BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Admissions Exam 2022
Written test in MATHEMATICS

IMPORTANT NOTE: Problems can have one or more correct answers, which candidates should indicate on their exam sheet. The grading system of the multiple choice exam can be found in the set of rules of the competition.

1. The value of the limit $\lim_{n\to\infty}\left(\dfrac{n^2-3n+1}{n^2+3n+2}\right)^{n/3}$ is

A e^2; B $\mathrm{e}-2$; C $\dfrac{1}{\mathrm{e}}$; D e^{-2}.

2. The value of the limit $\lim_{x\to 0}\dfrac{\mathrm{e}^x-1-\operatorname{arctg} x}{x^2}$ is

A 0; B $\dfrac{1}{2}$; C 1; D $\dfrac{3}{2}$.

3. The equation of the line tangent to the graph of the function $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\mathrm{e}^{3x}+2x+1$ at the point $x=0$ is

A $5x-y+2=0$; B $5x+y-2=0$; C $x-5y+2=0$; D $x+5y-2=0$.

4. In the Cartesian coordinate system xOy consider the point $M(1,-1)$. The equation of the line passing through the point M and having slope 3 is:

A $3x-y+4=0$; B $3x-y-4=0$; C $3x+y+4=0$; D $-3x+y-4=0$.

5. If the measure of the angle A is between $540°$ and $720°$, and $\cos A=-\dfrac{7}{25}$, then which of the following statements are true?

A $\sin\dfrac{A}{2}=-\dfrac{4}{5}$; B $\cos\dfrac{A}{2}=\dfrac{3}{5}$; C $\sin\dfrac{A}{2}=\dfrac{4}{5}$; D $\cos\dfrac{A}{2}=-\dfrac{3}{5}$.

6. Consider the vectors $\overrightarrow{u}=\overrightarrow{i}-(a+2)\overrightarrow{j}$ and $\overrightarrow{v}=a\overrightarrow{i}-3\overrightarrow{j}$, where $\overrightarrow{i}$ and $\overrightarrow{j}$ are the versors of the coordinate axes Ox and Oy, respectively, in the Cartesian coordinate system xOy. If the vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ are colinear, then the value of the parameter $a\in\mathbb{R}$ can be:

A $a=-3$; B $a=1$; C $a=-1$; D $a=3$.

7. The sum of the coefficients of the odd powers of x in the binomial expansion $(1+x)^{1011}$ is:

A 2^{2022}; B 2^{505}; C 2^{1010}; D 2^{1011}.

8. If $1+5+9+\ldots+x=496$, where the terms on the left hand side are in an arithmetic progression, then:

A $x=21$; B $x=41$; C $x=61$; D $x=81$.

9. The complex number $(1-i)^{2022}$ is equal to:

A 2^{1011}; B -2^{1011}; C $-2^{1011}i$; D $2^{1011}i$.

10. The value of the integral $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin^2 x + 1}}\,\mathrm{d}x$ is

A 0; B $\dfrac{\pi}{2}$; C $\ln\left(1+\sqrt{2}\right)$; D $1+\sqrt{2}$.

11. The function $f : [0, \infty) \to \mathbb{R}$, defined by $f(x) = x + \sqrt{x^2 + 2x}$, has as asymptote to $+\infty$ the line of equation

A $y = -1$; B $y = 2x + 1$; C $y = 0$; D $y = 2x + 2$.

12. The value of the limit $\displaystyle\lim_{n\to\infty} \frac{1}{n^2}\left(\mathrm{e}^{\frac{1}{n}} + 2\mathrm{e}^{\frac{2}{n}} + \ldots + n\mathrm{e}^{\frac{n}{n}}\right)$ is

A 1; B e; C $\dfrac{1}{\mathrm{e}}$; D $\mathrm{e} - 1$.

13. Let $ABCD$ be a parallelogram. Consider the points E and F such that $\overrightarrow{AE} = \frac{3}{2}\overrightarrow{AB}$ and $\overrightarrow{AF} = 3\,\overrightarrow{AD}$. Which of the following statements are true?

A $\overrightarrow{FE} = 3\,\overrightarrow{CE}$; B $\overrightarrow{FE} = 2\,\overrightarrow{CE}$; C $\overrightarrow{FC} = 2\,\overrightarrow{CE}$; D $\overrightarrow{FC} = \dfrac{3}{2}\overrightarrow{CE}$.

14. Let ABC be an arbitrary triangle with $a > b > c$, where $BC = a$, $CA = b$, $AB = c$. Which of the following statements are true?

A $\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{a-b}{a+b}$; B $\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{a^2-b^2}{a^2+b^2}$;

C $\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{a^2-b^2}{c^2}$; D $\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{a-b}{c}$.

15. In a triangle ABC the measures of the angles $\widehat{A}, \widehat{B}$ and $\widehat{C}$ (in this order) are in an arithmetic progression, and, if a, b, c denote the lengths of the sides opposite to these angles, we have $3a^2 = 2b^2$. Which of the following statements are true?

A $\widehat{A} = 45°$; B $\widehat{C} = 75°$; C $\widehat{C} = 45°$; D $\widehat{A} = 60°$.

16. Consider in $\mathbb{R}$ the equation $2\sqrt[3]{(x^2+a)^2} - 3\sqrt[3]{x^2+a} - 2 = 0$, where $a \in \mathbb{R}$. Which of the following statements are true?

A If $a = 8$, then $x = 0$ is a solution of the equation;

B The equation has solutions for every $a \leq -\frac{1}{8}$;

C The equation has solutions for every $a \geq -8$;

D The equation has solutions for every $a \leq 8$.

17. Consider the system:

$$\begin{cases} 3x + 2y - z = 1 \\ x + ay + z = 2 \\ -4x + y = 3, \end{cases}$$

where $a \in \mathbb{R}$. Which of the following statements are true?

A The system is determinate compatible for any $a > 0$;

B When the system is determinate compatible, its solution does not depend on a;

C There exists a value a for which the system is indeterminate compatible;

D There exists a value a for which the system is incompatible.

18. A group of 11 children want to play football. To this end, the children choose a referee amongst them and then the others form two teams called X and Y, each having 5 players. In how many ways can they do that?

A 462; B 2310; C 2772; D 5082.

19. Let $m \in \mathbb{R}$ be a parameter and $f\colon (0,\infty) \to \mathbb{R}$ be the function defined by $f(x) = x^2 - 2\ln x + m$. Which of the following statements are true?

A The function f has exactly one point of global maximum.

B If $m = -2022$, then the equation $f(x) = 0$ has exactly two real solutions.

C There exists $m \in \mathbb{R}$ such that f is injective.

D There exists a minimum $m \in \mathbb{R}$ with the property that $f(x) \geq 0$ for any $x \in (0,\infty)$.

20. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = ax^3 + bx + c$, where $a, b, c \in \mathbb{R}$. The variation table of f is given below.

x	$-\infty$			-1			1			$+\infty$
$f(x)$	$-\infty$	$\nearrow$	$\nearrow$	10	$\searrow$	$\searrow$	2	$\nearrow$	$\nearrow$	$+\infty$

Then the value of the sum $|a| + |b| + |c|$ is

A 11; B 13; C 20; D 14.

21. The value of the limit $\lim\limits_{x \searrow 0} \dfrac{\int_0^{x^2} \ln\left(1 - \sqrt{t}\right) \mathrm{d}t}{x^3}$ is

A 0; B $-\dfrac{1}{3}$; C $-\dfrac{2}{3}$; D $-\infty$.

22. Let $ABCDEF$ be a regular hexagon, and let a and b be two real numbers so that $\overrightarrow{AD} = a\overrightarrow{BE} + b\overrightarrow{CF}$. Then the number $b - 2a$ is equal to:

A -3; B 3; C -1; D 1.

23. Consider the triangle ABC with vertices $A(1,3)$, $B(-1,-5)$ and $C(2,1)$. Let D be a point on BC so that $\dfrac{BD}{DC} = 2$. Denote by d the distance from the point D to the altitude from the vertex B of the triangle ABC. Which of the following statements are true?

A The coordinates of the point D are $(1,-1)$;

B The coordinates of the point D are $(0,-3)$;

C $d = \dfrac{6\sqrt{5}}{5}$;

D $d = \dfrac{3\sqrt{5}}{5}$.

24. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{vmatrix} x & 1 & 4 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix}, \quad \forall x \in \mathbb{R}.$$

Knowing that $x = 2$ is a solution of the equation $f(x) = 0$, which of the following numbers are also solutions of this equation?

A $-1 - \sqrt{11}$; B $-1 + \sqrt{11}$; C $1 - \sqrt{13}$; D $1 + \sqrt{13}$.

25. Consider in $\mathbb{R}$ the equation

$$\left[\frac{x+2}{3}\right] = \frac{x+1}{4},$$

where $[a]$ represents the integer part of the real number a. If S denotes the set of solutions of this equation, which of the following statements are true?

$\boxed{\text{A}}$ $S = [-9, 3]$; $\boxed{\text{B}}$ $S = \{-9, -5, -1, 3\}$;
$\boxed{\text{C}}$ $S = [-5, 3]$; $\boxed{\text{D}}$ $S = \{-5, -1, 3\}$.

26. On the set $\mathbb{Z}$ of integer numbers consider the composition law $x * y = xy - x - y + 2$.
Knowing that the law is associative, which of the following statements are true?

$\boxed{\text{A}}$ $(1 * 2) * 4 = 8$;

$\boxed{\text{B}}$ The operation has identity element;

$\boxed{\text{C}}$ There exist exactly two invertible elements in $(\mathbb{Z}, *)$;

$\boxed{\text{D}}$ $(\mathbb{Z}, *)$ is a group.

27. On the set $\mathbb{R}$ of real numbers consider the operations: $x \perp y = x + y - 1$, $x * y = x + y - xy$. Knowing that $(\mathbb{R}, \perp, *)$ is a field and that the function $f : (\mathbb{R}, +, \cdot) \to (\mathbb{R}, \perp, *)$, $f(x) = ax + b$, is an isomorphism of fields, where $a, b \in \mathbb{R}$, which of the following statements are true?

$\boxed{\text{A}}$ $a = b = 1$; $\boxed{\text{B}}$ $a = -1,\ b = 1$; $\boxed{\text{C}}$ $a = 1,\ b = -1$; $\boxed{\text{D}}$ $a = 0,\ b = 1$.

28. For each $n \in \mathbb{N}^*$, let $I_n = \displaystyle\int_0^1 x^n e^{-x} dx$. Which of the following statements are true?

$\boxed{\text{A}}$ The sequence $(I_n)_{n\in\mathbb{N}^*}$ is monotone; $\boxed{\text{B}}$ $\lim\limits_{n\to\infty} I_n = 0$; $\boxed{\text{C}}$ $\lim\limits_{n\to\infty} nI_n = 0$; $\boxed{\text{D}}$ $\lim\limits_{n\to\infty} \dfrac{I_n}{I_{n+1}} = 1$.

29. In a parallelogram the lengths of the sides are 5 and 3, respectively, while the product of the lengths of the diagonals is 32. Denote by α the measure of the acute angle of the parallelogram. Which of the following statements are true?

$\boxed{\text{A}}$ The sum of the squares of the diagonals is 68. $\boxed{\text{B}}$ $\cos\alpha = \dfrac{\sqrt{33}}{15}$.

$\boxed{\text{C}}$ The sum of the squares of the diagonals is 34. $\boxed{\text{D}}$ $\cos\alpha = \dfrac{\sqrt{33}}{20}$.

30. Consider the line (d) $ax + by + c = 0$ $(abc \neq 0)$ and the points

$$M_1\left(\frac{b-c}{a}, 0\right), \quad M_2\left(-\frac{b+c}{a}, 0\right), \quad N\left(0, -\frac{c}{b}\right).$$

Which of the following statements are true?

$\boxed{\text{A}}$ The point of intersection of d with the Ox axis is the midpoint of the segment $[M_1M_2]$;

$\boxed{\text{B}}$ The line d is parallel to the Ox axis;

$\boxed{\text{C}}$ The point of intersection of d with the Oy axis is N;

$\boxed{\text{D}}$ The area of the triangle M_1M_2N is $\left|\dfrac{c}{a}\right|$.