşti, 1981.

4. WERNER D.: Funktionalanalysis. Vierte, überarbeitete Auflage, Springer-Verlag, Berlin - Heidelberg - New York, 2002 .

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

Functional analysis is one of the most important branches of mathematics, having applications in various domains (numerical analysis, approximation theory, optimization, PDEs, probability theory, mathematical and theoretical physics). This discipline both provides a theoretical background for such applications.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Knowledge of concepts and basic results	Midterm written test	40%
	Ability to perform proofs	Final written exam	50%
10.5 Seminar/lab activities	Ability to apply concepts and results acquired in the lecture	Own contributions to the exercise classes	10%
	There are valid the official rules of the faculty concerning the attendance of students to teaching activities.		
10.6 Minimum performance standards			
• Ability for showing that a certain functional is a norm/seminorm; • Ability for proving the linearity and continuity of an operator or a functional, and for computing its norm; • Knowledge of basic notions and results.			

Date	Signature of course coordinator	Signature of seminar coordinator
25.04.2021	Conf. univ. dr. Brigitte E. Breckner	Conf. univ. dr. Brigitte E. Breckner

Date of approval	Signature of the head of department
28.04.2021	Prof. univ. dr. Agratini Octavian