SYLLABUS

1. Information regarding the programme

1.1 Higher education institution	Babeş-Bolyai University Cluj-Napoca
1.2 Faculty	Faculty of Mathematics and Computer Science
1.3 Department	Department of Mathematics
1.4 Field of study	Mathematics
1.5 Study cycle	Master
1.6 Study programme / Qualification	Applied Mathematics

2. Information regarding the discipline

2.1 Name of the discipline	**Special Topics in Fluid Mechanics**						
2.2 Course coordinator	Prof. dr. Mirela Kohr						
2.3 Seminar coordinator	Prof. dr. Mirela Kohr						
2.4. Year of study	**1**	2.5 Semester	**1**	2.6. Type of evaluation	**E**	2.7 Type of discipline	**DC**

3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	**3**	Of which: 3.2 course	**2**	3.3 seminar/laboratory	**1 sem**
3.4 Total hours in the curriculum	**42**	Of which: 3.5 course	**28**	3.6 seminar/laboratory	**14**
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					42
Additional documentation (in libraries, on electronic platforms, field documentation)					32
Preparation for seminars/labs, homework, papers, portfolios and essays					42
Tutorship					9
Evaluations					8
Other activities:					-

3.7 Total individual study hours	133
3.8 Total hours per semester	175
3.9 Number of ECTS credits	7

4. Prerequisites (if necessary)

4.1. curriculum	• Theoretical Mechanics; Partial Differential Equations; Real Functions; Functional Analysis
4.2. competencies	• There are useful logical thinking and mathematical notions and results from the above mentioned fields

5. Conditions (if necessary)

5.1. for the course	Classroom with blackboard/video projector
5.2. for the seminar /lab activities	Classroom with blackboard/video projector

6. Specific competencies acquired

Professional competencies	• Ability to understand and manipulate concepts, results and advanced mathematical theories. • Ability to model and analyze from the mathematical point of view real processes from other sciences, economics, and engineering. • Ability to use the scientific language and to write scientific reports and papers. • Acquiring specific methods in potential theory in the study of boundary value problems for linear elliptic systems involved in fluid mechanics, but also in other areas of mathematics.
Transversal competencies	• Ability to inform themselves, to work independently or in a team in order to realize studies and to solve complex problems. • Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics, Applied Mathematics, or in other fields that use mathematical models. • Ability for continuous self-perfecting and study.

7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	• Knowledge, understanding and use of main concepts and results in fluid mechanics • Knowledge, understanding and use of methods of potential theory in the study of some boundary value problems in fluid mechanics • Ability to use concepts and fundamental results in the theory partial differential equations and the ability to apply these results in the study of specific problems of fluid mechanics
7.2 Specific objective of the discipline	• Acquiring basic and advanced knowledge in fluid mechanics. • Ability to apply and use mathematical models to describe and analyze problems concerning viscous incompressible fluid flows • Understanding of main concepts and results in the mathematical theory of viscous incompressible flows at low Reynolds numbers • Knowledge, understanding and use of specific methods of potential theory in the study of linear elliptic boundary value problems in fluid mechanics • The use of mathematical software in the numerical solution of problems in fluid mechanics • Ability student involvement in scientific research

8. Content

8.1 **Course**	Teaching methods	Remarks
1. Kinematics of fluids: fluid, configuration, motion. Velocity and acceleration fields. Spatial description of the motion of a fluid.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
2. Fluid Dynamics: Principle of mass conservation. The continuity equation.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative	

	explanations.	
3. The Cauchy equations. The constitutive equation of ideal fluid and the Euler equations. Applications	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
4. The constitutive equation of viscous Newtonian fluid. The Navier-Stokes equations. Non-dimensional analysis and special forms of the Navier-Stokes equations.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
5. Uniqueness results of the Dirichlet and Neumann problems for the Stokes system in Lyapunov/ Lipschitz domains in $\mathbf{R}^n$. The weak solution of the Stokes problem with homogeneous Dirichlet boundary condition.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
6. The method of fundamental solutions in fluid mechanics (I): The Green function, the pressure vector, and the stress tensor of Stokes flow due to a point force. Oseen-Burgers tensor in $\mathbf{R}^n$ (n=2, 3).	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
7. The method of fundamental solutions in fluid mechanics (II): Direct layer potential representations of the velocity and pressure fields of Stokes flow. Applications	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
8. The hydrodynamic layer potential theory (I): Bounded and compact operators, Fredholm operators on Banach spaces. The Fredholm alternative.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
9. The hydrodynamic layer potential theory (II): Single- and double layer potentials. Properties.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
10. Applications of the layer potential theory (I): Existence and uniqueness results for boundary value problems for the Stokes system in bounded Lipschitz domains in $\mathbf{R}^n$ ($n \geq 2$), with data in Sobolev spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
11. Applications of the layer potential theory (II): The study of the exterior Dirichlet problem for the Stokes system in various function spaces.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
12. Transmission problems for the Stokes system: Existence and uniqueness in Sobolev spaces. Applications to viscous incompressible flows in the presence of interfaces. Numerical results.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

13. Layer potential theory for the Brinkman system. Applications to viscous fluid flows in porous media. Analytical and numerical results.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	
14. Stokes and Brinkman models. Existence and uniqueness results in Hölder or Sobolev spaces for transmission problems associated with the Stokes and Brinkman systems in Lyapunov or Lipschitz domains.	Lectures, modeling, didactical demonstration, conversation. Presentation of alternative explanations.	

Bibliography

1. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004
2. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian)
3. Dragoş, L., *Fluid Mechanics*, Vol. I, Romanian Academy Press, Bucharest, 1999 (in Romanian)
4. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000
5. Pozrikidis, C., *Fluid Dynamics. Theory, Computation and Numerical Simulation,* Springer, 2009 (second edition)
6. Pozrikidis, C., *Introduction to Theoretical and Computational Fluid Dynamics*, Oxford University Press, 2011
7. Kiselev, S.P., Vorozhtsov, E.V., Fomin, V.M., *Foundations of Fluid Mechanics with Applications. Problem Solving Using Mathematica*, Birkhäuser, Boston, 1999
8. Taylor, M., *Partial Differential Equations*, Springer-Verlag, New York, 1996-1997, vol. 1-3
9. Wloka, J. T. , Rowley, B., Lawruk, B., *Boundary Value Problems for Elliptic Systems*, Cambridge University Press, Cambridge, 1995
10. Power, H., Wrobel, L.C., *Boundary Integral Methods in Fluid Mechanics*, WIT Press: Computational Mechanics Publications, Southampton (UK) – Boston, 1995
11. Mitrea, M. Wright, M., *Boundary value problems for the Stokes system in arbitrary Lipschitz domains*, Astérisque, 344 (2012): viii+241 pp.

8.2 **Seminar**	Teaching methods	Remarks
1. Differential operators. Material derivatives. The Euler Theorem. Applications	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
2. Second order Cartesian tensors in $\mathbf{R}^n$.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
3. Properties of the Cauchy stress tensor. Cauchy's fundamental theorem, and the symmetry property.	Applications of course concepts. Description of arguments and proofs for solving problems.	1 hour/week

	Homework assignments. Direct answers to students.	
4. Incompressible fluid. The incompressibility condition.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
5. The Killing theorem. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
6. The exterior Dirichlet problem for the Stokes system in Lyapunov/Lipschitz domains in $\mathbf{R}^n$ (n=2,3). Uniqueness results and applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
7. Properties of linearized viscous fluid flows. The Lorentz reciprocal identity for viscous incompressible fluid flows. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
8. Properties of two-dimensional bounded Stokes flows. The Stokes paradox.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
9. Analysis of some boundary value problems for the biharmonic equation in $\mathbf{R}^2$. Applications	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
10. The method of fundamental solutions in fluid mechanics. The Stokes flow past a solid (fluid) sphere. Applications and numerical results.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
11. Compactness properties of layer potential operators for the Stokes system in various function spaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
12. Transmission problems for the Stokes system. Applications to viscous incompressible fluid flows in the presence of interfaces.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct	1 hour/week

	answers to students.	
13. The Stokes and Brinkman models. Applications.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week
14. Existence and uniqueness in Hölder or Sobolev spaces for transmission problems for the Stokes and Brinkman systems in Lyapunov or Lipschitz domains.	Applications of course concepts. Description of arguments and proofs for solving problems. Homework assignments. Direct answers to students.	1 hour/week

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1. Kohr, M., Pop, I., *Viscous Incompressible Flow for Low Reynolds Numbers*, WIT Press (Wessex Institute of Technology Press), Southampton (UK) – Boston, 2004
2. Kohr, M., *Modern Problems in Viscous Fluid Mechanics*, Cluj University Press, Cluj-Napoca, 2 vols. 2000 (in Romanian)
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4. Truesdell, C., Rajagopal, K.R., *An Introduction to the Mechanics of Fluids*, Birkhäuser, Basel, 2000
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6. Kiselev, S.P., Vorozhtsov, E.V., Fomin, V.M., *Foundations of Fluid Mechanics with Applications. Problem Solving Using Mathematica*, Birkhäuser, Boston, 1999
7. Taylor, M., *Partial Differential Equations*, Springer-Verlag, New York, 1996-1997, vol. 1-3
8. Wloka, J. T. , Rowley, B., Lawruk, B., *Boundary Value Problems for Elliptic Systems*, Cambridge University Press, Cambridge, 1995

9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

The content of this discipline is in accordance with the curricula of the most important universities in Romania and abroad, where the applied mathematics plays an essential role. This discipline is useful in preparing future teachers and researchers in applied mathematics, as well as those who use mathematical models and advanced methods of study in other areas.

10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	Knowledge of concepts and basic results	Written exam at the end the semester	60%
	Ability to justify by proofs theoretical results		
10.5 Seminar/lab	Ability to apply concepts	Evaluation of reports and	40%

activities	and results acquired in the course in mathematical modeling and analysis of problems in fluid mechanics	homework during the semester, and active participation in the seminar activity. A midterm control work.	
	There are valid the official rules of the faculty concerning the attendance of students to teaching activities.		
10.6 Minimum performance standards			
At least grade 5 (from a scale of 1 to 10) at both written exam and seminar activity during the semester.			

Date | Signature of course coordinator | Signature of seminar coordinator

30.04.2014 | Prof. dr. Mirela KOHR | Prof. dr. Mirela KOHR

Date of approval | Signature of the head of department

Prof. dr. Octavian AGRATINI