BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

ADMISSION 2025
Written exam in MATHEMATICS

IMPORTANT NOTE: Questions may have one or more correct answers, which must be indicated by the candidate on the special form provided on the examination sheet. Grading of multiple-choice questions will be performed according to the partial scoring system detailed in the competition regulations.

1. Let $A(x) = \begin{pmatrix} x & 2x & 3 \\ 0 & 1 & 2 \\ x & 0 & 3 \end{pmatrix}$, where $x \in \mathbb{R}$. If $\det(A(x)) = 16$, the value of x can be

A -1; B 1; C 2; D -2.

2. In the rhombus $ABCD$, let $A(1,3)$ and $C(3,-2)$ be two opposed vertices. The equation of the diagonal BD is

A $4x-10y+1=0$; B $4x-10y-3=0$; C $10x+4y-22=0$; D $10x+4y+5=0$.

3. Consider the polynomial $P(X) = X^3 - aX^2 - 3X + a^2 + 2$, where a is a real parameter. The sum of all possible values of a for which $P(3) = 0$ is

A 8; B 9; C 20; D -20.

4. For the real numbers $x, y \neq 0$ we define the expression

$$x \star y = \frac{x-1}{y} - \frac{y-1}{x}.$$

Which of the following statements are true?

A If $x \star y = 0$, then $x = y$.
B $x \star y \in \mathbb{R}^*$, for all $x, y \in \mathbb{R}^*$.
C There exist $x, y \in \mathbb{R}^*$ for which $x \star y \neq y \star x$.
D $1 \star (2 \star 3) = \frac{5}{3}$.

5. If for the real numbers $a, b \in (0,1)$ we have $\log_a b > \log_b a$, then

A $\log_a b > 0$; B $\log_b a < 0$; C $a > b$; D $b > a$.

6. If $x \in \left(\pi, \frac{3\pi}{2}\right)$ and $\cos x = -\frac{4}{5}$, then the value of the expression $\sin(2x)$ is

A $-\frac{24}{25}$; B $\frac{24}{25}$; C $-\frac{7}{25}$; D $\frac{7}{25}$.

7. Let $\vec{i}$ and $\vec{j}$ be the versors of a Cartesian system and let m be a real number. The angle formed by the vectors $\vec{u} = \vec{i} - 3\vec{j}$ and $\vec{v} = m\vec{i} + \vec{j}$ is acute if

A $\vec{u} \cdot \vec{v} < 0$; B $\vec{u} \cdot \vec{v} > 0$; C $m > 3$; D $m < 3$.

8. The coefficient of the term x in the expansion of the binomial

$$\left(2\sqrt[3]{x}+3\frac{1}{\sqrt[6]{x}}\right)^6$$

is:

A 1080; B 2160; C 3240; D 4320.

9. Let $f:[-2,2]\to\mathbb{R}$ be the function defined by

$$f(x)=\begin{cases} x^3, & \text{if } x\in[0,1] \\ 0, & \text{if } x\in[-2,2]\setminus[0,1]. \end{cases}$$

Which of the following statements are true?

A f is continuous on $[-2,2]$;
B f is differentiable in the point 0;
C f is odd;
D f is integrable on $[-2,2]$ and $\int_{-2}^{2} f(x)\mathrm{d}x=\frac{1}{4}$.

10. Consider the function $f:(0,\infty)\to\mathbb{R}$, defined by $f(x)=\ln x-\ln^2 x, \forall x>0$. The point at which the tangent to the graph of f is parallel to the Ox axis has abscissa

A $\frac{1}{\mathrm{e}^2}$; B e^2; C $\frac{1}{\sqrt{\mathrm{e}}}$; D $\sqrt{\mathrm{e}}$.

11. The value of the limit $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin^2 x}$ is

A $\frac{1}{2}$; B $-\frac{1}{2}$; C 0; D ∞.

12. Consider the set $A=\{p\in\mathbb{R} \mid \lim\limits_{\substack{x\to 0\\ x>0}} x^p \operatorname{arctg} x\in(0,\infty)\}$. Which of the following statements are true?

A The set A is finite;
B The set A is infinite;
C $A=\emptyset$;
D $A\subseteq\mathbb{Q}$.

13. Consider $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}2 & 0\\ 1 & 3\end{pmatrix}$. If $X\in\mathcal{M}_2(\mathbb{R})$ is a matrix such that $AX-XB=4I_2$, then the sum of all the elements of the matrix X is

A 1; B 2; C 3; D 0.

14. If for a complex number z we have $(2+i)z+(1-3i)\overline{z}=2-3i$, then $|z|$ is equal to

A $\sqrt{2}$; B $\sqrt{5}$; C 3; D $2\sqrt{2}$.

15. The points $A(2,0)$, $B(-2,2)$ and the line $d: x+y-3=0$ are given. If C is a point on the line d such that the area of the triangle ABC is equal to 2, then the product of the coordinates of C can be

A -18; B -8; C -2; D 2.

16. Consider the triangle ABC for which $BC=\sqrt{37}$, $AC=7$, $AB=4$ and R denotes the radius of its circumscribed circle. Which of the following statements are true?

A $\widehat{A}=30^\circ$; B $\widehat{A}=60^\circ$; C $R=\frac{\sqrt{111}}{3}$; D $R=\frac{\sqrt{74}}{2}$.

17. Consider the triangle ABC and the points M, N such that $\overrightarrow{BM} = \frac{2}{5}\overrightarrow{BC}$ and $\overrightarrow{AN} = \frac{1}{4}\overrightarrow{AC}$. If we denote by G the centroid (center of mass) of the triangle ABC, then which of the following statements are true?

A $\overrightarrow{NG} = \frac{1}{3}\overrightarrow{AB} + \frac{1}{12}\overrightarrow{AC}$; B $\overrightarrow{NG} = \frac{1}{3}\overrightarrow{AB} - \frac{1}{4}\overrightarrow{AC}$;

C $\overrightarrow{NM} = \frac{3}{5}\overrightarrow{AB} + \frac{3}{20}\overrightarrow{AC}$; D the points M, N, G are collinear.

18. How many real numbers in the interval $[0, 2\pi]$ satisfy the equation $\operatorname{tg} x + \sin 2x = 3\sin x$?

A 2; B 3; C 4; D 5.

19. The endpoints of the hypotenuse of a right triangle ABC are the points A and B. Point A has coordinates $(2, -2)$, and point B has ordinate 4. The equation of one of the legs (catheti) is $x + y = 10$. The length of the leg BC is

A 2; B $\sqrt{3}$; C $\sqrt{2}$; D 1.

20. Let $f : \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be the function defined by $f(x) = \sin x - \mathrm{e}^{-x}, \forall x \in \left[0, \frac{\pi}{2}\right]$. Which of the following statements are true?

A f is strictly monotone on the interval $\left[0, \frac{\pi}{2}\right]$;

B f is concave on the interval $\left[0, \frac{\pi}{2}\right]$;

C For all $x \in \left[0, \frac{\pi}{2}\right]$ the inequality $f(x) \geq 0$ holds;

D The equation $f(x) = 0$ has exactly one solution in the interval $\left[0, \frac{\pi}{2}\right]$.

21. The value of the integral $\displaystyle\int_1^{\mathrm{e}} \frac{\mathrm{d}x}{x\sqrt{4 - \ln^2 x}}$ is

A $\frac{\pi}{12}$; B $\frac{\pi}{3}$; C $\frac{\pi}{6}$; D $\ln\sqrt{\frac{3}{4}}$.

22. Let $(g_n)_{n\geq 1}$ be a geometric progression such that $g_2 + g_4 = 10$ and $g_1 + g_2 + g_3 = 7$. The value of the term g_3 is

A 2; B 4; C 8; D 16.

23. For every real number $a > 1$ we write $I(a) = \displaystyle\int_1^a \frac{\operatorname{arctg} x}{x^2}\,\mathrm{d}x$. The value of the limit $\lim\limits_{a\to\infty} I(a)$ is

A $\frac{\pi}{4} + 2\ln 2$; B $\frac{\pi}{4} - \ln\sqrt{2}$; C $\frac{\pi}{4} + \ln\sqrt{2}$; D $\frac{\pi}{4} + \ln 2$.

24. The value of the limit $\lim\limits_{n\to\infty} \frac{\frac{n}{\sqrt{1}} + \frac{n}{\sqrt{2}} + \cdots + \frac{n}{\sqrt{n}}}{\sqrt{1} + \sqrt{2} + \cdots + \sqrt{n}}$ is

A 1; B 2; C 3; D ∞.

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Correct Answers

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1. **C**, **D**
2. **B**
3. **B**
4. **C**, **D**
5. **A**, **C**
6. **B**
7. **B**, **C**
8. **B**
9. **B**, **D**
10. **D**
11. **A**
12. **A**, **D**
13. **C**
14. **B**
15. **A**, **D**
16. **B**, **C**
17. **A**, **C**, **D**
18. **D**
19. **C**
20. **A**, **B**, **D**
21. **C**
22. **B**
23. **C**
24. **C**