SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	Babeş-Bolyai University, Cluj-Napoca
1.2 Faculty	Mathematics and Computer Science
1.3 Department	Mathematics
1.4 Field of study	Mathematics
1.5 Study cycle	Licence
1.6 Study programme / Qualification	Mathematics and Computer Science

2. Information regarding the discipline

2.1 Name of the discipline (en) (ro)	Convex Analysis Analiză convexă						
2.2 Course coordinator	Trif Tiberiu-Vasile						
2.3 Seminar coordinator	Trif Tiberiu-Vasile						
2.4 Year of study	2	2.5 Semester	3	2.6. Type of evaluation	VP	2.7 Type of discipline	optional
2.8 Code of the discipline	MLR0072						

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	4	Of which: 3.2 course	2	3.3 seminar/laboratory	2
3.4 Total hours in the curriculum	56	Of which: 3.5 course	28	3.6 seminar/laboratory	28

Time allotment:	hours
Learning using manual, course support, bibliography, course notes	25
Additional documentation (in libraries, on electronic platforms, field documentation)	14
Preparation for seminars/labs, homework, papers, portfolios and essays	20
Tutorship	15
Evaluation	20
Other activities:	

3.7 Total individual study hours	94
3.8 Total hours per semester	150
3.9 Number of ECTS credits	6

4. Prerequisites (if necessary)

4.1 curriculum	• Calculus 1 (Calculus in R) • Calculus 2 (Calculus in R^n)
4.2 competencies	• Logical thinking abilities, problematisation

5. Conditions (if necessary)

5.1 For the course	• Classroom with adequate infrastrusture
5.2 For the seminar/lab activities	• Classroom with adequate infrastrusture

6. Specific competencies aquired

Professional competencies	• C1.4 Recognizing the main classes /types of mathematical problems and selecting the appropriate methods and techniques for their solving • C2.1 Identifying the basic notions used to describe some processes and phenomena
Transversal competencies	• CT1 Application of efficient and rigorous working rules, manifest responsible attitudes towards the scientific and didactic fields, respecting the professional and ethical principles

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Getting to know some basic notions and results concerning convex functions • Getting to know some basic notions and results from convex analysis
7.2 Specific objectives of the discipline	• Presenting various characterization theorems of convex functions; based on them the student will be able to establish wether a given function is convex or not • Getting to know some specific fundamental properties of convex functions • Aplying the general inequalities specific to convex functions in proving other concrete inequalities • Solving some concrete convex optimization problems

8. Content

8.1 Course	Teaching methods	Remarks
1. Convex functions of a real variable: the modern definition of convexity, characterizations of convex real valued functions of a real variable and their regularity properties (existence of side derivatives, continuity, Lipschitz continuity).	Lecture, discussion, proof, problematisation	[4], pp. 93 – 102 [9], pp. 3 – 7
2. Convex functions of a real variable: characterization by means of support line, the Hermite-Hadamard inequality, characterizations of convexity by means of the first order side derivatives and by means of the second derivative, connection with harmonic functions.	Lecture, discussion, proof, problematisation	[4], pp. 102 – 103 pp. 107 – 108 pp. 136 – 139 [9], pp. 11 – 12
3. Means and their inequalities: weighted quasiarithmetic means and their comparison, weighted Hölder means and their comparison, Rado-Popoviciu type inequalities.	Lecture, discussion, proof, problematisation	[4], pp. 115 – 122
4. Generalizations of convex functions: Jensen-convex functions, log-convex functions and multiplicatively-convex functions.	Lecture, discussion, proof, problematisation	[4], pp. 124 – 132 [9], pp. 218 – 223
5. Convex functions on vector spaces: definition, characterizations, examples (affine functions, sublinear functions, indicator functions, quadratic forms, support functions).	Lecture, discussion, proof, problematisation	[4], pp. 72 – 79
6+7. Continuity of convex functions on normed spaces:	Lecture, discussion,	[4], pp. 24 – 29

semicontinuous functions, characterization of semicontinuity by means of sequences, the lower and the upper limit of a function at a point and their relationship with semicontinuity, the connection between continuity, Lipschitz-continuity and local boundedness in the case of convex functions defined on normed spaces, continuity of convex functions on finite dimensional normed spaces, continuity vs. Lower semicontinuity for convex functions defined on Banach spaces.	proof, problematisation	pp. 147 – 153 [7], pp. 119 – 123 [9], pp. 91 – 94
8. Directional differentiability and algebraic subdifferentiability of convex functions defined on vector spaces: side directional derivatives and their properties, algebraic subgradients and their characterization, algebraic subdifferentiability of convex functions.	Lecture, discussion, proof, problematisation	[4], pp. 154 – 159
9. Subdifferentiability of convex functions on normed spaces: the definition of subgradients and of the subdifferential, subdifferentiability vs. algebraic subdifferentiability vs. semicontinuity, the relative interior of a set, subdifferentiability of convex functions at relatively interior points to the effective domain.	Lecture, discussion, proof, problematisation	[4], pp. 159 – 163
10. Differentiable convex functions of several variables: characterization of convexity for differentiable and twice differentiable functions of *n* real variables.	Lecture, discussion, proof, problematisation	[4], pp. 163 – 174 [7], pp. 135 – 145 [9], pp. 97 – 103
11. Convex optimization problems: feasible points, optimal solutions, Lagrange's function, necessary and sufficient optimality conditions.	Lecture, discussion, proof, problematisation	[1], pp. 43 – 45 [4], pp. 193 – 197 [7], pp. 145 – 152 [9], pp. 171 – 176
12+13. The Fenchel conjugate and the Fenchel biconjugate: the Fenchel-Young inequality, the Fenchel duality theorem, closed convex functions and their characterizations, calculation of conjugates and of biconjugates of certain concrete functions.	Lecture, discussion, proof, problematisation	[1], pp. 49 – 63 pp. 76 – 87 [4], pp. 198 – 208
14. Checking the homeworks, dicussing the midterm test papers, establishing the final grades.	Discussion	

Bibliography

1. BORWEIN J. M., LEWIS A. S.: Convex Analysis and Nonlinear Optimization. Theory and Examples. CMS Books in Mathematics, Springer-Verlag, 2000.
2. BRECKNER B. E., POPOVICI N.: Convexity and Optimization. An Introduction. Editura Fundaţiei pentru Studii Europene, Cluj-Napoca, 2006.
3. BRECKNER W. W.: Introducere in teoria problemelor de optimizare convexa cu restrictii. Editura Dacia, Cluj, 1974.
4. BRECKNER W. W., TRIF T.: Convex Functions and Related Functional Equations. Selected Topics. Cluj University Press, Cluj-Napoca, 2008.
5. HIRIART-URRUTY J. B., LEMARECHAL C.: Convex Analysis and Minimization Algorithms. Springer-Verlag, 1993.
6. KUCZMA M.: An Introduction to the Theory of Functional Equations and Inequalities. Panstwowe Wydawnictwo Naukowe, Warszawa-Krakow-Katowice, 1985.
7. NICULESCU C. P., PERSSON L.-E.: Convex Functions and Their Applications. A Contemporary Approach. Springer-Verlag, New York, 2006.
8. PRECUPANU T.: Spatii liniare topologice si elemente de analiza convexa. Editura Academiei Romane, Bucuresti, 1992.
9. ROBERTS A. W., VARBERG D. E.: Convex Functions. Academic Press, 1973.

10. ROCKAFELLAR R. T.: Convex Analysis. Princeton University Press, 1970.		
8.2 Seminar / laboratory	Teaching methods	Remarks
1+2. Study of the convexity for certain concrete functions, applications of Jensen's inequality in proving other inequalities, the AM-GM inequality as a corollary of convexity.	Discussion, problematisation	[2], pp. 104 – 107 [4], pp. 189 – 191
3+4. Applications of the Hermite-Hadamard inequality (inequalities between the geometric mean, the logarithmic mean and the arithmetic mean, Stirling's formula), characterizatio of convex functions by means of the Hermite-Hadamard inequality.	Discussion, problematisation	[2], pp. 137 – 139 [3], pp. 73 – 74
5+6. Ky Fan type inequalities, the Hardy-Littlewood-Pólya majorization theorem and its applications (Popoviciu's and Petrović's inequalities).	Discussion, problematisation	[2], pp. 121 – 122 pp. 109 – 115
7+8. Log-convexity of the gamma function, the Bohr-Mollerup theorem, multiplicative convexity of the gamma function.	Discussion, problematisation	[2], pp. 126 – 129 [3], pp. 68 – 71
9+10. Jensen-convexity vs convexity on normed spaces, Bernstein-Doetsch type theorems.	Discussion, problematisation	[4], pp. 211 – 216
11+12. Calculation of the subgradients for certain concrete functions on normed spaces, study of the convexity of certian functions of *n* real variables.	Discussion, problematisation	[2], pp. 172 – 176
13+14. Solving some convex optimization problems.	Discussion, problematisation	[1], pp. 43 – 45 [2], p. 197

Bibliography

1. BORWEIN J. M., LEWIS A. S.: Convex Analysis and Nonlinear Optimization. Theory and Examples. CMS Books in Mathematics, Springer-Verlag, 2000.
2. BRECKNER W. W., TRIF T.: Convex Functions and Related Functional Equations. Selected Topics. Cluj University Press, Cluj-Napoca, 2008.
3. NICULESCU C. P., PERSSON L.-E.: Convex Functions and Their Applications. A Contemporary Approach. Springer-Verlag, New York, 2006.
4. ROBERTS A. W., VARBERG D. E.: Convex Functions. Academic Press, 1973.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the programme

- Convex functions are useful tools, helping the future math teacher in proving inequalities that occur in elementary mathematics
- Convex optimization knowledge will be useful to the future graduate who will work in a software company

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in grade
10.4 Course	- knowledge of notions and basic results - applying the basic theoretical results to solving concrete problems	Three test papers during the semester	75%
10.5 Seminar/lab	- solving concrete problems with the help of theoretical results from the course	Solving some problems during the semester	25%
10.6 Minimum performance standards			
Active participation in course and seminar activities			

Date	Signature of course coordinator	Signature of seminar coordinator
28.4.2021	....	....

Date of approval	Signature of the head of departament
....	…...........................