BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

ADMISSION 2025
Written exam in MATHEMATICS

IMPORTANT NOTE: Questions may have one or more correct answers, which must be indicated by the candidate on the special form provided on the examination sheet. Grading of multiple-choice questions will be performed according to the partial scoring system detailed in the competition regulations.

1. Let $A(x) = \begin{pmatrix} x & x & 3 \\ 0 & 1 & 2 \\ x & 0 & 2 \end{pmatrix}$, where $x \in \mathbb{R}$. If $\det(A(x)) = 6$, then the value of x can be

$\boxed{A}$ -2; $\boxed{B}$ 2; $\boxed{C}$ $-\frac{3}{2}$; $\boxed{D}$ 1.

2. The center of symmetry of the square $ABCD$ is the point $M(1,1)$. The vertex B has coordinates $(3,2)$. The equation of the diagonal AC is

$\boxed{A}$ $2x - y - 1 = 0$; $\boxed{B}$ $2x + y - 3 = 0$; $\boxed{C}$ $x + 2y - 1 = 0$; $\boxed{D}$ $x - 2y + 1 = 0$.

3. Consider the polynomial $P(X) = X^3 + aX^2 - 5X + a^2 - 3$, where a is a real parameter. The sum of all possible values of a for which $P(2) = 0$ is

$\boxed{A}$ -6; $\boxed{B}$ -4; $\boxed{C}$ 4; $\boxed{D}$ 6.

4. For the real numbers $x, y \neq 0$ we define the expression

$$x \star y = \frac{x+1}{y} + \frac{y+1}{x}.$$

Which of the following statements are true?

$\boxed{A}$ $x \star y \in \mathbb{R}^*$, for all $x, y \in \mathbb{R}^*$; $\boxed{B}$ $x \star y = y \star x$, for all $x, y \in \mathbb{R}^*$;

$\boxed{C}$ $1 \star (2 \star 3) = \frac{14}{3}$; $\boxed{D}$ There exists $e \in \mathbb{R}^*$ such that $x \star e = x$, for all $x \in \mathbb{R}^*$.

5. If for the real numbers $a, b \in (0,1)$ we have $a^b > b^a$, then

$\boxed{A}$ $\ln a < \ln b$; $\boxed{B}$ $\frac{\ln a}{a} > \frac{\ln b}{b}$; $\boxed{C}$ $a > b$; $\boxed{D}$ $b > a$.

6. Suppose that $x \in \left(\frac{\pi}{2}, \pi\right)$ and $y \in \left(0, \frac{\pi}{2}\right)$, and moreover $\sin x = \frac{5}{13}$ and $\sin y = \frac{3}{5}$. Then the value of the expression $\cos(x+y)$ is

$\boxed{A}$ $-\frac{33}{65}$; $\boxed{B}$ $-\frac{16}{65}$; $\boxed{C}$ $-\frac{63}{65}$; $\boxed{D}$ $\frac{56}{65}$.

7. Consider the triangle ABC in which $AB = 4$, $AC = 6$ and $BC = 8$. Which of the following statements are true?

$\boxed{A}$ $\cos A = -\frac{1}{4}$; $\boxed{B}$ $\cos A = \frac{1}{2}$; $\boxed{C}$ $\overrightarrow{AB} \cdot \overrightarrow{AC} = -12$; $\boxed{D}$ $\overrightarrow{AB} \cdot \overrightarrow{AC} = -6$.

8. If for the natural number n we have $5C_{n+3}^{n} = A_{n+2}^{3}$, then n is

$\boxed{\text{A}}$ 15; $\boxed{\text{B}}$ 16; $\boxed{\text{C}}$ 17; $\boxed{\text{D}}$ 18.

9. For every $a \in [-2, 2]$ we consider the sequence defined by

$$S_n(a) = \frac{a}{2} + \frac{a^2}{2^2} + \cdots + \frac{a^n}{2^n}, \qquad n \in \mathbb{N}^*.$$

Which of the following statements are true?

$\boxed{\text{A}}$ $\lim_{n\to\infty} S_n(a) = \dfrac{a}{2-a}$, for all $a \in (-2, 2)$;

$\boxed{\text{B}}$ $\big(S_n(a)\big)_{n\geq 1}$ is a bounded sequence for every $a \in [-2, 2]$;

$\boxed{\text{C}}$ $\big(S_n(a)\big)_{n\geq 1}$ is a strictly increasing sequence for every $a \in [0, 2]$;

$\boxed{\text{D}}$ $\lim_{n\to\infty} \dfrac{1}{S_n(2) + S_n(-2)} = 0$.

10. Consider the function $f : (0, \infty) \to \mathbb{R}$, defined by $f(x) = ax \ln x - 3x$. If $x = \sqrt{\mathrm{e}}$ is a point of global minimum for f, then the value of a is

$\boxed{\text{A}}$ 1; $\boxed{\text{B}}$ 2; $\boxed{\text{C}}$ 3; $\boxed{\text{D}}$ $\dfrac{3}{2}$.

11. The value of the limit $\lim_{x\to 1} \dfrac{\sqrt{x^2 + x + 2} - 2}{x^2 - 1}$ is

$\boxed{\text{A}}$ $\dfrac{3}{4}$; $\boxed{\text{B}}$ $\dfrac{1}{4}$; $\boxed{\text{C}}$ $\dfrac{1}{8}$; $\boxed{\text{D}}$ $\dfrac{3}{8}$.

12. Consider the set $A = \{p \in \mathbb{R} \mid \lim_{x\to\infty} x^p \operatorname{arctg} x \in (0, \infty)\}$. Which of the following statements are true?

$\boxed{\text{A}}$ The set A is infinite; $\boxed{\text{B}}$ The set A is finite;

$\boxed{\text{C}}$ $A = \emptyset$; $\boxed{\text{D}}$ $A \subseteq \mathbb{Q}$.

13. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. If $X \in \mathcal{M}_2(\mathbb{R})$ is a matrix such that $AX + XB = 3I_2$, then the sum of all elements of the matrix X is

$\boxed{\text{A}}$ 1; $\boxed{\text{B}}$ 2; $\boxed{\text{C}}$ 3; $\boxed{\text{D}}$ 4.

14. If for the complex number z we have $(3 + i)z + (1 - 2i)\overline{z} = 11 - 4i$, then $|z|$ is equal to

$\boxed{\text{A}}$ 5; $\boxed{\text{B}}$ 3; $\boxed{\text{C}}$ $\sqrt{5}$; $\boxed{\text{D}}$ $\sqrt{3}$.

15. Let $(a_n)_{n\geq 1}$ be an arithmetic progression such that $a_5 + a_{10} = 70$ and $a_1 + a_2 + \cdots + a_{15} = 600$. The value of the term a_3 is

$\boxed{\text{A}}$ -10; $\boxed{\text{B}}$ 0; $\boxed{\text{C}}$ 10; $\boxed{\text{D}}$ -20.

16. The side AB of rectangle $ABCD$ lies on the line with equation $5x + 12y - 2 = 0$, and the length of side BC is 2. Which of the following lines could contain the side CD of the rectangle?

$\boxed{\text{A}}$ $d_1 : 5x + 12y + 10 = 0$; $\boxed{\text{B}}$ $d_2 : 5x + 12y - 36 = 0$;

$\boxed{\text{C}}$ $d_3 : 5x + 12y + 24 = 0$; $\boxed{\text{D}}$ $d_4 : 5x + 12y - 28 = 0$.

17. The area of a rhombus is 18, and the length of each of its sides is 6. The measure of the acute angle of the rhombus is

$\boxed{A}$ $30°$; $\boxed{B}$ $45°$; $\boxed{C}$ $60°$; $\boxed{D}$ $75°$.

18. Consider the parallelogram $ABCD$ and the points M, N, P such that $\overrightarrow{AM} = \frac{2}{3}\overrightarrow{AB}$, $\overrightarrow{BN} = \frac{3}{4}\overrightarrow{BC}$ and $\overrightarrow{DP} = x \cdot \overrightarrow{DC}$, where $x \in \mathbb{R}$. The value of x for which the points M, N, P are collinear is

$\boxed{A}$ $\frac{9}{10}$; $\boxed{B}$ $\frac{4}{5}$; $\boxed{C}$ $\frac{5}{4}$; $\boxed{D}$ $\frac{10}{9}$.

19. How many numbers in the interval $[0, 2\pi]$ satisfy the equation $\operatorname{ctg} x + 2\sin 2x = 4\cos x$?

$\boxed{A}$ 2; $\boxed{B}$ 3; $\boxed{C}$ 4; $\boxed{D}$ 5.

20. The endpoints of the hypotenuse of a right triangle are $A(-3, 4)$ and $B(8, 2)$. The leg AC is parallel to the line with equation $3x + 4y = 0$. The area of triangle ABC is

$\boxed{A}$ 10; $\boxed{B}$ 15; $\boxed{C}$ 20; $\boxed{D}$ 25.

21. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be the functions defined by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} \qquad \text{and respectively} \qquad g(x) = x^2 - 4x + 4, \ \forall x \in \mathbb{R}.$$

Consider then $h : [0, 3] \to \mathbb{R}$ the function defined by $h = f \circ g$. Which of the following statements are true?

$\boxed{A}$ h is continuous on $[0, 3]$; $\boxed{B}$ h has finite one-sided (lateral) limits at every point of $[0, 3]$;
$\boxed{C}$ h is integrable on $[0, 3]$; $\boxed{D}$ h admits a primitive on $[0, 3]$. .

22. Let $f : [0, \pi] \to \mathbb{R}$ be the function defined by $f(x) = \ln(1 + x) - \cos x$, $\forall x \in [0, \pi]$. Which of the following statements are true?

$\boxed{A}$ f is strictly monotone on the interval $[0, \pi]$; $\boxed{B}$ $f''\left(\frac{\pi}{3}\right) > 0$;
$\boxed{C}$ f is concave on the interval $[0, \pi]$; $\boxed{D}$ f is convex on the interval $[0, \pi]$.

23. The value of the integral $\displaystyle\int_1^8 \frac{x}{\sqrt{3x + 1}}\,\mathrm{d}x$ is

$\boxed{A}$ 7; $\boxed{B}$ 8; $\boxed{C}$ 6; $\boxed{D}$ 4.

24. The value of the integral $\displaystyle\int_0^{\pi/4} \frac{\mathrm{d}x}{1 + 2\cos^2 x}$ is

$\boxed{A}$ $\frac{\pi}{6\sqrt{3}}$; $\boxed{B}$ $\frac{\pi}{6}$; $\boxed{C}$ $\frac{\pi}{3\sqrt{3}}$; $\boxed{D}$ $\frac{\pi}{12\sqrt{3}}$.

BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Correct Answers

ADMISSIONS EXAM, July 2025

Written test in MATHEMATICS

1. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

2. $\boxed{\textbf{B}}$

3. $\boxed{\textbf{B}}$

4. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

5. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

6. $\boxed{\textbf{C}}$

7. $\boxed{\textbf{A}}$, $\boxed{\textbf{D}}$

8. $\boxed{\textbf{A}}$

9. $\boxed{\textbf{A}}$, $\boxed{\textbf{D}}$

10. $\boxed{\textbf{B}}$

11. $\boxed{\textbf{D}}$

12. $\boxed{\textbf{B}}$, $\boxed{\textbf{D}}$

13. $\boxed{\textbf{B}}$

14. $\boxed{\textbf{C}}$

15. $\boxed{\textbf{A}}$

16. $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$

17. $\boxed{\textbf{A}}$

18. $\boxed{\textbf{D}}$

19. $\boxed{\textbf{C}}$

20. $\boxed{\textbf{D}}$

21. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

22. $\boxed{\textbf{A}}$, $\boxed{\textbf{B}}$

23. $\boxed{\textbf{B}}$

24. $\boxed{\textbf{A}}$