BABEŞ-BOLYAI UNIVERSITY, CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

**Admission 2023**
**Written test in MATHEMATICS**

**IMPORTANT NOTE: Problems can have one or more correct answers, which the candidate should indicate on the test form. The grading system of the multiple choice exam can be found in the set of rules of the competition.**

**1.** If $f, g\colon \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 + x + 1$ and $g(x) = 2x + 1$, then $g(f(1))$ is equal to

A 6; B 7; C 8; D 9.

**2.** If the roots of the second degree equation of parameter $m \in \mathbb{R}$

$$x^2 - mx + m = 0$$

are not real numbers, then

A $m \in (-\infty, -4]$; B $m \in (-4, 0]$; C $m \in (0, 4)$; D such $m \in \mathbb{R}$ does not exist.

**3.** The value of the limit $\lim\limits_{n\to\infty} \left(\sqrt{n^2 + 2n} - n\right)$ is:

A $\frac{1}{2}$; B 1; C 2; D $+\infty$.

**4.** The vertices of the triangle $ABC$ are $A(-2, -1)$, $B(-1, 2)$ and $C(6, -3)$. The median from $B$ has length

A 4; B $2\sqrt{3}$; C $3\sqrt{2}$; D 5.

**5.** Consider the vectors $\vec{x} = 2\vec{i} + a\vec{j}$ and $\vec{y} = b\vec{i} + 3\vec{j}$, where $a, b \in \mathbb{R}$ and the unit vectors $\vec{i}$ and $\vec{j}$ are perpendicular. Which of the following statements imply the colliniarity of the vectors $\vec{x}$ and $\vec{y}$ ?

A $a = 1,\ b = 6$; B $a \cdot b = 6$; C $\vec{x} \cdot \vec{y} = 0$; D $\vec{x} \cdot \vec{y} = \sqrt{a^2 + 4} \cdot \sqrt{b^2 + 9}$.

**6.** The value of the expression $a = \sqrt{2 + \sqrt{3}} - \sqrt{2 - \sqrt{3}}$ is equal to

A $\sqrt{3}$; B $\sqrt{2}$; C $2(\sqrt{3} - \sqrt{2})$; D 2.

**7.** Let $f\colon \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} |x|^3, & x < 0 \\ 2x^2, & x \geq 0. \end{cases}$$

Which of the following statements are true?

A $f(-2) = f(2)$; B $f$ is injective; C $f$ is surjective; D $\mathrm{Im} f = [0, +\infty)$.

**8.** Consider the line $d : x - 2y = 0$ and the points $M(3, 4)$, $O(0, 0)$. The coordinates of the point $N$, which belongs to the line $d$, and is such that the triangle $MNO$ is right in $N$, can be

A $N(4, 2)$; B $N(6, 3)$; C $N(5, \frac{5}{2})$; D $N(2\sqrt{5}, 2\sqrt{5})$.

Problems **9** and **10** refer to the function $f:[0,2\pi]\to\mathbb{R},\ f(x)=3\cos x+\cos(2x)$.

**9.** $f(\pi)$ is

$\boxed{\text{A}}$ $-3$; $\boxed{\text{B}}$ $-2$; $\boxed{\text{C}}$ $2$; $\boxed{\text{D}}$ $3$.

**10.** The number of solutions of the equation $f(x)=1$ is

$\boxed{\text{A}}$ $0$; $\boxed{\text{B}}$ $1$; $\boxed{\text{C}}$ $2$; $\boxed{\text{D}}$ $4$.

**11.** The value of the limit $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\ln(\sin x)}{(\pi-2x)^2}$ is:

$\boxed{\text{A}}$ $+\infty$; $\boxed{\text{B}}$ $-\dfrac{1}{4}$; $\boxed{\text{C}}$ $\dfrac{1}{8}$; $\boxed{\text{D}}$ $-\dfrac{1}{8}$.

**12.** Denote by $S$ the set of solutions of the equation

$$\overline{z}z^2=2+2i,$$

where $z$ is a complex number. Which of the following statements are true?

$\boxed{\text{A}}$ $1+i\in S$; $\boxed{\text{B}}$ If $w\in S$, then $|w|=\sqrt{2}$;
$\boxed{\text{C}}$ If $w\in S$, then $\overline{w}\in S$; $\boxed{\text{D}}$ The number of elements in $S$ is equal to 2.

**13.** Let $a$ be a real parameter and consider the system of equations

$$\begin{cases} x-\ y+\ z &=0\\ -x+ay+az &=0\\ x-\ y+az &=0.\end{cases}$$

Which of the following statements are correct?

$\boxed{\text{A}}$ The system is compatible for every value of $a$.
$\boxed{\text{B}}$ There exists $a\in\mathbb{R}$ for which the system is incompatible.
$\boxed{\text{C}}$ There exists a unique number $a$ for which the determinant of the matrix of the system is 0.
$\boxed{\text{D}}$ $z$ takes the same value for every $a$ for which the system is compatible.

**14.** In the acute triangle $ABC$ we know the length of the sides $a=BC=2$, $b=AC=\sqrt{6}$ and the length of the radius of its circumscribed circle $R=\sqrt{2}$. Which of the following statements are true?

$\boxed{\text{A}}$ $A=45^\circ$; $\boxed{\text{B}}$ $B=30^\circ$; $\boxed{\text{C}}$ $B=60^\circ$; $\boxed{\text{D}}$ $C=75^\circ$.

**15.** The line that passes through $C(3,-1)$ and lies at the same distance from the points $A(2,3)$ and $B(6,1)$ can have the equation:

$\boxed{\text{A}}$ $4x+y-11=0$; $\boxed{\text{B}}$ $x+2y-1=0$; $\boxed{\text{C}}$ $x-4y=7$; $\boxed{\text{D}}$ $3x-y-10=0$.

**16.** Consider the triangle $ABC$ and $D$ a point on the side $BC$ such that $AD$ is the bisector of the angle $BAC$. If the lengths of the sides of the triangle are $BC=4, CA=5, AB=6$, which of the following statements are true?

$\boxed{\text{A}}$ $\overrightarrow{BD}=\dfrac{5}{6}\overrightarrow{DC}$; $\boxed{\text{B}}$ $\overrightarrow{BD}=\dfrac{6}{5}\overrightarrow{DC}$;

$\boxed{\text{C}}$ $\overrightarrow{AD}=\dfrac{5}{11}\overrightarrow{AB}+\dfrac{6}{11}\overrightarrow{AC}$; $\boxed{\text{D}}$ $\overrightarrow{AD}=\dfrac{6}{11}\overrightarrow{AB}+\dfrac{5}{11}\overrightarrow{AC}$.

**17.** The value of the integral $\displaystyle\int_0^{\pi/3} \sin x \cdot \ln(\cos x)\, \mathrm{d}x$ is:

$\boxed{\text{A}}$ $\ln\sqrt{2} - \dfrac{1}{2}$;  $\boxed{\text{B}}$ $\ln 2 - \dfrac{1}{2}$;  $\boxed{\text{C}}$ $\ln\sqrt{3} - \dfrac{1}{2}$;  $\boxed{\text{D}}$ $\ln\sqrt{2} + \dfrac{1}{2}$.

**18.** Let $S$ be the set of four digit numbers. The number of elements in $S$ which are divisible by 3 is

$\boxed{\text{A}}$ 2999;  $\boxed{\text{B}}$ 3000;  $\boxed{\text{C}}$ 3001;  $\boxed{\text{D}}$ 3333.

**19.** On the set $G = (0, +\infty)$, we are given the operation $x * y = x + y + |x - y|$. Which of the following statements are true?

$\boxed{\text{A}}$ the operation "$*$" is commutative;  $\boxed{\text{B}}$ $1 * (2 * 3) = (1 * 2) * 3$;

$\boxed{\text{C}}$ $x * y \geq 1$ for every $x, y \in G$;  $\boxed{\text{D}}$ the operation "$*$" admits a neutral element.

Problems **20**, **21**, **22** and **23** refer to the function $f : (0, +\infty) \to \mathbb{R}$, defined by $f(x) = -x \ln x$.

**20.** Which of the following statements are true?

$\boxed{\text{A}}$ $f$ has a finite lateral limit in the point $x_0 = 0$;

$\boxed{\text{B}}$ $f$ is strictly increasing on the interval $(0, 1]$;

$\boxed{\text{C}}$ $f$ is strictly decreasing on the interval $[1, +\infty)$;

$\boxed{\text{D}}$ $f$ is concave.

**21.** The number of solutions of the equation $f(x) = \dfrac{1}{3}$ is:

$\boxed{\text{A}}$ 3;  $\boxed{\text{B}}$ 2;  $\boxed{\text{C}}$ 1;  $\boxed{\text{D}}$ 0.

**22.** If $d$ is the tangent line to the graph of $f$ which passes through the point $(0, 1)$, and $m$ is the slope of $d$, which of the following statements are true?

$\boxed{\text{A}}$ $m \in [-2, 0]$;  $\boxed{\text{B}}$ $m \in (-\infty, -2]$;

$\boxed{\text{C}}$ $d$ passes through the point $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$;  $\boxed{\text{D}}$ $d$ passes through the point $\left(\dfrac{1}{\mathrm{e}}, \dfrac{1}{\mathrm{e}}\right)$.

**23.** The smallest real number $a$ for which the inequality $f(x) \leq a - x$ takes place for every $x \in (0, +\infty)$ is:

$\boxed{\text{A}}$ $\dfrac{1}{\mathrm{e}}$;  $\boxed{\text{B}}$ $\dfrac{2}{\mathrm{e}}$;  $\boxed{\text{C}}$ 2;  $\boxed{\text{D}}$ 1.

**24.** Let $f : [-1, 1] \to \mathbb{R}$ be a continuous function with the property that

$$6 + f(x) = 2f(-x) + 3x^2 \left( \int_{-1}^{1} f(t)\, \mathrm{d}t \right) \quad \text{for all } x \in [-1, 1].$$

Which of the following statements are true?

$\boxed{\text{A}}$ $\displaystyle\int_{-1}^{1} f(x)\, \mathrm{d}x = 6$;  $\boxed{\text{B}}$ $\displaystyle\int_{-1}^{1} f(x)\, \mathrm{d}x = 4$;  $\boxed{\text{C}}$ $f\left(\dfrac{1}{2}\right) = 3$;  $\boxed{\text{D}}$ $f(-1) = -6$.

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# Correct Answers

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**1.** $\boxed{\textbf{B}}$

**2.** $\boxed{\textbf{C}}$

**3.** $\boxed{\textbf{B}}$

**4.** $\boxed{\textbf{D}}$

**5.** $\boxed{\textbf{A}}$, $\boxed{\textbf{B}}$, $\boxed{\textbf{D}}$,

**6.** $\boxed{\textbf{B}}$

**7.** $\boxed{\textbf{A}}$, $\boxed{\textbf{D}}$

**8.** $\boxed{\textbf{A}}$

**9.** $\boxed{\textbf{B}}$

**10.** $\boxed{\textbf{C}}$

**11.** $\boxed{\textbf{D}}$

**12.** $\boxed{\textbf{A}}$, $\boxed{\textbf{B}}$

**13.** $\boxed{\textbf{A}}$, $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$

**14.** $\boxed{\textbf{A}}$, $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$

**15.** $\boxed{\textbf{B}}$, $\boxed{\textbf{D}}$

**16.** $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

**17.** $\boxed{\textbf{A}}$

**18.** $\boxed{\textbf{B}}$

**19.** $\boxed{\textbf{A}}$

**20.** $\boxed{\textbf{A}}$, $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$

**21.** $\boxed{\textbf{B}}$

**22.** $\boxed{\textbf{A}}$, $\boxed{\textbf{C}}$

**23.** $\boxed{\textbf{D}}$

**24.** $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$