BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

ADMISSIONS EXAM 2021
September Session
Written test in MATHEMATICS

1. Consider the equation $x^2 + ax + 3 = 0$ with positive solutions x_1 and x_2 such that $x_1^2, x_2, 1$ are in geometric progression (in this order). Then the value of $a \in \mathbb{R}$ can be

A $2\sqrt{3}$; **B** $-2\sqrt{3}$; **C** $\sqrt{3}$; **D** $-\sqrt{3}$.

2. Let $(x_n)_{n\geq 1}$ be the sequence defined by $x_n = \dfrac{1}{4^n} C_{2n}^n$. Indicate which of the following statements are true.

A The sequence $(x_n)_{n\geq 1}$ is strictly increasing.
B The sequence $(x_n)_{n\geq 1}$ is strictly decreasing.
C The sequence $(x_n)_{n\geq 1}$ is bounded.
D The sequence $(x_n)_{n\geq 1}$ is convergent.

3. The value of the limit $\lim\limits_{x\to\infty} \left(\dfrac{x-\sqrt{x}}{x+\sqrt{x}}\right)^{\sqrt{x}}$ is

A 1; B e^2; C $\dfrac{1}{e^2}$; D 0.

4. The value of the limit $\lim\limits_{x\to 0} \dfrac{x \sin x}{e^x + e^{-x} - 2}$ is

A 1; B 2; C $\dfrac{1}{2}$; D 0.

5. Consider the vectors $\vec{u} = a\vec{i} + 3\vec{j}$ and $\vec{v} = 2\vec{i} + b\vec{j}$, where the unit vectors $\vec{i}$ and $\vec{j}$ are perpendicular. Indicate which of the following statements are true.

A For $a = -1$ and $b = -6$ the vectors $\vec{u}$ and $\vec{v}$ are colinear.
B Vectors $\vec{u}$ and $\vec{v}$ have the same length only if $a = 2$ and $b = 3$.
C There exist infinitely many real numbers a, b for which the vectors $\vec{u}$ and $\vec{v}$ have the same length.
D Vectors $\vec{u}$ and $\vec{v}$ are perpendicular, if $2a = 3b$.

6. Let $ABCDEF$ be a regular hexagon. Denote $\overrightarrow{AB} = \overrightarrow{u}$ and $\overrightarrow{AE} = \overrightarrow{v}$. Indicate which of the following statements are true.

A $\overrightarrow{AC} = \frac{3}{2}\overrightarrow{u} + \overrightarrow{v}$. B $\overrightarrow{AC} = \frac{3}{2}\overrightarrow{u} + \frac{1}{2}\overrightarrow{v}$. C $\overrightarrow{AC} = \frac{1}{2}\overrightarrow{u} + \frac{3}{2}\overrightarrow{v}$. D $\overrightarrow{AC} = \frac{3}{2}\overrightarrow{u} - \frac{1}{2}\overrightarrow{v}$.

7. If S is the set of solutions of equation $\dfrac{2\lg(x+3)}{\lg(5x+11)} = 1$, then:

A $S \subseteq [-2, 0]$; B $S \subseteq [-2, 1]$; C $S \subseteq [0, 1]$; D $S \subseteq [-2, -1]$.

8. Consider in $\mathbb{R}$ the equation

$$\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1.$$

The set of its solutions is:

A $S = [3, 12]$; B $S = [1, \infty)$; C $S = [5, 10]$; D $S = \{4, 11\}$.

9. Let $n \in \mathbb{N}$, $n \geq 3$. In the xOy plane consider a set $\mathcal{M}$ consisting of n distinct points none of which is O. The number of all triangles having two vertices in $\mathcal{M}$ and one in O is:

A A_n^2 ; B 2^n ; C C_{n+1}^3 ; D at most C_n^2 .

10. Consider the system of equations

$$\begin{cases} (a-1)x + 2y + 3z &= 1 \\ x + 2y + 3z &= 1 \\ x + 2y + (a+1)z &= 1, \end{cases}$$

where a is a real parameter. Which of the following statements are true?

A There exist several values of a for which the determinant of the system is 0.
B There exists a single value of a for which the system is compatible.
C There exist several values of a for which the system is compatible.
D If the system is compatible, then it has a unique solution.

11. If $\sin x = a$ and $x \in \left(\frac{3\pi}{2}, 2\pi\right)$, then:

A $\sin 2x = 2a$; B $\sin 2x = 2a\sqrt{1-a^2}$;

C $\sin 2x = -2a\sqrt{1-a^2}$; D $\cos 2x = 1 - 2a^2$.

12. If $A(-2,-1)$, $B(2,1)$ and $C(-1,2)$ are points in a Cartesian coordinate system, then the triangle ABC is:

A obtuse; B isosceles; C right; D equilateral.

13. In a Cartesian coordinate system consider the lines

$$d_1 : (m-1)x + (3m-7)y - 5 = 0 \text{ and } d_2 : (m-2)x + (2m-5)y = 0,$$

where m is a real parameter. Indicate which of the following statements are true.

A The lines are parallel for $m = 3$.
B There exists a value of the parameter m for which the lines coincide.
C The lines are perpendicular for a single value of m.
D The lines are not perpendicular for any value of m.

14. In a Cartesian coordinate system xOy consider the points $A(0,1)$ and $H(3,2)$, where H is the orthocenter of the triangle ABC. The slope of the line BC is equal to:

A 3; B -3; C $\frac{1}{3}$; D $-\frac{1}{3}$.

15. Let $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ be defined by $f(x) = (x+2)e^{1/x}$. The equation of the oblique asymptote to the graph of f at $+\infty$ is:

A $y = x$; B $y = x + 1$; C $y = x + 2$; D $y = x + 3$.

16. Let $a \in \mathbb{R}$ and let $f : (-1,1) \to \mathbb{R}$ be defined by $f(x) = \dfrac{x+a}{\sqrt{1-x^2}}$. The value of a for which the tangent to the graph of f at the point with x-coordinate 0 passes through the point $(-2, 5)$ is:

$\boxed{\text{A}}$ 9; $\boxed{\text{B}}$ 7; $\boxed{\text{C}}$ -5; $\boxed{\text{D}}$ 5.

17. The value of the integral $\displaystyle\int_0^{\pi/2} \frac{\sin 2x}{3-\cos 2x}\,\mathrm{d}x$ is:

$\boxed{\text{A}}$ $\ln 2$; $\boxed{\text{B}}$ $\dfrac{1}{4}\ln 2$; $\boxed{\text{C}}$ $\dfrac{1}{2}\ln 2$; $\boxed{\text{D}}$ $\dfrac{1}{2}$.

18. The value of the integral $\displaystyle\int_1^{e^2} \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x$ is:

$\boxed{\text{A}}$ 4; $\boxed{\text{B}}$ 4e; $\boxed{\text{C}}$ 8e $-$ 4; $\boxed{\text{D}}$ 2e $+$ 2.

19. In the field $(\mathbb{Z}_5, +, \cdot)$ the system of equations

$$\begin{cases} x+\widehat{2}y & =\widehat{3} \\ \widehat{2}x+\widehat{4}y & =\widehat{2} \end{cases}$$

$\boxed{\text{A}}$ has no solutions;
$\boxed{\text{B}}$ has a unique solution;
$\boxed{\text{C}}$ has exactly five distinct solutions;
$\boxed{\text{D}}$ has exactly ten distinct solutions.

20. Consider the function $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x-[x]-\dfrac{1}{2}$, where $[x]$ denotes the integer part of the number x. Which of the following statements are true?

$\boxed{\text{A}}$ The graph of the function f intersects the Oy axis in at least 2 points.
$\boxed{\text{B}}$ The graph of the function f does not intersect the Ox axis.
$\boxed{\text{C}}$ The graph of the function f intersects the Ox axis in infinitely many points.
$\boxed{\text{D}}$ The graph of the function f does not intersect the Oy axis.

21. Consider the matrix equation $X^2=\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ in $\mathcal{M}_2(\mathbb{Z})$. Which of the following statements are true?

$\boxed{\text{A}}$ The equation has a unique solution.
$\boxed{\text{B}}$ The equation has exactly two solutions.
$\boxed{\text{C}}$ The equation has more than two solutions.
$\boxed{\text{D}}$ The sum of all solutions of the equation is O_2.

22. The points $A(0,2)$, $B(2,1)$ are vertices of the parallelogram $ABCD$ and $G(2,0)$ is the gravity center of the triangle ABD. Indicate which of the following statements are true.

$\boxed{\text{A}}$ The aria of the triangle ABD is equal to 3.
$\boxed{\text{B}}$ The aria of the triangle ABD is equal to 6.
$\boxed{\text{C}}$ The aria of the parallelogram $ABCD$ is equal to 6.
$\boxed{\text{D}}$ The aria of the parallelogram $ABCD$ is equal to 12.

23. The real numbers a and b satisfy the equality $(\cos\ a+\cos\ b)^2+(\sin\ a+\sin\ b)^2=2\cos^2\dfrac{a-b}{2}$. Indicate which of the following statements are true.

$\boxed{\text{A}}$ $a-b\in\{2k\pi | k\in\mathbb{Z}\}$; $\boxed{\text{B}}$ $a-b\in\{(2k+1)\pi | k\in\mathbb{Z}\}$;

$\boxed{\text{C}}$ $a-b\in\left\{\dfrac{\pi}{2}+2k\pi \,\middle|\, k\in\mathbb{Z}\right\}$; $\boxed{\text{D}}$ $a-b\in\left\{\dfrac{\pi}{2}+k\pi \,\middle|\, k\in\mathbb{Z}\right\}$.

24. With the usual notations in a triangle ABC, let $b = 5$, $c = 7$ and $m(\widehat{B}) = 45^\circ$. Indicate which of the following statements are true.

$\boxed{\text{A}}$ There exists a single triangle ABC with this data.
$\boxed{\text{B}}$ There exist two triangles ABC with this data.
$\boxed{\text{C}}$ $\sin A \in \left\{\frac{3}{5}, \frac{4}{5}\right\}$.
$\boxed{\text{D}}$ $a \in \{2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}\}$.

25. The set of values of the real parameter m for which the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \ln\left(1 + x^2\right) - mx$ is increasing on $\mathbb{R}$ is:

$\boxed{\text{A}}$ $(0, \infty)$; $\boxed{\text{B}}$ $(-\infty, -1]$; $\boxed{\text{C}}$ $[-1, 1]$; $\boxed{\text{D}}$ $[-1, \infty)$.

26. For a matrix $X = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$ denote by $\operatorname{tr}(X) = x + t$, the sum of all elements on the main diagonal of X. For the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$ the limit $\lim\limits_{n\to\infty} \dfrac{\operatorname{tr}(A^n)}{\det(A^n)}$ is equal cu:

$\boxed{\text{A}}$ 0; $\boxed{\text{B}}$ 2; $\boxed{\text{C}}$ 1; $\boxed{\text{D}}$ $+\infty$.

27. Let a be a real parameter and consider the composition law

$$x * y = xy + ax + ay + 1$$

on $\mathbb{R}$. Which of the following statements are true?

$\boxed{\text{A}}$ If $*$ has an identity element, then a is uniquely determined.
$\boxed{\text{B}}$ There exist several values of a for which $*$ has an identity element.
$\boxed{\text{C}}$ If e is an identity element for $*$, then $e = \frac{1}{a}$.
$\boxed{\text{D}}$ If e is an identity element for $*$, then $e = -\frac{1}{a}$.

28. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = \mathrm{e}^x + \mathrm{e}^{-x} - x^2$. Indicate which of the following statements are true.

$\boxed{\textbf{A}}$ $f'(0) = f''(0) = 0$.
$\boxed{\textbf{B}}$ 0 is not a point of local extrema for f.
$\boxed{\textbf{C}}$ The function f' is strictly monotone.
$\boxed{\textbf{D}}$ 0 is a point of global extrema for f''.

29. For each $n \in \mathbb{N}^*$ denote $I_n = \displaystyle\int_1^{\mathrm{e}} \left(\ln x\right)^n \mathrm{d}x$. Indicate which of the following statements are true.

$\boxed{\text{A}}$ $I_{n+1} + (n+1)I_n = \mathrm{e}, \ \ \forall n \in \mathbb{N}^*$. $\boxed{\text{B}}$ $\lim\limits_{n\to\infty} I_n = 0$. $\boxed{\text{C}}$ $\lim\limits_{n\to\infty} nI_n = \mathrm{e}$. $\boxed{\text{D}}$ $\lim\limits_{n\to\infty} nI_n = 1$.

30. The diagonals of a square $\mathcal{P}_\alpha$ of side α are on the coordinate axes of a Cartesian coordinate system ($\alpha \in (0, \infty)$). Let N_α be the number of points inside the square $\mathcal{P}_\alpha$ with integer coordinates. Indicate which of the following statements are true.

$\boxed{\textbf{A}}$ For $\alpha = 3$ we have $N_3 = 5$.
$\boxed{\textbf{B}}$ For $\alpha = 3$ we have $N_3 = 13$.
$\boxed{\textbf{C}}$ There exists α such that $N_\alpha = 41$.
$\boxed{\textbf{D}}$ There exists α such that $N_\alpha = 67$.

BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Correct Answers

ADMISSIONS EXAM 2021
Written test in MATHEMATICS
September Session

1. $\boxed{\textbf{B}}$

2. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$

3. $\boxed{\textbf{C}}$

4. $\boxed{\textbf{A}}$

5. $\boxed{\textbf{A}}$, $\boxed{\textbf{C}}$

6. $\boxed{\textbf{B}}$

7. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

8. $\boxed{\textbf{C}}$

9. $\boxed{\textbf{D}}$

10. $\boxed{\textbf{C}}$

11. $\boxed{\textbf{B}}$, $\boxed{\textbf{D}}$

12. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

13. $\boxed{\textbf{A}}$, $\boxed{\textbf{D}}$

14. $\boxed{\textbf{B}}$

15. $\boxed{\textbf{D}}$

16. $\boxed{\textbf{B}}$

17. $\boxed{\textbf{C}}$

18. $\boxed{\textbf{A}}$

19. $\boxed{\textbf{A}}$

20. $\boxed{\textbf{C}}$

21. $\boxed{\textbf{B}}$, $\boxed{\textbf{D}}$

22. $\boxed{\textbf{A}}$, $\boxed{\textbf{C}}$

23. $\boxed{\textbf{B}}$

24. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$

25. $\boxed{\textbf{B}}$

26. $\boxed{\textbf{C}}$

27. $\boxed{\textbf{B}}$, $\boxed{\textbf{D}}$

28. $\boxed{\textbf{A}}$, $\boxed{\textbf{C}}$, $\boxed{\textbf{D}}$

29. $\boxed{\textbf{A}}$, $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$

30. $\boxed{\textbf{B}}$, $\boxed{\textbf{C}}$