BABEŞ-BOLYAI UNIVERSITY, CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Admission 2023
Written test in MATHEMATICS

IMPORTANT NOTE: Problems can have one or more correct answers, which the candidate should indicate on the test form. The grading system of the multiple choice exam can be found in the set of rules of the competition.

1. If $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 + 3x - 2$, then $f(f(1))$ is equal to

A 6; B 7; C 8; D 9.

2. If the roots of the second degree equation of parameter $m \in \mathbb{R}$

$$x^2 - (m+1)x + m = 0$$

coincide, then

A $m \in (-\infty, -2)$; B $m \in [-2, 2]$; C $m \in (2, +\infty)$; D such $m \in \mathbb{R}$ does not exist.

3. The slope of a line parallel to the line $d : x - 3y + 4 = 0$ is equal to

A -3; B $-\frac{1}{3}$; C $\frac{1}{3}$; D 3.

4. Consider the vectors $\vec{u} = a\vec{i} + \vec{j}$ and $\vec{v} = b\vec{i} - 3\vec{j}$, where $a, b \in \mathbb{R}$ and $\vec{i}$, $\vec{j}$ are perpendicular unit vectors. Which of the following statements imply the perpendicularity of the vectors $\vec{u}$ and $\vec{v}$?

A $a = -2,\ b = -\frac{3}{2}$; B $a \cdot b = 3$; C $3a + b = 0$; D $a = 1,\ b = -3$.

5. The value of the expression $a = \sqrt{2 + \sqrt{3}} + \sqrt{2 - \sqrt{3}}$ is equal to

A $\sqrt{6}$; B $2\sqrt{2}$; C $2\sqrt{3}$; D $\sqrt{3} + \sqrt{2}$.

6. The value of the limit $\lim\limits_{n\to\infty} \left(\frac{2n+1}{2n-1}\right)^n$ is:

A $\frac{1}{\mathrm{e}}$; B e; C e^2; D e^{-2}.

7. Let $f\colon \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} x^3, & x < 0 \\ -x^2, & x \geq 0. \end{cases}$$

Which of the following statements are true?

A $f(-1) = f(1)$; B f is injective; C f is surjective; D $\mathrm{Im} f = (-\infty, 0]$.

8. About the triangle MNP it is known that its area is equal to 10, the points $M(-2, 1)$, $N(2, 5)$, and the point P belongs to the Ox axis. The coordinates of the point P can be

A $P(-8, 0)$; B $P(-2, 0)$; C $P(2, 0)$; D $P(8, 0)$.

Problems **9** and **10** refer to the function $f : [0, \pi] \to \mathbb{R},\ f(x) = \sin x + \cos(2x)$.

9. $f\left(\frac{\pi}{4}\right)$ is

A $\frac{\sqrt{3}}{2}$; B $\frac{\sqrt{3}}{2} + 1$; C $\frac{\sqrt{2}}{2}$; D $\frac{\sqrt{2}}{2} + 1$.

10. The number of solutions of the equation $f(x) = 1$ is

A 1; B 2; C 3; D 4.

11. The value of the limit $\lim\limits_{x \to +\infty} x \cdot \left(\frac{\pi}{2} - \operatorname{arctg} x\right)$ is:

A 0; B -1; C 1; D $+\infty$.

12. Denote by S the set of solutions of the equation

$$x^{\frac{1}{1+\log_4 x}} = 4x^4.$$

Which of the following statements are true?

A S has exactly one element;
B S has exactly two elements;
C There exists a unique $a \in S$ such that $a < 1$;
D There exists a unique $a \in S$ such that $a \geq 1$.

13. If $X, Y \in \mathcal{M}_2(\mathbb{Z}_7)$ are such that

$$\begin{cases} X + \hat{2}Y = \begin{pmatrix} \hat{2} & \hat{3} \\ \hat{4} & \hat{1} \end{pmatrix} \\ \hat{2}X - Y = \begin{pmatrix} \hat{5} & \hat{1} \\ \hat{2} & \hat{6} \end{pmatrix}, \end{cases}$$

then $\det X \cdot \det Y$ is equal to

A $\hat{0}$; B $\hat{1}$; C $\hat{2}$; D $\hat{4}$.

14. Consider the parallelogram $ABCD$ and the points $M \in AB$, $N \in AC$ such that $\overrightarrow{AM} = \frac{3}{2}\overrightarrow{AB}$ and $\overrightarrow{AN} = m\overrightarrow{AC}$, where $m \in \mathbb{R}^*$. The points D, N and M are collinear if

A $m = \frac{1}{2}$; B $m = \frac{3}{5}$; C $m = \frac{2}{3}$; D $m = \frac{4}{7}$.

15. Denote by $\mathcal{C}_t$ the circle centered in the point $M(t, 0)$, and which passes through the points $A(1, 1)$ and $B(1, -1)$. Denote the radius of $\mathcal{C}_t$ by r_t. Which of the following statements are true?

A For $t = 0$ we have $r_t = \sqrt{2}$.
B For every $t \in (0, 4)$, we have $r_t \in (\sqrt{2}, \sqrt{10})$.
C There exists t such that $r_t = \frac{1}{2}$.
D For $t = 2$ the triangle AMB is a right triangle.

16. In the triangle ABC, we have that $m(\hat{A}) = 45°$, $AB = c$, $AC = \dfrac{2\sqrt{2}\,c}{3}$ and $BC = a$. Which of the following statements are true?

A $\sin B = \dfrac{2c}{3a}$; B $\cos B = \dfrac{c}{3a}$; C $\mathrm{tg}B = 2$; D $\mathrm{tg}B = \sqrt{2}$.

17. The value of the integral $\displaystyle\int_1^4 \frac{\mathrm{d}x}{\sqrt{5-x}}$ is:

A 1; B 2; C -1; D -2.

18. If S is the set of even numbers which have four digits, then the number of elements of S is:

A 4500; B 4499; C 4501; D 5000.

19. On the set $G = (0, +\infty)$, we are given the operation $x * y = \frac{|x-y|}{x+y}$. Which of the following statements are true?

A The operation "$*$" is commutative; B $1 * (2 * 3) = (1 * 2) * 3$;
C $x * y < 1$ for every $x, y \in G$; D the operation "$*$" admits a neutral element.

Problems **20**, **21**, **22** and **23** refer to the function $f : [0, +\infty) \to \mathbb{R}$, defined by

$$f(x) = \begin{cases} x^2 \ln^2 x, & \text{if } x \in (0, +\infty) \\ 0, & \text{if } x = 0. \end{cases}$$

20. Which of the following statements are true?

A f is continuous at the point $x_0 = 0$;
B f is discontinuous at the point $x_0 = 0$;
C f is strictly decreasing on the interval $[0, 1]$;
D f is strictly increasing on the interval $[1, +\infty)$.

21. The number of local extremum points of f is:

A 0; B 1; C 2; D 3.

22. The number of inflection points of the graph of f is:

A 0; B 1; C 2; D 3.

23. If d is the tangent line to the graph of f in the point of abscissa e, and m is the slope of the line d, which of the following statements are true?

A $m = 4\mathrm{e}$; B d intersects the Ox axis in the point with abscissa $\dfrac{3\mathrm{e}}{4}$;
C $m = 2\mathrm{e}$; D d intersects the Oy axis in the point with ordinate $-4\mathrm{e}^2$.

24. The value of the integral $\displaystyle\int_{-\pi/6}^{\pi/2} \frac{\cos\left(x + \frac{\pi}{6}\right)}{\sin\left(x + \frac{\pi}{3}\right)}\,\mathrm{d}x$ is:

A $\dfrac{\pi}{6}$; B $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{2}\ln\sqrt{3}$; C $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{2}\ln 3$; D $\dfrac{\pi}{3}$.

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Correct Answers

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1. **C**
2. **B**
3. **C**
4. **A**, **B**
5. **A**
6. **B**
7. **A**, **D**
8. **A**, **C**
9. **C**
10. **D**
11. **C**
12. **A**, **C**
13. **D**
14. **B**
15. **A**, **D**
16. **A**, **B**, **C**
17. **B**
18. **A**
19. **A**, **C**
20. **A**, **D**
21. **D**
22. **C**
23. **A**, **B**
24. **D**