SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	**Babeş-Bolyai University Cluj-Napoca**
1.2 Faculty	**Faculty of Mathematics and Computer Science**
1.3 Department	**Department of Mathematics**
1.4 Field of study	**Mathematics**
1.5 Study cycle	**Master**
1.6 Study programme / Qualification	**Advanced Mathematics**

2. Information regarding the discipline

2.1 Name of the discipline	**Potential theory and elliptic boundary value problems (Teoria potențialului şi probleme eliptice pe frontieră)**						
2.2 Course coordinator	**Professor PhD Mirela KOHR**						
2.3 Seminar coordinator	**Professor PhD Mirela KOHR**						
2.4. Year of study	**2**	2.5 Semester	**4**	2.6. Type of evaluation	**E**	2.7 Type of discipline	**DS/Optional**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	**3**	Of which: 3.2 course	**2**	3.3 seminar/laboratory	**1 sem**
3.4 Total hours in the curriculum	**36**	Of which: 3.5 course	**24**	3.6 seminar/laboratory	**12**
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					45
Additional documentation (in libraries, on electronic platforms, field documentation)					45
Preparation for seminars/labs, homework, papers, portfolios and essays					45
Tutorship					34
Evaluations					20
Other activities:					-

3.7 Total individual study hours	189
3.8 Total hours per semester	225
3.9 Number of ECTS credits	9

4. Prerequisites (if necessary)

4.1. curriculum	• Mathematical Methods in Fluid Mechanics; Nonlinear Partial Differential Equations; Nonlinear Applied Analysis
4.2. competencies	• There are useful logical thinking and mathematical notions and results from the above mentioned fields

5. Conditions (if necessary)

5.1. for the course	Classroom with blackboard/video projector
5.2. for the seminar /lab activities	Classroom with blackboard/video projector

6. Specific competencies acquired

Professional competencies	• Ability to understand and manipulate concepts, individual results and advanced mathematical theories. • Ability to model and analyze from the mathematical point of view real processes from other sciences, fluid mechanics and porous media, economics, and engineering. • Ability to use scientific language and to write scientific reports and papers.
Transversal competencies	• Ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics, Applied Mathematics, or in other fields that use mathematical models. • Ability for continuous self-perfecting and study.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Knowledge, understanding and use of main concepts and results of potential theory in the study of linear elliptic boundary value problems. • Knowledge, understanding and combine advances mathematical methods, potential theory, the fixed point theory and topological degree theory in the study of nonlinear elliptic boundary value problems in fluid mechanics, porous media, and other sciences.
7.2 Specific objective of the discipline	• Acquiring basic and advanced knowledge in potential theory. • Knowledge, understanding and use of advanced topics in mathematics in the study of elliptic boundary value problems. • Ability student involvement in scientific research.

8. Content

8.1 **Course**	Teaching methods	Remarks
1. Boundary value problems for the Laplace operator. Classical solutions and layer potential representations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Basic theory of pseudo-differential operators on $\mathbf{R}^n$: The class S^m. The definition of pseudo-differential operator of order m. Continuity of pseudo-differential operators in Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Elliptic pseudo-differential operators on $\mathbf{R}^n$. Parametrix and fundamental solution.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
4. Strongly elliptic operators and elliptic systems in the sense of Agmon-Douglis-Nirenberg on $\mathbf{R}^n$. The Stokes and Brinkman systems with constant/variable coefficients. Fredholm operators.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Potential theory for the Stokes and Brinkman	Lectures, modeling, didactical	

systems with constant/variable coefficients on Lipschitz domains in $\mathbf{R}^n$ (I): Related layer potential operators. Boundedness and compactness results in the scale of L^p and Sobolev spaces.	demonstration, conversation. Presentation of alternative explanations.	
6. Potential theory for the Stokes and Brinkman systems with constant/variable coefficients on Lipschitz domains in $\mathbf{R}^n$ (II): Fredholm and invertibility results for related layer potential operators in L^p and Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. Linear elliptic boundary value problems on Lipschitz domains in $\mathbf{R}^n$. Variational and potential approach. Well-posedness results in L^p and Sobolev spaces (I).	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. Linear elliptic boundary value problems on Lipschitz domains in $\mathbf{R}^n$. Variational and potential approach. Well-posedness results in L^p and Sobolev spaces (II).	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations	
9. Boundary value problems for linear elliptic systems with nonlinear boundary conditions on Lipschitz domains in $\mathbf{R}^n$. Existence and uniqueness based on the results in the linear PDE theory and topological degree theory.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. Semilinear elliptic boundary value problems on bounded Lipschitz domains with arbitrary data in L^p and Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Boundary value problems for nonlinear elliptic systems on Lipschitz domains in $\mathbf{R}^n$, with nonlinear boundary conditions. Existence and uniquness based on the results in the linear PDE theory and fixed point theorems.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Transmission problems for the Navier-Stokes and Darcy-Forchheimer-Brinkman systems with variable coefficients on Lipschitz domains in $\mathbf{R}^n$ (n=2,3). Applications to porous media flow problems.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.

2. Kohr, M., *Probleme Moderne în Mecanica Fluidelor Vâscoase*, Presa Universitară Clujeană, Cluj-Napoca, 2 vols. 2000.

3. Kohr, M., Lanza de Cristoforis, M., Wendland, W.L., *Nonlinear Neumann-transmission problems for Stokes and Brinkman equations on Euclidean Lipschitz domains*, Potential Analysis, **38** (2013), 1123-1171.

4. Kohr, M., Wendland, W.L., *Layer potentials and Poisson problems for the nonsmooth coefficient Brinkman system in Sobolev and Besov spaces*, Journal of Mathematical Fluid Mechanics, **20** (2018), 1921-1965.

5. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 2008.

6. McLean, W., *Strongly Elliptic Systems and Boundary Integral Equations*, Cambridge University Press, Cambridge, UK, 2000.

7. Sayas, F-J., Brown, T.S., Hassell, M.E., *Variational Techniques for Elliptic Partial Differential*

Equations: Theoretical Tools and Advanced Applications, CRC Press, Boca Raton, FL, 2019.

8. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, 344 (2012): viii+241 pp.
9. Mitrea, I., Mitrea, M., *Multi-Layer Potentials and Boundary Problems for Higher-Order Elliptic Systems in Lipschitz Domains*, Lecture Notes in Mathematics, 2063. Springer, Heidelberg, 2013. x+424 pp.
10. Galdi, G.P., *An Introduction to the Mathematical Theory of the Navier–Stokes Equations*. Second Edition. Springer, Berlin, 2011.
11. Agranovich, M.S., *Sobolev Spaces, Their Generalizations, and Elliptic Problems in Smooth and Lipschitz Domains,* Springer, Heidelberg, 2015.
12. Wloka, J. T. , Rowley, B., Lawruk, B., *Boundary Value Problems for Elliptic Systems*, Cambridge University Press, Cambridge, 1995.

8.2 **Seminar**	Teaching methods	Remarks
1. Sobolev spaces. Trace theorems and Green's functions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
2. Boundary value problems for the Laplace operator. The variational solution for the Dirichlet and Neumann problems.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
3. Basic theory of pseudo-differential operators on $\mathbf{R}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
4. The construction of a parametrix for the Brinkman system in $\mathbf{R}^n$. Properties and related results.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
5. Fredholm operators.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
6. Fredholm and invertibility properties of layer potential operators for the Stokes and Brinkman systems in L^p and Sobolev spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
7. Well-posedness results for linear elliptic boundary value problems on Lipschitz domains in $\mathbf{R}^n$, with data in L^p and Sobolev spaces (I).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
8. Well-posedness results for linear elliptic boundary value problems on Lipschitz domains in $\mathbf{R}^n$, with data in L^p and Sobolev spaces (II).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	

9. Existence and uniqueness for boundary value problems for linear elliptic systems with nonlinear boundary conditions on Lipschitz domains in $\mathbf{R}^n$, and data in L^p and Sobolev spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
10. Semilinear elliptic boundary value problems on bounded Lipschitz domains with arbitrary data in L^p and Sobolev spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
11. Boundary value problems for nonlinear elliptic systems on Lipschitz domains in $\mathbf{R}^n$, with nonlinear boundary conditions. Existence results in various function spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	
12. Transmission problems for the Navier-Stokes and Darcy-Forchheimer-Brinkman systems with variable coefficients in Lipschitz domains in $\mathbf{R}^n$ (n=2,3).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	

Bibliography

1. Kohr., M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.

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4. Kohr, M., Lanza de Cristoforis, M., Wendland, W.L., *Nonlinear Neumann-transmission problems for Stokes and Brinkman equations on Euclidean Lipschitz domains*, Potential Analysis, **38** (2013), 1123-1171.

5. Kohr, M., Lanza de Cristoforis, M., Mikhailov S.E., Wendland, W.L., *Integral potential method for a transmission problem with Lipschitz interface in* $\mathbf{R}^3$ *for the Stokes and Darcy-Forchheimer-Brinkman PDE systems*, Zeitschrift für Angewandte Mathematik und Physik, **67**:116, no. 5, 1-30, 2016.

6. Kohr, M., Wendland, W.L., *Variational approach for the Stokes and Navier-Stokes systems with nonsmooth coefficients in Lipschitz domains on compact Riemannian manifolds*, Calculus of Variations and Partial Differential Equations, **57**:165 (2018), 1-41.

7. Kohr, M., Wendland, W.L., *Boundary value problems for the Brinkman system with* L^∞ *coefficients in Lipschitz domains on compact Riemannian manifolds. A variational approach*, Journal de Mathématiques Pures et Appliquées, **131** (2019), 17-63.

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10. Agranovich, M.S., *Sobolev Spaces, Their Generalizations, and Elliptic Problems in Smooth and Lipschitz Domains,* Springer, Heidelberg, 2015.

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12. Grisvard, P., *Elliptic Problems in Nonsmooth Domains*, Pitman Advanced Pub. Program, Boston, 1985.

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The content of this discipline is in accordance with the curricula of the most important universities in Romania and abroad, where the advanced mathematics plays an essential role. This discipline is useful in specifical PhD research activities, in preparing future researchers in pure and applied mathematics, and for those who use mathematical models and advanced methods of study in other areas.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Knowledge of concepts and basic results.	Written exam.	60%
	Ability to justify by proofs theoretical results.		
10.5 Seminar/lab activities	Ability to apply concepts and results acquired in the course in the analysis of elliptic boundary value problems.	Evaluation of reports and homework during the semester, and active participation in the seminar activity.	15%
		A midterm written test.	25%
10.6 Minimum performance standards			
➢ The final grade should be at least 5 (from a scale of 1 to 10).			

Date	Signature of course coordinator	Signature of seminar coordinator
29.04.2020	Professor PhD Mirela KOHR	Professor PhD Mirela KOHR

Date of approval	Signature of the head of department
	Professor PhD Octavian AGRATINI