#### **SYLLABUS**

# 1. Information regarding the programme

1.1 Higher education	Babeş Bolyai University of Cluj-Napoca
institution	
1.2 Faculty	Faculty of Mathematics and Computer Science
1.3 Department	Department of Mathematics
1.4 Field of study	Mathematics
1.5 Study cycle	Master
1.6 Study programme /	Master of Advanced Mathematics
Qualification	

## 2. Information regarding the discipline

2.1 Name of the discipline Nonlinear partial differential equations							
2.2 Course coordinator Prof.PhD. Radu Precup							
2.3 Seminar coordinator				Prof.PhD. Radu Precup			
2.4. Year of	1	2.5	2 2.6. Type of E 2.7 Type of Compulsory				
study Semester evaluation discipline							

## **3. Total estimated time** (hours/semester of didactic activities)

3.1 Hours per week	3	Of which: 3.2 course	2	3.3	1 sem
				seminar/laboratory	
3.4 Total hours in the curriculum	42	Of which: 3.5 course	28	3.6	14
				seminar/laboratory	
Time allotment:					
Learning using manual, course support, bibliography, course notes					
Additional documentation (in libraries, on electronic platforms, field documentation)					
Preparation for seminars/labs, homework, papers, portfolios and essays					20
Tutorship					8
Evaluations					
Other activities:					-
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3.7 Total individual study hours	90
3.8 Total hours per semester	132
3.9 Number of ECTS credits	8

# **4. Prerequisites** (if necessary)

4.1. curriculum	•
4.2. competencies	•

## **5. Conditions** (if necessary)

5.1. for the course	<ul> <li>Partial differential equations; Functional analysis</li> </ul>
5.2. for the seminar /lab	Partial differential equations; Functional analysis
activities	

6. Specific competencies acquired

	<u> </u>		omposencies acquired
Professional	competencies	•	Use of the theory of linear partial differential equations and of the basic principles of functional analysis for the investigation of nonlinear boundary value problems  Ability to apply abstract principles of nonlinear analysis to elliptic boundary value problems
Transversal	competencies	•	Understand the role of partial differential equations in mathematical modelling of real phenomena

# **7. Objectives of the discipline** (outcome of the acquired competencies)

7.1 General objective of the discipline	Acquire knowledge about some main techniques of investigation of nonlinear boundary value problems
7.2 Specific objective of the discipline	<ul> <li>Rewrite boundary value problems as operator equations</li> <li>Apply general fixed point principles to the operator equations associated to boundary value problems</li> <li>Apply basic variational methods to elliptic equations</li> <li>Compare different methods by analysing the sufficient conditions and the conclusions of the theorems</li> </ul>

## 8. Content

8.1 Course	Teaching methods	Remarks
1. Summary basic notions and results from the	Exposure: description,	
theory of linear partial differential equations	explanation, dialogue,	
	examples	
2. Sobolev spaces: definition, properties	Exposure: description,	
	explanation, dialogue,	
	examples	
3. Operator formulation of semilinear elliptic	Exposure: description,	
boundary value problems	explanation, dialogue,	
	examples	
4. The Nemytskii superposition operator.	Exposure: description,	
Properties; Elliptic problems with Lipschitz	explanation, examples,	
nonlinearities	proof, dialogue	
5. Elliptic problems with nonlinearities having a	Exposure: description,	
growth at most linear. Application of	explanation, examples,	
Schauder's fixed point theorem	proof	
6. The Leray-Schauder continuation principle.	Exposure: description,	
Method of "a priori" bounds	explanation, examples,	
7. 0	proof, dialogue	
7. Comparison of different existence results by	Exposure: explanation,	
analysing their hypotheses and conclusions	examples, dialogue	
8. Supersolutions, subsolutions, monotone	Exposure: description,	
iterations	explanation, examples	
9. Variational methods. The Frechet derivative.	Exposure: description,	

	explanation, examples,
	proofs
10. Variational structure of elliptic boundary value	Exposure: description,
problems	explanation, examples
11. Ekeland's variational principle	Exposure: description,
	explanation, examples,
	discussion of case studies
12. The Palais-Smale compactness condition	Exposure: description,
	explanation, examples
13. Schechter's critical point theorem in a ball	Exposure: description,
	explanation, proofs,
	examples
14. Applications to elliptic problems	Exposure: description,
	examples, dialogue

### Bibliography

- 1. R. Precup, Linear and Semilinear Partial Differential Equations, De Gruyter, Berlin, 2012.
- 2. R. Precup, Methods in Nonlinear Integral Equations, Kluwer, 2002.
- 3. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, New York, 2011
- 4. M. Struwe, Variational Methods, Springer, 1990.

8.2 Seminar	To a alice a mostly a de	Damadra
	Teaching methods	Remarks
1. Exemplification of some basic notions and	Exercise, dialogue, team	
results from the theory of linear partial	work	
differential equations		
2. Sobolev spaces	Exercise, dialogue, team	
	work	
3. Operator formulation of boundary value	Exercise, explanation,	
problems. Case of ordinary differential	dialogue, team work	
equations; The Nemytskii superposition		
operator. Examples		
4. Examples of elliptic problems with Lipschitz	Exercise, explanation,	
nonlinearities	dialogue, team work	
5. Examples of elliptic problems with	Exercise, explanation,	
nonlinearities having a growth at most linear	dialogue, team work	
6. Applications of the homotopy principle to two-	Exercise, explanation,	
point boundary value problems	dialogue, team work	
7. Comparison of different existence results in	Exercise, explanation,	
case of two-point boundary value problems	dialogue, team work	
8. Applications of super and subsolutions method	Exercise, explanation,	
	dialogue, team work	
9. Examples of cones	Exercise, explanation,	
	dialogue	
10. The Frechet derivative. Examples	Exercise, explanation,	
•	dialogue, team work	
11. Ekeland's variational principle. Consequences	Exercise, explanation,	
and applications	dialogue, team work	
12. The Palais-Smale condition. Case of one-	Exercise, explanation,	
dimensional problems	dialogue	
13. Schechter's critical point theorem.	Exercise, explanation,	
Applications	dialogue, team work	
14. Case of systems without a variational structure	Exercise, explanation,	
	dialogue, team work	
	dialogue, team work	

#### Bibliography

- 5. R. Precup, Linear and Semilinear Partial Differential Equations, De Gruyter, Berlin, 2012.
- 6. R. Precup, Methods in Nonlinear Integral Equations, Kluwer, 2002.
- 7. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, New York, 2011.
- 8. M. Struwe, Variational Methods, Springer, 1990.

# 9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

• The contents of the course correspond to current research themes in nonlinear boundary value problems and make connexion to mathematical models from physics, biology, medicine etc.

#### 10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the			
			grade (%)			
10.4 Course		Written exam	60%			
		Continuous observations	10%			
10.5 Seminar/lab activities		-Practical examination	20%			
		-continuous observations	10%			
10.6 Minimum performance standards						
At least grade 5 (from a scale of 1 to 10) at both written exam and seminar practical examination						

Date Signature of course coordinator Signature of seminar coordinator

April 9, 2016 Prof.PhD. Radu Precup Prof.PhD. Radu Precup

Date of approval Signature of the head of department

April 15, 2016 Prof.PhD. Octavian Agratini