BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

ADMISSIONS EXAM 2021
Written test in MATHEMATICS
Version 1.

1. In a Cartesian coordinate system xOy consider the points $A(-1,1), B(1,3), C(3,2)$. The equation of the line OG, where G is the gravity center of the triangle ABC is:

A $y=-2x$; B $y=-\frac{x}{2}$; C $y=2x$; D $y=\frac{x}{2}$.

2. Relative to a Cartesian coordinate system consider the vector $\vec{v}(t,t^2)$ with $t \in \mathbb{R} \setminus \{0\}$. Indicate which of the following statements are true.

A For $t=2$ the vector $\vec{v}$ is perpendicular to the vector $\vec{a}(-1,\frac{1}{2})$.
B There exists t such that $\vec{v}$ is collinear to the vector $\vec{b}(17,19)$.
C There exists t such that $\vec{v}$ is collinear to the vector $\vec{c}(-1,-1)$.
D There exists t such that $\vec{v}$ is collinear to the vector $\vec{d}(0,1)$.

3. If $(-4,0)$ and $(1,-1)$ are two vertices of a triangle with area 4, then the third vertex may belong to the line:

A $x+5y=0$; B $x+5y+8=0$; C $x+5y-4=0$; D $x+5y+12=0$.

4. If the line of equation $ax+cy-2b=0$, $a,b,c>0$ forms a triangle with area 2 with the coordinate axes, then:

A a,b,c are in geometric progression; B $a,-b,c$ are in geometric progression;
C $a,2b,c$ are in geometric progression; D $a,-2b,c$ are in geometric progression.

5. Consider the function

$$f\colon \mathbb{R}\to\mathbb{R},\ f(x)=\begin{cases}\left|\frac{1}{2}x-3\right|, & \text{if } x\in(-\infty,-2]\\ x+3, & \text{if } x\in(-2,1)\\ 3-2x, & \text{if } x\in[1,\infty).\end{cases}$$

Possibly using the graph of the function, indicate which of the following statements are true.

A f is surjective, but not injective.
B f is bijective.
C f is injective, but not surjective.
D f is neither surjective, nor injective.

6. Consider the family of quadratic functions $f_m:\mathbb{R}\to\mathbb{R}$, $f_m(x)=mx^2-(2m+1)x+m+1, \forall x\in\mathbb{R}$, with $m\in\mathbb{R}\setminus\{0\}$. The value of $m\in\mathbb{R}\setminus\{0\}$ for which the vertex of the parabola associated with the function f_m lies on the line of equation $2x+3y+6=0$ is:

A $\frac{1}{16}$; B $-\frac{1}{32}$; C $-\frac{1}{24}$; D $-\frac{5}{32}$.

7. Consider the system of equations

$$\begin{cases} ax + y + z &= 1 \\ x + ay + z &= 2 \\ x + y + z &= 4, \end{cases}$$

where a is a real parameter. Which of the following statements are true?

- A The determinant of the system does not depend on the parameter a.
- B For $a < 0$ the system is determinate compatible.
- C For $a = 1$ the system is indeterminate compatible.
- D For $a = 1$ the system is incompatible.

8. Let $A = \begin{pmatrix} 1 & a \\ a & 1 \end{pmatrix}$, with $a \in \mathbb{R}$. Which of the following statements are true?

- A There exists $a \in \mathbb{R}$ such that $A^2 = O_2$.
- B There exists $a \in \mathbb{R}$ such that $A^2 = I_2$.
- C There exists $a \in \mathbb{R}$ such that $A^2 = A$.
- D There exist $a, b \in \mathbb{R}$ such that $A^2 = \begin{pmatrix} b & b \\ b & b \end{pmatrix}$.

9. Let $(x_n)_{n\geq 1}$ be the sequence defined by $x_n = \dfrac{3^n}{(n+1)!}$. Which of the following statements are true?

- A The sequence $(x_n)_{n\geq 1}$ is decreasing.
- B $0 < x_n < 1$ for any $n \geq 1$.
- C $\lim\limits_{n\to\infty} \dfrac{x_{n+1}}{x_n} = 0$.
- D $\lim\limits_{n\to\infty} x_n = 0$.

10. Given $a \in \mathbb{R} \setminus \{0\}$, the value of the limit $\lim\limits_{x\to\infty} \left(\dfrac{x+a}{x-a}\right)^x$ is

- A e^{-2a};
- B 1;
- C e^{2a};
- D ∞.

11. Let $L = \lim\limits_{a\to\infty} \displaystyle\int_0^a x\mathrm{e}^{-x}\mathrm{d}x$. Indicate which of the following statements are true.

- A $L = \infty$.
- B $L = 1$.
- C $L < \mathrm{e}$.
- D L does not exist.

12. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} x^4 + ax + 1, & \text{if } x < 1 \\ b + \ln x, & \text{if } x \geq 1. \end{cases}$$

The values of the real parameters a and b for which f is differentiable on $\mathbb{R}$ are:

- A $a = 3,\ b = 1$;
- B $a = -3,\ b = 1$;
- C $a = -3,\ b = -1$;
- D $a = 1,\ b = 3$.

13. If r and R are the radii of the inscribed and circumscribed circles, respectively, to a triangle whose sides have lengths 3, 4 and 5, then the ratio $\dfrac{r}{R}$ is:

- A $\dfrac{2}{5}$;
- B $\dfrac{5}{2}$;
- C $\dfrac{4}{5}$;
- D $\dfrac{1}{5}$.

14. Consider the triangle ABC and the points M, N, P such that M is the midpoint of AB, $\overrightarrow{AP} = 2\overrightarrow{AC}$ and $\overrightarrow{BN} = k\overrightarrow{BC}$. The value of the real parameter k, for which $\overrightarrow{MP} = 3\overrightarrow{MN}$ is

- A $\dfrac{3}{2}$;
- B $\dfrac{1}{3}$;
- C $\dfrac{1}{2}$;
- D $\dfrac{2}{3}$.

15. Consider in $\mathbb{R}$ the equation

$$\log_3 \sqrt{3+x} + \log_9(3-x) = \frac{1}{2}.$$

The set of its solutions is:

[A] $S = \{0\}$; [B] $S = \{-\sqrt{3}, \sqrt{3}\}$; [C] $S = \{-\sqrt{6}, \sqrt{6}\}$; [D] $S = \{-\sqrt{12}, \sqrt{12}\}$.

16. The product of the real solutions of the equation $x^2 + x + 4 = 2\sqrt{x^2 + x + 7}$ is:

[A] 12; [B] -12; [C] 2; [D] -2.

17. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x + |x^3 - x^2| + \max\{x^3, x^4\}$. Indicate which of the following statements are true.

[A] $f(x) = x^4 - x^3 + x^2 + x$ for any $x \in (-\infty, 0)$.
[B] $f(x) = x^2 - x$ for any $x \in [0, 1]$.
[C] The function f is not differentiable at 0.
[D] The tangent to the graph of f at the point $O(0,0)$ is the first bisector.

18. Consider the set

$$A := \{a \in \mathbb{R} \mid \text{function } f\colon [a, \infty) \to \mathbb{R},\ f(x) = x^4 - 10x^2 + 2021 \text{ is strictly increasing}\}.$$

Indicate which of the following statements are true.

[A] $A = \emptyset$. [B] $[3, \infty) \subseteq A$. [C] The set A has a smallest element. [D] $2 \notin A$.

19. The set of values of the real parameter a for which the equation $x^2(1 - \ln x) = a$ has two distinct real solutions is:

[A] $(\sqrt{\mathrm{e}}, \mathrm{e})$; [B] $\left(-\infty, \frac{\mathrm{e}}{2}\right)$; [C] $\left(0, \frac{\mathrm{e}}{2}\right)$; [D] $\left[0, \frac{\mathrm{e}}{2}\right]$.

20. If $\cos x = -\dfrac{7}{25}$ and $x \in \left(\dfrac{5\pi}{2}, 3\pi\right)$, then:

[A] $\cos\frac{x}{2} = \frac{3}{5}$; [B] $\cos\frac{x}{2} = \frac{4}{5}$; [C] $\cos\frac{x}{2} = -\frac{3}{5}$; [D] $\cos\frac{x}{2} = -\frac{4}{5}$.

21. With the usual notations in a triangle ABC, let $a = 13$, $b = 1$ and $\operatorname{tg}\frac{C}{2} = \frac{2}{3}$. Indicate which of the following statements is true.

[A] $c = 4\sqrt{10}$. [B] $c = 6\sqrt{5}$. [C] $\sin C = \dfrac{12}{13}$. [D] $\mathcal{A}rea\,(ABC) = 6$.

22. If a is a parameter and the equation $\cos 2x + a \sin x - 2a + 7 = 0$ has solutions, then:

[A] $0 < a \leq 5$;
[B] $2 \leq a \leq 6$;
[C] for $a = 5$ the set of solutions is $S = \left\{(-1)^k \frac{\pi}{6} + k\pi | k \in \mathbb{Z}\right\}$;
[D] for $a = 5$ the set of solutions is $S = \left\{(-1)^k \frac{\pi}{3} + k\pi | k \in \mathbb{Z}\right\}$.

23. Let $\alpha \neq 1$ be a root of the equation $z^3 = 1$. Indicate which of the following statements are true.

[A] $|\alpha| = 1$. [B] $1 + \dfrac{1}{\alpha} + \dfrac{1}{\alpha^2} \notin \mathbb{R}$.
[C] $\alpha^{2021} = -\alpha - 1$. [D] the number $-\alpha$ is a root of the equation $z^2 - z + 1 = 0$.

24. In the ring $(\mathbb{Z}_{12}, +, \cdot)$ the equation $x^2 + \widehat{4}x + \widehat{3} = \widehat{0}$

A has no solutions;

B does not have a unique solution;

C has exactly two distinct solutions;

D has exactly four distinct solutions.

25. Consider the expression

$$x * y = \frac{xy - 2}{x + y - 4}.$$

Which of the following statements are true?

A $*$ is a composition law on $\mathbb{R}$.

B $*$ is a composition law on $(2, +\infty)$.

C $3 * (3 * 3) = \frac{18}{5}$.

D $x * 4 = x$, for any $x > 3$.

26. The value of the integral $\displaystyle\int_{-\pi/3}^{\pi/3} \sqrt{(1 - \cos x)(1 - \cos 2x)}\, \mathrm{d}x$ is:

A 0; B $\dfrac{4}{3}$; C $\dfrac{2}{3}$; D $\dfrac{4\sqrt{2}}{3}$.

27. Let $f\colon (-1, \infty) \to \mathbb{R}$ be the function defined by $f(x) = \mathrm{e}^x - 1 - \ln(1 + x)$ and let a, $b \in (-1, \infty)$ such that $a < b$. Indicate which of the following statements are true.

A Function f has a single global minimum point.

B Function f is injective.

C $\displaystyle\int_a^b (1 + \ln(1 + x))\mathrm{d}x < \int_a^b \mathrm{e}^x \mathrm{d}x$.

D Function f has at least one global maximum point.

28. In the xOy coordinate system consider the points $A(-6, 2)$, $B(4, -3)$, $M(\alpha, 0)$ and $N(0, \beta)$. If the sum $AM + MB + BN + NA$ is minimum, then:

A $MN = 0$; B $MN = 1$; C $MN = \sqrt{2}$; D $MN = \sqrt{5}$.

29. Let $A = \{1, 2, \ldots, 99, 100\}$. How many sums equal to 5044 can be formed with the elements of the set A (sums that do not contain repetitive elements)?

A 3; B 4; C 5; D 6.

30. For each $n \in \mathbb{N}^*$, denote $I_n = \displaystyle\int_0^1 \frac{x^{2n}}{1 + x^2}\, \mathrm{d}x$. Indicate which of the following statements are true.

A $I_n + I_{n+1} = \dfrac{1}{2n + 1}, \quad \forall n \in \mathbb{N}^*$. B $\displaystyle\lim_{n\to\infty} I_n = 0$. C $\displaystyle\lim_{n\to\infty} nI_n = \frac{1}{2}$. D $\displaystyle\lim_{n\to\infty} nI_n = \frac{1}{4}$.

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Correct Answers

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1. C
2. A, B, C
3. C, D
4. A, B
5. B
6. B
7. B, D
8. B, C, D
9. A, C, D
10. C
11. B, C
12. C
13. A
14. D
15. C
16. D
17. A, D
18. B, C, D
19. C
20. C
21. A, C, D
22. B, C
23. A, C, D
24. B, D
25. B
26. C
27. A, C
28. D
29. B
30. A, B, D