SYLLABUS

${\bf 1.}\ Information\ regarding\ the\ programme$

| 1.1 Higher education | Universitatea Babeș-Bolyai Cluj-Napoca | |
|-----------------------|--|--|
| institution | | |
| 1.2 Faculty | Matematică și Informatică | |
| 1.3 Department | Matematică | |
| 1.4 Field of study | Matematică | |
| 1.5 Study cycle | Master | |
| 1.6 Study programme / | Advanced Mathematics | |
| Qualification | | |

2. Information regarding the discipline

| 2.1 Name of the discipline Homological Algebra | | | | | | | |
|--|-----|----------|---|---------------------------|---|-------------|----|
| 2.2 Course coor | din | ator | | Prof. Simion Breaz | 1 | | |
| 2.3 Seminar coordinator | | | | Prof. Simion Breaz | | | |
| 2.4. Year of | 2 | 2.5 | 3 | 2.6. Type of | E | 2.7 Type of | DF |
| study | | Semester | | evaluation | | discipline | |

3. Total estimated time (hours/semester of didactic activities)

| 3.1 Hours per week | 3 | Of which: 3.2 course | 2 | 3.3 | 1 |
|---|----|----------------------|----|--------------------|-------|
| | | | | seminar/laboratory | |
| 3.4 Total hours in the curriculum | 42 | Of which: 3.5 course | 28 | 3.6 | 14 |
| | | | | seminar/laboratory | |
| Time allotment: | | | | | hours |
| Learning using manual, course support, bibliography, course notes | | | | | 46 |
| Additional documentation (in libraries, on electronic platforms, field documentation) | | | | | 36 |
| Preparation for seminars/labs, homework, papers, portfolios and essays | | | | | 32 |
| Tutorship | | | | | 14 |
| Evaluations | | | | 20 | |
| Other activities: | | | | | |
| 2777 (1: 1: 1: 1 1 1 1 | | 1.40 | | | |

| 3.7 Total individual study hours | 148 |
|----------------------------------|-----|
| 3.8 Total hours per semester | 200 |
| 3.9 Number of ECTS credits | 8 |

4. Prerequisites (if necessary)

| 4.1. curriculum | |
|-------------------|--|
| 4.2. competencies | |

5. Conditions (if necessary)

| 5.1. for the course | |
|---------------------------|--|
| 5.2. for the seminar /lab | |
| activities | |

6. Specific competencies acquired

| Professional competencies ' | Knowledge, understanding and use of main concepts and results in Homological Algebra (complexes, homology and cohomology, derived functors) Ability to use fundamental theoretical concepts and in various fields of mathematics fields of mathematics (Algebra, Topology, Banach Spaces, Fixed Point Theory) |
|-----------------------------|--|
| Transversal competencies | Ability to inform themselves, to work independently or in a team; Ability to approach complex problems and to use information from various specific fields; Ability to identify and use advanced techniques and methods in order to realize a specific research. |

7. Objectives of the discipline (outcome of the acquired competencies)

| 7.1 General objective of the | Knowledge, understanding and use of main concepts and results in |
|-------------------------------|---|
| discipline | Homological Algebra |
| | |
| | Ability to use concepts and fundamental results in some specific fields of |
| | mathematics (module theory, topological spaces, Banach spaces) |
| 7.2 Specific objective of the | Understanding the basic concepts about categories, complexes, resolutions, |
| discipline | sheaves. |
| | Ability to use specific derived functors (Ext, Tor, Pext) in concrete situations. |

8. Content

| 8. Content | | |
|-------------------------------------|----------------------|---------|
| 8.1 Course | Teaching methods | Remarks |
| 1. Preliminaries | Lectures, didactical | |
| | demonstration, | |
| | conversation. | |
| 2. Modules | Lectures, didactical | |
| | demonstration, | |
| | conversation. | |
| | | |
| 3. Categories | Lectures, didactical | |
| | demonstration, | |
| | conversation. | |
| | | |
| 4. Limits and colimits | Lectures, didactical | |
| | demonstration, | |
| | conversation. | |
| | | |
| 5. Functors | Lectures, didactical | |
| | demonstration, | |
| | conversation. | |
| | | |
| 6. Injective and projective modules | Lectures, didactical | |
| | demonstration, | |
| | conversation. | |
| | | |

| 7. Flat modules | Lectures, didactical |
|----------------------|----------------------|
| , | demonstration, |
| | conversation. |
| | Conversation. |
| 8. Complexes | Lectures, didactical |
| | demonstration, |
| | conversation. |
| | |
| 9. Homology functors | Lectures, didactical |
| | demonstration, |
| | conversation. |
| 10. Derived functors | Lectures, didactical |
| | demonstration, |
| | conversation. |
| | |
| 11. Ext | Lectures, didactical |
| | demonstration, |
| | conversation. |
| 12. Tor | Lectures, didactical |
| | demonstration, |
| | conversation. |
| | |
| 13. Sheaves | Lectures, didactical |
| | demonstration, |
| | conversation. |
| 14. Sheaf cohomology | Lectures, didactical |
| | demonstration, |
| | conversation. |
| | |
| Bibliography | |

- I. Moerdijk: Notes on Homological Algebra, course notes, <u>www.math.ru.nl/topology/Notes%20on%20Homological%20Algebra.pdf</u>
 J.J. Rotman: An Introduction to Homological Algebra, Springer, 2009

| 8.2 Seminar / laboratory | Teaching methods | Remarks |
|--------------------------------|--------------------|---------|
| 1. The fundamental group | problematization, | |
| | exercises, problem | |
| | solving, | |
| 2. Modules | problematization, | |
| | exercises, problem | |
| | solving, | |
| | | |
| 3. Example of Categories | problematization, | |
| | exercises, problem | |
| | solving, | |
| | | |
| 4. Categories of Banach spaces | problematization, | |
| | exercises, problem | |

| | solving, |
|--|---|
| 5. The additive category of Banach spaces | problematization, exercises, problem solving, |
| 6. The category of Abelian groups | problematization, exercises, problem solving, |
| 7. Flat modules | problematization, exercises, problem solving, |
| 8. Directed limits | problematization, exercises, problem solving, |
| 9. Inverse limits | problematization, exercises, problem solving, |
| 10. Functors | problematization, exercises, problem solving, |
| 11. Ext and Tor | problematization, exercises, problem solving, |
| 12. Ext and Tor for abelian groups | problematization, exercises, problem solving, |
| 13. Relative homological algebra | problematization, exercises, problem solving, |
| 14. Projective, injective and flat Banach spaces | problematization, exercises, problem solving, |

Bibliography

- 1. S. Breaz, G. Calugareanu, G. Modoi, D. Valcan: Exercices in Abelian Group Theory, Kluwer 2003.
- 2. J. Cigler, V. Losert, P. Michor: Banach Modules and Functors on Cateories of Banach Specaes, Marcel Dekker, 1979.
- 3. A. Hatcher: Algebraic Topology, Cambridge University Press, 2001, http://www.math.cornell.edu/~hatcher/AT/AT.pdf
- 4. C. Schochet: A Pext primer: Pure extensions and lim¹ for infinite abelian groups, NYJM Monographs, 2003, http://nyjm.albany.edu/m/2003/1v.pdf

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The content is in accordance with the curricula of many important universities where pure mathematics plays important places in their research.

This discipline is useful since it realizes connections between various mathematical domains, and it is well known that the methods of homological algebra were used during the time to solve important problems in mathematics.

The methods and tools presented here are often used in specifical PhD research activities.

10. Evaluation

| Type of activity | 10.1 Evaluation criteria | 10.2 Evaluation methods | 10.3 Share in the | |
|------------------------------------|-----------------------------|--------------------------|-------------------|--|
| | | | grade (%) | |
| 10.4 Course | Concepts and basic results | Final exam | 50% | |
| | Standard examples | | | |
| 10.5 Seminar/lab activities | Ability to use the concepts | Final exam and a midterm | 25%+25% | |
| | in order to solve problems | test. | | |
| | | | | |
| 10.6 Minimum performance standards | | | | |
| At least grade 5 from 10. | | | | |

| Date | Signature of course coordinator | Signature of seminar coordinator | |
|------------------|-------------------------------------|----------------------------------|--|
| 30.04.2019 | Prof. Simion Breaz | Prof. Simion Breaz | |
| Date of approval | Signature of the head of department | | |
| | Prof. Octavian Agratini | | |