**SYLLABUS**

**1. Information regarding the programme**

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| 1.1 Higher education institution | **Babeş Bolyai University of Cluj-Napoca** |
| 1.2 Faculty | **Faculty of Mathematics and Computer Science** |
| 1.3 Department | **Department of Mathematics** |
| 1.4 Field of study | **Mathematics** |
| 1.5 Study cycle | **Master** |
| 1.6 Study programme / Qualification | **Master of Advanced Mathematics** |

**2. Information regarding the discipline**

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|---|---|---|---|---|---|---|---|
| 2.1 Name of the discipline | **Nonlinear partial differential equations** | | | | | | |
| 2.2 Course coordinator | **Prof.PhD. Radu Precup** | | | | | | |
| 2.3 Seminar coordinator | **Prof.PhD. Radu Precup** | | | | | | |
| 2.4. Year of study | **1** | 2.5 Semester | **2** | 2.6. Type of evaluation | **E** | 2.7 Type of discipline | **Compulsory** |

**3. Total estimated time** (hours/semester of didactic activities)

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|---|---|---|---|---|---|
| 3.1 Hours per week | 3 | Of which: 3.2 course | 2 | 3.3 seminar/laboratory | 1 sem |
| 3.4 Total hours in the curriculum | 42 | Of which: 3.5 course | 28 | 3.6 seminar/laboratory | 14 |
| Time allotment: | | | | | hours |
| Learning using manual, course support, bibliography, course notes | | | | | 41 |
| Additional documentation (in libraries, on electronic platforms, field documentation) | | | | | 43 |
| Preparation for seminars/labs, homework, papers, portfolios and essays | | | | | 40 |
| Tutorship | | | | | 18 |
| Evaluations | | | | | 16 |
| Other activities: ..... | | | | | - |

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| 3.7 Total individual study hours | 158 |
| 3.8 Total hours per semester | 200 |
| 3.9 Number of ECTS credits | 8 |

**4. Prerequisites** (if necessary)

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| 4.1. curriculum | • |
| 4.2. competencies | • |

**5. Conditions** (if necessary)

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| 5.1. for the course | • Partial differential equations; Functional analysis |
| 5.2. for the seminar /lab activities | • Partial differential equations; Functional analysis |

**6. Specific competencies acquired**

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| **Professional competencies** | • Use of the theory of linear partial differential equations and of the basic principles of functional analysis for the investigation of nonlinear boundary value problems<br>• Ability to apply abstract principles of nonlinear analysis to elliptic boundary value problems |
| **Transversal competencies** | • Understand the role of partial differential equations in mathematical modelling of real phenomena |

**7. Objectives of the discipline** (outcome of the acquired competencies)

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| 7.1 General objective of the discipline | • Acquire knowledge about some main techniques of investigation of nonlinear boundary value problems |
| 7.2 Specific objective of the discipline | • Rewrite boundary value problems as operator equations<br>• Apply general fixed point principles to the operator equations associated to boundary value problems<br>• Apply basic variational methods to elliptic equations<br>• Compare different methods by analysing the sufficient conditions and the conclusions of the theorems |

**8. Content**

| 8.1 Course | Teaching methods | Remarks |
|---|---|---|
| 1. Summary basic notions and results from the theory of linear partial differential equations | Exposure: description, explanation, dialogue, examples | |
| 2. Sobolev spaces: definition, properties | Exposure: description, explanation, dialogue, examples | |
| 3. Operator formulation of semilinear elliptic boundary value problems | Exposure: description, explanation, dialogue, examples | |
| 4. The Nemytskii superposition operator. Properties; Elliptic problems with Lipschitz nonlinearities | Exposure: description, explanation, examples, proof, dialogue | |
| 5. Elliptic problems with nonlinearities having a growth at most linear. Application of Schauder's fixed point theorem | Exposure: description, explanation, examples, proof | |
| 6. The Leray-Schauder continuation principle. Method of „a priori" bounds | Exposure: description, explanation, examples, proof, dialogue | |
| 7. Comparison of different existence results by analysing their hypotheses and conclusions | Exposure: explanation, examples, dialogue | |
| 8. Supersolutions, subsolutions, monotone iterations | Exposure: description, explanation, examples | |
| 9. Variational methods. The Frechet derivative. | Exposure: description, | |

| | | |
|---|---|---|
| | explanation, examples, proofs | |
| 10. Variational structure of elliptic boundary value problems | Exposure: description, explanation, examples | |
| 11. Ekeland's variational principle | Exposure: description, explanation, examples, discussion of case studies | |
| 12. The Palais-Smale compactness condition | Exposure: description, explanation, examples | |
| 13. Schechter's critical point theorem in a ball | Exposure: description, explanation, proofs, examples | |
| 14. Applications to elliptic problems | Exposure: description, examples, dialogue | |

Bibliography


1. R. Precup, Linear and Semilinear Partial Differential Equations, De Gruyter, Berlin, 2012.
2. R. Precup, Methods in Nonlinear Integral Equations, Kluwer, 2002.
3. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, New York, 2011
4. M. Struwe, Variational Methods, Springer, 1990.


| 8.2 Seminar | Teaching methods | Remarks |
|---|---|---|
| 1. Exemplification of some basic notions and results from the theory of linear partial differential equations | Exercise, dialogue, team work | |
| 2. Sobolev spaces | Exercise, dialogue, team work | |
| 3. Operator formulation of boundary value problems. Case of ordinary differential equations; The Nemytskii superposition operator. Examples | Exercise, explanation, dialogue, team work | |
| 4. Examples of elliptic problems with Lipschitz nonlinearities | Exercise, explanation, dialogue, team work | |
| 5. Examples of elliptic problems with nonlinearities having a growth at most linear | Exercise, explanation, dialogue, team work | |
| 6. Applications of the homotopy principle to two-point boundary value problems | Exercise, explanation, dialogue, team work | |
| 7. Comparison of different existence results in case of two-point boundary value problems | Exercise, explanation, dialogue, team work | |
| 8. Applications of super and subsolutions method | Exercise, explanation, dialogue, team work | |
| 9. Examples of cones | Exercise, explanation, dialogue | |
| 10. The Frechet derivative. Examples | Exercise, explanation, dialogue, team work | |
| 11. Ekeland's variational principle. Consequences and applications | Exercise, explanation, dialogue, team work | |
| 12. The Palais-Smale condition. Case of one-dimensional problems | Exercise, explanation, dialogue | |
| 13. Schechter's critical point theorem. Applications | Exercise, explanation, dialogue, team work | |
| 14. Case of systems without a variational structure | Exercise, explanation, dialogue, team work | |

Bibliography
5. R. Precup, Linear and Semilinear Partial Differential Equations, De Gruyter, Berlin, 2012.
6. R. Precup, Methods in Nonlinear Integral Equations, Kluwer, 2002.
7. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, New York, 2011.
8. M. Struwe, Variational Methods, Springer, 1990.

**9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program**

- The contents of the course correspond to current research themes in nonlinear boundary value problems and make connexion to mathematical models from physics, biology, medicine etc.

**10. Evaluation**

| Type of activity | 10.1 Evaluation criteria | 10.2 Evaluation methods | 10.3 Share in the grade (%) |
|---|---|---|---|
| 10.4 Course | | Written exam<br>Continuous observations | 60%<br>10% |
| 10.5 Seminar/lab activities | | -Practical examination<br>-continuous observations | 20%<br>10% |
| 10.6 Minimum performance standards | | | |
| ➢ At least grade 5 (from a scale of 1 to 10) at both written exam and seminar practical examination | | | |

| Date | Signature of course coordinator | Signature of seminar coordinator |
|---|---|---|
| April 29, 2020 | Prof.PhD. Radu Precup | Prof.PhD. Radu Precup |
| Date of approval | | Signature of the head of department |
| May 4, 2020 | | Prof.PhD. Octavian Agratini |