SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	**Babeş-Bolyai University Cluj-Napoca**
1.2 Faculty	**Faculty of Mathematics and Computer Science**
1.3 Department	**Department of Mathematics**
1.4 Field of study	**Mathematics**
1.5 Study cycle	**Master**
1.6 Study programme / Qualification	**Advanced Mathematics**

2. Information regarding the discipline

2.1 Name of the discipline	**Geometric function theory in several complex variables (Teoria geometrică a funcțiilor de mai multe variabile complexe)**						
2.2 Course coordinator	**Professor PhD Mirela KOHR**						
2.3 Seminar coordinator	**Professor PhD Mirela KOHR**						
2.4. Year of study	**2**	2.5 Semester	**4**	2.6. Type of evaluation	**E**	2.7 Type of discipline	**DS/Optional**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	**3**	Of which: 3.2 course	**2**	3.3 seminar/laboratory	**1 sem**
3.4 Total hours in the curriculum	**36**	Of which: 3.5 course	**24**	3.6 seminar/laboratory	**12**
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					45
Additional documentation (in libraries, on electronic platforms, field documentation)					45
Preparation for seminars/labs, homework, papers, portfolios and essays					45
Tutorship					34
Evaluations					20
Other activities:					-

3.7 Total individual study hours	189
3.8 Total hours per semester	225
3.9 Number of ECTS credits	9

4. Prerequisites (if necessary)

4.1. curriculum	• Complex analysis; Complex analysis in one and higher dimensions; Real analysis; Partial differential equations
4.2. competencies	• The are useful logical thinking and mathematical notions and results from the above mentioned fields

5. Conditions (if necessary)

5.1. for the course	• Classroom with blackboard/video projector
5.2. for the seminar /lab activities	• Classroom with blackboard/video projector

6. Specific competencies acquired

Professional competencies	• Ability to understand and manipulate concepts, individual results and advanced mathematical theories. • Ability to use scientific language and to write scientific reports and papers.
Transversal competencies	• Ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics, Applied Mathematics, or in other fields that use concepts of complex analysis. • Ability for continuous self-perfecting and study.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Knowledge, understanding and use of main concepts and results of geometric function theory of several complex variables. • Knowledge, understanding and use of methods of complex analysis in one or higher dimensions in the study of special problems in pure and applied mathematics. • Ability to use and apply concepts and fundamental results of advanced mathematics in the study of specific problems of function theory in $\mathbf{C}^n$.
7.2 Specific objective of the discipline	• Acquiring basic and advanced knowledge in geometric function theory in $\mathbf{C}^n$. • Understanding of main concepts and results in the theory of holomorphic mappings on the unit ball in $\mathbf{C}^n$. • Knowledge, understanding and use of advanced topics in mathematics in the study of special problems in several complex variables. • Ability student involvement in scientific research.

8. Content

8.1 Course	Teaching methods	Remarks
1. The Carathéodory family M of holomorphic mappings in several complex variables. Growth and distortion results, coefficient bounds. Compactness of the family M.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Starlike mappings on the unit ball in $\mathbf{C}^n$. Necessary and sufficient conditions for starlikeness. Growth and distortion results and coefficient bounds.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Convex mappings on the unit ball in $\mathbf{C}^n$. Necessary and sufficient conditions for convexity on the Euclidean unit ball and the unit polydisc in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

4. Growth, distortion and coefficient bounds for convex mappings on the unit ball in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Loewner chains and transition mappings (evolution families) in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. Loewner chains, Herglotz vector fields and the generalized Loewner differential equation in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. Kernel convergence and biholomorphic mappings on the unit ball in $\mathbf{C}^n$. Applications in the theory of Loewner chains.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. The solutions of the generalized Loewner differential equation in $\mathbf{C}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The family $S^{\mathbf{0}}(B^{\mathbf{n}})$ of biholomorphic mappings with parametric representation on the unit ball in $\mathbf{C}^n$. Characterizations in terms of Loewner chains. Compactness of the family $S^{\mathbf{0}}(B^{\mathbf{n}})$. The Runge property. Open problems.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. Extreme points and support points associated with the family $S^{\mathbf{0}}(B^{\mathbf{n}})$. Approximation properties by automorphisms of the space $\mathbf{C}^n$. Open problems and conjectures.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Univalence criteria on the unit ball in $\mathbf{C}^n$ via the theory of Loewner chains. Parametric representation and asymptotic starlikeness in higher dimensions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Extension operators that preserve analytic and geometric properties (starlikeness, convexity, Loewner chains, parametric representation). Open problems, conjectures, and research directions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.

2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.

3. G. Kohr, *Geometric Function Theory in Several Complex Variables*, Lecture notes, 2020.

4. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex variables*, Mathematische Annalen, **347** (2010), 411-435.

5. I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* $\mathbf{C}^n$, Mathematische Annalen, **359** (2014), 61-99.

6. S. Gong, *Convex and Starlike Mappings in Several Complex Variables*, Kluwer Acad. Publ., Dordrecht, 1998.

7. P. Duren, *Univalent Functions*, Springer-Verlag, New York, 1983.

8. M. Elin, S. Reich. D. Shoikhet, *Numerical Range of Holomorphic Mappings and Applications*, Birkhäuser, Springer, Cham, 2019.

9. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea

Publishing, Providence, Rhode Island, 2001.

10. Ch. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.

11. T. Poreda, *On generalized differential equations in Banach spaces*, Dissertationes Mathematicae, **310** (1991), 1-50.

12. M. Range, *Holomorphic Functions and Integral Representations in Several Complex Variables* Springer-Verlag, New York, 1986.

13. W. Rudin, *Function Theory in the Unit Ball of* $\mathbf{C}^n$, Springer-Verlag, New York, 1980.

8.2 Seminar	Teaching methods	Remarks
1. Examples of mappings in the Carathéodory family *M*. Special subclasses of *M*. Distortion and coefficient bounds.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
2. Sufficient conditions of starlikeness on the unit ball in $\mathbf{C}^n$. Examples of starlike mappings.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
3. Sufficient conditions of convexity on the unit ball in $\mathbf{C}^n$. Examples of convex mappings.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
4. Starlike mappings of order α on the Euclidean unit ball in $\mathbf{C}^n$, $0\leq \alpha<1$. Growth and coefficient bounds. Examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
5. Loewner chains and transition mappings (evolution families) in several complex variables. Examples.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
6. Loewner chains and the associated Loewer PDE in higher dimensions. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
7. The analytical characterizations of starlikeness and spirallikeness of type α on the unit ball in $\mathbf{C}^n$ in terms of Loewner chains.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
8. Variation of Loewner chains in $\mathbf{C}^n$. Applications to extremal problems for univalent mappings with parametric representation on the unit ball in $\mathbf{C}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	

9. Bounded mappings with parametric representation on the unit ball in $\mathbf{C}^n$. Growth and coefficient bounds. Applications to extremal problems.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
10. Univalence criteria on the unit ball in $\mathbf{C}^n$ via the theory of Loewner chains.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
11. Kernel convergence and Loewner chains in $\mathbf{C}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
12. Extension operators that preserve analytic and geometric properties. Open problems, conjectures, and research directions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	

Bibliography

1. I. Graham, G. Kohr, *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. G. Kohr, *Basic Topics in Holomorphic Functions of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 2003.
3. G. Kohr, *Geometric Function Theory in Several Complex Variables*, Seminar notes, 2020.
4. F. Bracci, I. Graham, H. Hamada, G. Kohr, *Variation of Loewner chains, extreme and support points in the class* S^0 *in higher dimensions*, Constructive Approximation, **43** (2016), 231-251.
5. P. Duren, I. Graham, H. Hamada, G. Kohr, *Solutions for the generalized Loewner differential equation in several complex variables*, Mathematische Annalen, **347** (2010), 411-435.
6. I. Graham, H. Hamada, G. Kohr, M. Kohr, *Extremal properties associated with univalent subordination chains in* $\mathbf{C}^n$, Mathematische Annalen, **359** (2014), 61-99.
7. G. Kohr, P. Liczberski, *Univalent Mappings of Several Complex Variables*, Cluj University Press, Cluj-Napoca, 1998.
8. P. Curt, *Special Chapters in Geometric Function Theory of Several Complex Variables*, Editura Albastră, Cluj-Napoca, 2001 (in Romanian).
9. S. Gong, *Convex and Starlike Mappings in Several Complex Variables*, Kluwer Acad. Publ., Dordrecht, 1998.
10. S. Gong, The Bieberbach Conjecture, Amer. Math. Soc. Intern. Press, Providence, R.I., 1999.
11. S.G. Krantz, *Function Theory of Several Complex Variables*, Reprint of the 1992 Edition, AMS Chelsea Publishing, Providence, Rhode Island, 2001.
12. Ch. Pommerenke, *Univalent Functions*, Vandenhoeck & Ruprecht, Göttingen, 1975.
13. F. Bracci (Ed.), *Geometric Function Theory in Higher Dimension*, Springer INdAM Series, vol. **26** (2017), Springer International Publishing AG, Cham, Switzerland.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The content of this discipline is in accordance with the curricula of the most important universities in Romania and abroad, where the advanced mathematics plays an essential role. This discipline is useful in specifical PhD research activities and in preparing future researchers in pure and applied mathematics.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Knowledge of concepts and basic results	Written exam.	60%
	Ability to justify by proofs theoretical results		
10.5 Seminar/lab activities	Ability to apply concepts and results acquired in the course in the study of advanced topics of geometric function theory in $\mathbf{C}^n$ and related area.	Evaluation of reports and homework during the semester, and active participation in the seminar activity. A midterm written test.	15% 25%
	There are valid the official rules of the faculty concerning the attendance of students to teaching activities.		
10.6 Minimum performance standards			
➢ The final grade should be at least 5 (from a scale of 1 to 10).			

Date | Signature of course coordinator | Signature of seminar coordinator

23.04.2021 | Professor PhD Mirela KOHR | Professor PhD Mirela KOHR

Date of approval | Signature of the head of department

Professor PhD Octavian AGRATINI