SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	**Babeş-Bolyai University Cluj-Napoca**
1.2 Faculty	**Faculty of Mathematics and Computer Science**
1.3 Department	**Department of Mathematics**
1.4 Field of study	**Mathematics**
1.5 Study cycle	**Master**
1.6 Study programme / Qualification	**Advanced Mathematics**

2. Information regarding the discipline

2.1 Name of the discipline	**Mathematical methods in fluid mechanics (Metode matematice în mecanica fluidelor)**						
2.2 Course coordinator	**Professor PhD Mirela KOHR**						
2.3 Seminar coordinator	**Professor PhD Mirela KOHR**						
2.4. Year of study	**1**	2.5 Semester	**1**	2.6. Type of evaluation	**C**	2.7 Type of discipline	**DF/Compulsory**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	**3**	Of which: 3.2 course	**2**	3.3 seminar/laboratory	**1 sem**
3.4 Total hours in the curriculum	**42**	Of which: 3.5 course	**28**	3.6 seminar/laboratory	**14**
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					38
Additional documentation (in libraries, on electronic platforms, field documentation)					38
Preparation for seminars/labs, homework, papers, portfolios and essays					38
Tutorship					10
Evaluations					9
Other activities:					-

3.7 Total individual study hours	133
3.8 Total hours per semester	175
3.9 Number of ECTS credits	7

4. Prerequisites (if necessary)

4.1. curriculum	• Theoretical Mechanics; Partial Differential Equations; Real Functions; Numerical Analysis
4.2. competencies	• There are useful logical thinking and mathematical notions and results from the above mentioned fields

5. Conditions (if necessary)

5.1. for the course	Classroom with blackboard/video projector
5.2. for the seminar /lab activities	Classroom with blackboard/video projector

6. Specific competencies acquired

Professional competencies	• Ability to understand and manipulate concepts, individual results and advanced mathematical theories. • Ability to model and analyze from the mathematical point of view real processes from other sciences, economics, and engineering. • Ability to use scientific language and to write scientific reports and papers.
Transversal competencies	• Ability to inform themselves, to work independently or in a team in order to carry out studies and to solve complex problems. • Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics, Applied Mathematics, or in other fields that use mathematical models. • Ability for continuous self-perfecting and study.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Knowledge, understanding and use of main concepts and results of fluid mechanics. • Ability to use and apply concepts and fundamental results of the theory of partial differential equations in the study of specific problems of fluid mechanics. • Knowledge, understanding and use advanced mathematical methods in the study of special boundary value problems in fluid mechanics.
7.2 Specific objective of the discipline	• Acquiring basic and advanced knowledge in fluid mechanics. • Ability to apply and use mathematical models to describe and analyze problems concerning viscous incompressible fluid flows. • Understanding of main concepts and results in the mathematical theory of viscous incompressible flows at low Reynolds numbers. • Knowledge, understanding and use of advanced topics in mathematics in the study of special boundary value problems in fluid mechanics. • Ability student involvement in scientific research.

8. Content

8.1 **Course**	Teaching methods	Remarks
1. Introduction in the theory of Sobolev spaces (I): The fundamental spaces of the theory of distributions. Distributions.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Introduction in the theory of Sobolev spaces (II): Sobolev spaces on $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. The dual of a Sobolev space. The Sobolev continuous embedding theorem and the Rellich - Kondrachov compact embedding theorem.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
3. Kinematics of fluids: fluid, configuration, motion. Velocity and acceleration fields. Spatial description of the motion of	Lectures, modeling, didactical demonstration, conversation.	

a fluid.	Presentation of alternative explanations.	
4. Fluid Dynamics: Principle of mass conservation. The continuity equation.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Fluid Dynamics: The Cauchy stress tensor. The Cauchy equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. The constitutive equation of ideal fluid. The Euler equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. The mathematical model of viscous Newtonian fluid: The constitutive equation and the Navier-Stokes equations. Special forms of the Navier-Stokes equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. Uniqueness results of the Dirichlet and Neumann problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$. Variational approach for the weak solution of the Stokes problem in a bounded Lipschitz domain with Dirichlet boundary condition.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The method of fundamental solutions in fluid mechanics: The Oseen-Burgers tensor and the fundamental pressure vector for the Stokes system in $\mathbf{R}^n$ (n=2, 3).	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. The hydrodynamic layer potential theory (I): Bounded and compact operators, Fredholm operators on Banach spaces. The Fredholm alternative.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. The hydrodynamic layer potential theory (II): Single- and double layer potentials for the Stokes system. Boundedness, compactness, and Fredholm properties in Sobolev spaces	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Applications of the hydrodynamic layer potential theory (I): Well-posedness results for boundary value problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$, with data in Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
13. Applications of the layer potential theory for the Stokes system (II): Well-posedness results in weighted Sobolev spaces for the exterior Dirichlet problem for the Stokes system in $\mathbf{R}^n$.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
14. Transmission problems and layer potentials for the Stokes system with L^∞ coefficients in Lipschitz domains: Variational and layer potential approach. Well-posedness results in Sobolev spaces. Applications to viscous incompressible flows in the presence of interfaces. Numerical results.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.

2. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols.

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3. Kohr, M., *Special Topics of Mechanics*, Cluj University Press, Cluj-Napoca, 2005 (in Romanian).
4. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000.
5. Pozrikidis, C., *Introduction to Theoretical and Computational Fluid Dynamics*, Oxford University Press, Oxford, 2011.
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7. Galdi, G.P., *An Introduction to the Mathematical Theory of the Navier–Stokes Equations*. Second Edition, Springer, Berlin, 2011.
8. Adams, R. Fournier, J., *Sobolev Spaces*, 2nd edition, Pure and Applied Mathematics, vol. 140, Elsevier/Academic Press, Amsterdam, 2003.
9. Agranovich, M.S., *Sobolev Spaces, Their Generalizations, and Elliptic Problems in Smooth and Lipschitz Domains,* Springer, Heidelberg, 2015.
10. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 2008.
11. McLean, W., *Strongly Elliptic Systems and Boundary Integral Equations*, Cambridge University Press, Cambridge, UK, 2000.
12. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, 344 (2012): viii+241 pp.

8.2 **Seminar**	Teaching methods	Remarks
1. Introduction in the theory of Sobolev spaces (I): The fundamental spaces of the theory of distributions. Distributions.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
2. Introduction in the theory of Sobolev spaces (II): Sobolev spaces over $\mathbf{R}^n$. Sobolev spaces on Lipschitz domains in $\mathbf{R}^n$ and on Lipschitz boundaries. Trace theorems.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
3. Differential operators. Material derivatives. The Euler theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
4. Second order Cartesian tensors in $\mathbf{R}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
5. Properties of the Cauchy stress tensor: Cauchy's fundamental theorem, and the symmetry property.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
6. The mathematical model of incompressible fluid.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week

7. The Killing theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
8. Variational approach for the weak solution of the Stokes problem in a bounded Lipschitz domain with homogeneous Dirichlet boundary condition.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
9. The exterior Dirichlet problem for the Stokes system in Lipschitz domains in $\mathbf{R}^n$ (n=2,3). Uniqueness results and applications. The Stokes Paradox.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
10. The method of fundamental solutions in fluid mechanics (I): Direct layer potential representations of the velocity field of Stokes flow.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
11. The method of fundamental solutions in fluid mechanics (II): The Stokes flow past a solid (fluid) sphere.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
12. Well-posedness results in Sobolev spaces for boundary value problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$ ($n\geq 2$).	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
13. Well-posedness results for the exterior Dirichlet problem for the Stokes system in $\mathbf{R}^n$, with data in Sobolev spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
14. Well-posedness results for transmission problems for the Stokes system. Applications to viscous incompressible fluid flows in the presence of interfaces. Numerical results based on the boundary element method.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week

Bibliography

1. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004.

2. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian).

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7. Precup, R., *Lectures on Partial Differential Equations*, Cluj University Press, Cluj-Napoca, 2004 (in Romanian).
8. Hsiao, G.C., Wendland W.L., *Boundary Integral Equations*, Springer-Verlag, Heidelberg, 2008.
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9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The content of this discipline is in accordance with the curricula of the most important universities in Romania and abroad, where the advanced mathematics plays an essential role. This discipline is useful in specifical PhD research activities, in preparing future researchers in pure and applied mathematics, and for those who use mathematical models and advanced methods of study in other areas.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Knowledge of concepts and basic results	Colloquium	60%
	Ability to justify by proofs theoretical results		
10.5 Seminar/lab activities	Ability to apply concepts and results acquired in the course in mathematical modeling and analysis of problems in fluid mechanics	Evaluation of reports and homework during the semester, and active participation in the seminar activity. A midterm written test.	15% 25%
	There are valid the official rules of the faculty concerning the attendance of students to teaching activities		
10.6 Minimum performance standards			
➢ The final grade should be at least 5 (from a scale of 1 to 10).			

Date | Signature of course coordinator | Signature of seminar coordinator

5.05.2019 | Professor PhD Mirela KOHR | Professor PhD Mirela KOHR

Date of approval | Signature of the head of department

Professor PhD Octavian AGRATINI