BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Admissions Exam 2022
Written test in MATHEMATICS

IMPORTANT NOTE: Problems can have one or more correct answers, which candidates should indicate on their exam sheet. The grading system of the multiple choice exam can be found in the set of rules of the competition.

1. The value of the limit $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+n-2}{2n^2+3n+1}\right)^{n+2}$ is

A e; B $e-1$; C $\dfrac{1}{e}$; D e^{-2}.

2. The value of the limit $\lim\limits_{x\to 0}\dfrac{e^{x^2}-\cos x}{x^2}$ is

A 0; B $\dfrac{1}{2}$; C 1; D $\dfrac{3}{2}$.

3. Let $f:\mathbb{R}\to\mathbb{R}$ be the function defined by $f(x)=(x+1)e^x$ and consider the point $P(0,f(0))$. Denote by d the line tangent to the graph of f in P. Which of the following statements are true?

A d has equation $y=2x+1$; B d contains the point $Q(3,7)$;

C d has equation $y=2x-1$; D d contains the point $S(-3,-4)$.

4. The trapezoid $ABCD$ has $m(\widehat{A})=m(\widehat{D})=90^\circ$, $AB\parallel CD$ and AC perpendicular to BD. Which of the following statements are true?

A $\overrightarrow{AB}\cdot\overrightarrow{AD}=0$; B $\overrightarrow{AB}\cdot\overrightarrow{BC}=0$; C $\overrightarrow{AD}\cdot\overrightarrow{DC}=0$; D $\overrightarrow{AC}\cdot\overrightarrow{BD}=0$.

5. In the Cartesian coordinate system xOy consider the point $M(1,1)$. The equation of the line passing through the point M and having slope 2 is:

A $2x-y-1=0$; B $2x+y-1=0$; C $2x+y+1=0$; D $-2x+y-1=0$.

6. If the measure of the angle A is between 450° and 540°, and $\cos A=-\dfrac{7}{25}$, then which of the following statements are true?

A $\sin\dfrac{A}{2}=-\dfrac{4}{5}$; B $\cos\dfrac{A}{2}=-\dfrac{3}{5}$; C $\sin\dfrac{A}{2}=-\dfrac{3}{5}$; D $\cos\dfrac{A}{2}=-\dfrac{4}{5}$.

7. The imaginary part of the number $(1+i)^{2022}$ is equal to :

A 0; B 2^{1011}; C -2^{1011}; D 2^{2022}.

8. The set of real solutions of the inequality

$$\log_9(5x+3)>\frac{1}{2}\log_3(x-1)$$

is:

A $(-1,+\infty)$; B $(-\frac{3}{5},+\infty)$; C $(1,+\infty)$; D $(2,+\infty)$.

9. The sum of the first 8 terms of an arithmetic progression is 64, while the sum of the first 19 terms is 361. Which of the following statements are true?

A the ratio is 2; B the first term is 1;
C the ratio is 4; D the first term is -8.

10. The value of the integral $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1+\cos^2 x}\,\mathrm{d}x$ is

A $\frac{\pi}{4}$; B $\frac{\pi}{2}$; C $\ln 2$; D $\ln\left(1+\sqrt{2}\right)$.

11. The function $f : [0,\infty) \to \mathbb{R}$, defined by $f(x) = x - \sqrt{x^2+2x}$, has as asymptote to $+\infty$ the line of equation

A $y = -2$; B $y = 2x - 1$; C $y = -1$; D $y = 2x + 1$.

12. The value of the limit $\lim\limits_{n\to\infty} \frac{1}{n^3}\left(\mathrm{e}^{\frac{1}{n}} + 2^2\mathrm{e}^{\frac{2}{n}} + \ldots + n^2\mathrm{e}^{\frac{n}{n}}\right)$ is

A 1; B $\mathrm{e}-2$; C e^2; D $\mathrm{e}-1$.

13. Consider the vectors $\vec{u} = a\vec{i} + \vec{j}$ and $\vec{v} = 2\vec{i} + (a+1)\vec{j}$, where $\vec{i}$ and $\vec{j}$ are the versors of the coordinate axes Ox and Oy, respectively, in the Cartesian coordinate system xOy. If the vectors $\vec{u}$ and $\vec{v}$ are colinear, then the value of the parameter $a \in \mathbb{R}$ can be:
A $a = -2$; B $a = 1$; C $a = -1$; D $a = 2$.

14. Which of the following relations are true?

A $\frac{1}{\sin^2 15^\circ} + \frac{1}{\cos^2 15^\circ} = 8\sqrt{3}$; B $\frac{1}{\sin^2 15^\circ} - \frac{1}{\cos^2 15^\circ} = 8\sqrt{3}$;
C $\frac{1}{\sin^2 15^\circ} + \frac{1}{\cos^2 15^\circ} = 16$; D $\frac{1}{\sin^2 15^\circ} - \frac{1}{\cos^2 15^\circ} = 16$.

15. If in the acute triangle ABC we have $BC = 2AC\sin\frac{A}{2}$, then:

A $AB = \frac{AC}{2}$; B $AB = AC$; C $AB = \sqrt{2}AC$; D $AB = 2AC$.

16. The sum of the real solutions of the equation $\sqrt{x^2+x+3} + \sqrt{x^2+x-1} = 2$ is

A 0; B 1; C 2; D -1.

17. Consider the system:

$$\begin{cases} 2x + y - 3z = 4 \\ x \qquad - z = 5 \\ -3x - y + az = -9 \end{cases}$$

where $a \in \mathbb{R}$. Which of the following statements are true?

A The system is determinate compatible for any $a < 0$;

B When the system is determinate compatible, its solution depends on a;

C There exists a value a for which the system is indeterminate compatible;

D There exists a value a for which the system is incompatible.

18. In how many ways can 5 persons be seated in a car with 7 seats, if only 3 of them have a driver's license and one person with a driver's license is seating at the wheel?

$\boxed{\text{A}}$ 120; $\boxed{\text{B}}$ 1080; $\boxed{\text{C}}$ 2520; $\boxed{\text{D}}$ 5040.

19. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = |x|(e^x - 1)$. Which of the following statements are true?

$\boxed{\text{A}}$ The function f is not differentiable in 0;

$\boxed{\text{B}}$ The function f is continuous in 0;

$\boxed{\text{C}}$ The function f is injective;

$\boxed{\text{D}}$ The function f is surjective.

20. Let $f : \mathbb{R} \setminus \{1\} \to \mathbb{R}$ be the function defined by $f(x) = ax + b + \dfrac{c}{x-1}$, where $a, b, c \in \mathbb{R}$. The variation table of f is given below.

x	$-\infty$			0			1			2			$+\infty$
$f(x)$	$-\infty$	$\nearrow$	$\nearrow$	-5	$\searrow$	$\searrow$	$-\infty \mid +\infty$	$\searrow$	$\searrow$	7	$\nearrow$	$\nearrow$	$+\infty$

Then the value of the sum $|a| + |b| + |c|$ is

$\boxed{\text{A}}$ 7; $\boxed{\text{B}}$ 5; $\boxed{\text{C}}$ 10; $\boxed{\text{D}}$ 8.

21. The value of the limit $\lim\limits_{x \searrow 0} \dfrac{\int_0^{x^2} \sin\sqrt{t}\,\mathrm{d}t}{x^3}$ is

$\boxed{\text{A}}$ 0; $\boxed{\text{B}}$ $\dfrac{1}{3}$; $\boxed{\text{C}}$ $\dfrac{2}{3}$; $\boxed{\text{D}}$ $+\infty$.

22. In the Cartesian coordinate system xOy consider the points $A(4, 4)$, $B(7, 0)$ and $C(-1, -8)$. Let D be the point where the interior bisector of the angle A intersects the opposite side in the triangle ABC. Then the sum of the $x-$ and $y-$ coordinates of the point D is:

$\boxed{\text{A}}$ $\dfrac{36}{17}$; $\boxed{\text{B}}$ 2; $\boxed{\text{C}}$ $\dfrac{23}{9}$; $\boxed{\text{D}}$ $\dfrac{32}{19}$.

23. In the parallelogram $ABCD$ denote by $AB = a$, $AD = b$, $BD = d_1$, $AC = d_2$, $m\left(\widehat{DAB}\right) = \alpha$. If $\alpha \neq 90°$, then which of the following statements are true?

$\boxed{\text{A}}$ $d_1^2 = a^2 + b^2 - 2ab\cos\alpha$; $\boxed{\text{B}}$ $d_2^2 = a^2 + b^2 - 2ab\cos\alpha$;

$\boxed{\text{C}}$ $d_1^2 + d_2^2 = a^2 + b^2$; $\boxed{\text{D}}$ $d_1^2 + d_2^2 = 2(a^2 + b^2)$.

24. Let $x_n = \cos\left(\dfrac{\pi}{4}\right) \cdot \cos\left(\dfrac{\pi}{8}\right) \cdot \ldots \cdot \cos\left(\dfrac{\pi}{2^{n+1}}\right)$ for each $n \in \mathbb{N}, n \geq 2$. Which of the following statements are true?

$\boxed{\text{A}}$ $x_{n+1} < x_n$ for each $n \in \mathbb{N}, n \geq 2$; $\boxed{\text{B}}$ $\frac{\sqrt{2}}{2} < x_n < 1$ for any $n \in \mathbb{N}, n \geq 2$;

$\boxed{\text{C}}$ $x_2 = \sqrt{1 + \frac{\sqrt{2}}{2}}$; $\boxed{\text{D}}$ $2^n x_n \sin\left(\dfrac{\pi}{2^{n+1}}\right) = 1$ for each $n \in \mathbb{N}, n \geq 2$.

25. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{vmatrix} x & 1 & 3 \\ -2 & x & 2 \\ 2 & 2 & x \end{vmatrix}.$$

Knowing that $x = -2$ is a solution of the equation $f(x) = 0$, which of the following numbers are also solutions of this equation?

$\boxed{\text{A}}$ $1 - \sqrt{5}$; $\boxed{\text{B}}$ $1 + \sqrt{5}$; $\boxed{\text{C}}$ $1 - \sqrt{7}$; $\boxed{\text{D}}$ $1 + \sqrt{7}$.

26. Consider in $\mathbb{R}$ the equation

$$\left[\frac{x+1}{2}\right] = \frac{x+1}{3},$$

where $[a]$ represents the integer part of the real number a. If S denotes the set of solutions of this equation, which of the following statements are true?

$\boxed{\text{A}}$ $S = \left(-3, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, 3\right)$; $\boxed{\text{B}}$ $S = \{-1, 2\}$;
$\boxed{\text{C}}$ $S = \left[-2, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, 2\right]$; $\boxed{\text{D}}$ $S = \{-4, -1, 2\}$.

27. On the set $\mathbb{R}$ of real numbers consider the composition law $x * y = xy - 2x - 2y + 6$.
Knowing that the law is associative, which of the following statements are true?

$\boxed{\text{A}}$ $1 * (2 * 3) = 2$;

$\boxed{\text{B}}$ The subset $[0, +\infty)$ is closed with respect to $*$;

$\boxed{\text{C}}$ There exists $a \in \mathbb{R}$ such that $a * x = a$, for any $x \in \mathbb{R}$;

$\boxed{\text{D}}$ $(\mathbb{R}, *)$ is a group.

28. Consider the ring $(R, +, \cdot)$, where $R = \{a + bi\sqrt{2} \mid a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$. Which of the following statements are true?

$\boxed{\text{A}}$ $(R, +, \cdot)$ has at least 3 invertible elements;

$\boxed{\text{B}}$ The sum of the invertible elements of $(R, +, \cdot)$ is 0;

$\boxed{\text{C}}$ $(R, +, \cdot)$ has at least one invertible element which is not a real number;

$\boxed{\text{D}}$ $(R, +, \cdot)$ is a field.

29. For each $n \in \mathbb{N}^*$, let $I_n = \displaystyle\int_0^1 x^n \mathrm{e}^x \mathrm{d}x$. Which of the following statements are true?

$\boxed{\text{A}}$ $\lim\limits_{n\to\infty} I_n = 1$; $\boxed{\text{B}}$ $\lim\limits_{n\to\infty} I_n = 0$; $\boxed{\text{C}}$ $\lim\limits_{n\to\infty} nI_n = \infty$; $\boxed{\text{D}}$ $\lim\limits_{n\to\infty} nI_n > 2$.

30. Consider the right triangle ABC $(m(\widehat{C}) = 90^\circ)$ and let a, b and c denote the lengths of the sides and the hypotenuse of this triangle. Consider, in addition, the points $E(-1, 0)$, $F(1, 0)$, $M\left(\dfrac{b-c}{a}, 0\right)$ and the line (d) $ax + by + c = 0$. Denote by $\mathrm{dist}(X, d)$ the distance from an arbitrary point X to the line d. Which of the following statements are true?

$\boxed{\text{A}}$ The points $E(-1, 0)$, $F(1, 0)$ belong to the line d; $\boxed{\text{B}}$ $\mathrm{dist}(M, d) = \dfrac{b}{c}$;

$\boxed{\text{C}}$ The point $M\left(\dfrac{b-c}{a}, 0\right)$ belongs to the line d. $\boxed{\text{D}}$ $\mathrm{dist}(E, d) \cdot \mathrm{dist}(F, d) = \dfrac{b^2}{c^2}$.