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Coefficient bounds for new subclasses of analytic and m-fold symmetric bi-univalent functions

Abbas Kareem Wanas and Ágnes Orsolya Páll-Szabó

Abstract. In the present paper, we introduce and study two new subclasses of analytic and m-fold symmetric bi-univalent functions defined in the open unit disk U. Furthermore, for functions in each of the subclasses introduced here, we obtain upper bounds for the initial coefficients $|a_{m+1}|$ and $|a_{2m+1}|$. Also, we indicate certain special cases for our results.

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1. Introduction

Denote by $\mathcal{A}$ the class of functions f of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

which are analytic in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$. Let S be the subclass of $\mathcal{A}$ consisting in functions of the form (1.1) which are also univalent in U. The Koebe one-quarter theorem (see [4]) states that the image of U under every function $f \in S$ contains a disk of radius $\frac{1}{4}$. Therefore, every function $f \in S$ has an inverse f^{-1} which satisfies $f^{-1}(f(z)) = z$, $(z \in U)$ and $f(f^{-1}(w)) = w$, $(|w| < r_0(f), r_0(f) \geq \frac{1}{4})$, where

$$g(w) = f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots . \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in U if both f and f^{-1} are univalent in U. We denote by Σ the class of bi-univalent functions in U given by (1.1). For a brief history and interesting examples in the class Σ see [14], (see also [6, 7, 10, 11, 12]).

For each function $f \in S$, the function $h(z) = (f(z^m))^{\frac{1}{m}}$, $(z \in U, m \in \mathbb{N})$ is univalent and maps the unit disk U into a region with m-fold symmetry. A function is said to be m-fold symmetric (see [8]) if it has the following normalized form:

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1}, \ (z \in U, m \in \mathbb{N}). \tag{1.3}$$

Let S_m stands for the class of m-fold symmetric univalent functions in U, which are normalized by the series expansion (1.3). In fact, the functions in the class S are one-fold symmetric.

In [15] Srivastava et al. defined m-fold symmetric bi-univalent functions analogues to the concept of m-fold symmetric univalent functions. They gave some important results, such as each function $f \in \Sigma$ generates an m-fold symmetric bi-univalent function for each $m \in \mathbb{N}$. Furthermore, for the normalized form of f given by (1.3), they obtained the series expansion for f^{-1} as follows:

$$\begin{aligned} g(w) = w - a_{m+1} w^{m+1} + \left[(m+1)a_{m+1}^2 - a_{2m+1}\right] w^{2m+1} \\ - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right] w^{3m+1} + \cdots , \end{aligned} \tag{1.4}$$

where $f^{-1} = g$. We denote by Σ_m the class of m-fold symmetric bi-univalent functions in U. It is easily seen that for $m = 1$, the formula (1.4) coincides with the formula (1.2) of the class Σ. Some examples of m-fold symmetric bi-univalent functions are given as follows:

$$\left(\frac{z^m}{1-z^m}\right)^{\frac{1}{m}}, \ \left[\frac{1}{2}\log\left(\frac{1+z^m}{1-z^m}\right)\right]^{\frac{1}{m}} \ \textit{and} \ \left[-\log\left(1-z^m\right)\right]^{\frac{1}{m}}$$

with the corresponding inverse functions

$$\left(\frac{w^m}{1+w^m}\right)^{\frac{1}{m}}, \ \left(\frac{e^{2w^m}-1}{e^{2w^m}+1}\right)^{\frac{1}{m}} \ \textit{and} \ \left(\frac{e^{w^m}-1}{e^{w^m}}\right)^{\frac{1}{m}},$$

respectively.

Recently, many authors investigated bounds for various subclasses of m-fold bi-univalent functions (see [1, 2, 5, 13, 15, 16, 17]).

The purpose of the present investigation is to introduce the new subclasses $\mathcal{AS}_{\Sigma_m}(\gamma, \lambda; \alpha)$ and $\mathcal{AS}^*_{\Sigma_m}(\gamma, \lambda; \beta)$ of Σ_m and find estimates on the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in each of these new subclasses.

We will require the following lemma in proving our main results.

Lemma 1.1. [3] *If $h \in \mathcal{P}$, then $|c_k| \le 2$ for each $k \in \mathbb{N}$, where $\mathcal{P}$ is the family of all functions h analytic in U for which*

$$Re\,(h(z)) > 0, \quad (z \in U),$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots , \quad (z \in U).$$

2. Coefficient bounds for the function class $\mathcal{AS}_{\Sigma_m}(\gamma,\lambda;\alpha)$

Definition 2.1. A function $f\in\Sigma_m$ given by (1.3) is said to be in the class $\mathcal{AS}_{\Sigma_m}(\gamma,\lambda;\alpha)$ if it satisfies the following conditions:

$$\left|arg\left(\left[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^{\gamma}\right)\right|<\frac{\alpha\pi}{2} \tag{2.1}$$

and

$$\left|arg\left(\left[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^{\gamma}\right)\right|<\frac{\alpha\pi}{2}, \tag{2.2}$$

$$(z,w\in U, 0<\alpha\le 1,\ 0\le\gamma\le 1,\ 0\le\lambda\le 1,\ m\in\mathbb{N}),$$

where the function $g=f^{-1}$ is given by (1.4).

In particular, for one-fold symmetric bi-univalent functions, we denote the class $\mathcal{AS}_{\Sigma_1}(\gamma,\lambda;\alpha)=\mathcal{AS}_{\Sigma}(\gamma,\lambda;\alpha)$.

Remark 2.2. It should be remarked that the classes $\mathcal{AS}_{\Sigma_m}(\gamma,\lambda;\alpha)$ and $\mathcal{AS}_{\Sigma}(\gamma,\lambda;\alpha)$ are a generalization of well-known classes consider earlier. These classes are:
(1) For $\lambda=0$ and $\gamma=1$, the class $\mathcal{AS}_{\Sigma_m}(\gamma,\lambda;\alpha)$ reduce to the class $S^{\alpha}_{\Sigma_m}$ which was considered by Altinkaya and Yalçin [1];
(2) For $\gamma=1$, the class $\mathcal{AS}_{\Sigma}(\gamma,\lambda;\alpha)$ reduce to the class $M_{\Sigma}(\alpha,\lambda)$ which was introduced by Liu and Wang [9];
(3) For $\lambda=0$ and $\gamma=1$, the class $\mathcal{AS}_{\Sigma}(\gamma,\lambda;\alpha)$ reduce to the class $S^{*}_{\Sigma}(\alpha)$ which was given by Brannan and Taha [3].

Theorem 2.3. *Let $f\in\mathcal{AS}_{\Sigma_m}(\gamma,\lambda;\alpha)$ $(0<\alpha\le 1,\ 0\le\gamma\le 1,\ 0\le\lambda\le 1,\ m\in\mathbb{N})$ be given by (1.3). Then*

$$|a_{m+1}|\le\frac{2\alpha}{m\sqrt{2\alpha\gamma(1+\lambda m)+\gamma(\gamma-\alpha)\left(1+\lambda m\right)^2}} \tag{2.3}$$

and

$$|a_{2m+1}|\le\frac{2\alpha^2(m+1)}{m^2\gamma^2\left(1+\lambda m\right)^2}+\frac{\alpha}{m\gamma(1+2\lambda m)}. \tag{2.4}$$

Proof. It follows from conditions (2.1) and (2.2) that

$$\left[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^{\gamma}=[p(z)]^{\alpha} \tag{2.5}$$

and

$$\left[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^{\gamma}=[q(w)]^{\alpha}, \tag{2.6}$$

where $g=f^{-1}$ and p, q in $\mathcal{P}$ have the following series representations:

$$p(z)=1+p_m z^m+p_{2m}z^{2m}+p_{3m}z^{3m}+\cdots \tag{2.7}$$

and

$$q(w)=1+q_m w^m+q_{2m}w^{2m}+q_{3m}w^{3m}+\cdots. \tag{2.8}$$

Comparing the corresponding coefficients of (2.5) and (2.6) yields

$$m\gamma(1+\lambda m)a_{m+1} = \alpha p_m, \tag{2.9}$$

$$\begin{aligned} &m\left[2\gamma(1+2\lambda m)a_{2m+1} - \gamma\left(\lambda m^2+2\lambda m+1\right)a_{m+1}^2\right] \\ &+\frac{m^2}{2}\gamma(1+\lambda m)(\gamma-1)(1+\lambda m)a_{m+1}^2 \\ &= \alpha p_{2m} + \frac{\alpha(\alpha-1)}{2}p_m^2, \end{aligned} \tag{2.10}$$

$$-m\gamma(1+\lambda m)a_{m+1} = \alpha q_m \tag{2.11}$$

and

$$\begin{aligned} &m\left[\gamma\left(3\lambda m^2+2(\lambda+1)m+1\right)a_{m+1}^2 - 2\gamma(1+2\lambda m)a_{2m+1}\right] \\ &+\frac{m^2}{2}\gamma(1+\lambda m)(\gamma-1)(1+\lambda m)a_{m+1}^2 = \alpha q_{2m} + \frac{\alpha(\alpha-1)}{2}q_m^2. \end{aligned} \tag{2.12}$$

Making use of (2.9) and (2.11), we obtain

$$p_m = -q_m \tag{2.13}$$

and

$$2m^2\gamma^2\left(1+\lambda m\right)^2 a_{m+1}^2 = \alpha^2(p_m^2+q_m^2). \tag{2.14}$$

Also, from (2.10), (2.12) and (2.14), we find that

$$\begin{aligned} &m^2\left[2\gamma\left(1+\lambda m\right)+\gamma(\gamma-1)\left(1+\lambda m\right)^2\right]a_{m+1}^2 \\ &= \alpha(p_{2m}+q_{2m}) + \frac{\alpha(\alpha-1)}{2}\left(p_m^2+q_m^2\right) \\ &= \alpha(p_{2m}+q_{2m}) + \frac{m^2\gamma^2(\alpha-1)\left(1+\lambda m\right)^2}{\alpha}a_{m+1}^2. \end{aligned}$$

Therefore, we have

$$a_{m+1}^2 = \frac{\alpha^2(p_{2m}+q_{2m})}{m^2\left[2\alpha\gamma(1+\lambda m)+\gamma(\gamma-\alpha)\left(1+\lambda m\right)^2\right]}. \tag{2.15}$$

Now, taking the absolute value of (2.15) and applying Lemma 1.1 for the coefficients p_{2m} and q_{2m}, we deduce that

$$|a_{m+1}| \le \frac{2\alpha}{m\sqrt{2\alpha\gamma(1+\lambda m)+\gamma(\gamma-\alpha)\left(1+\lambda m\right)^2}}.$$

This gives the desired estimate for $|a_{m+1}|$ as asserted in (2.3).
In order to find the bound on $|a_{2m+1}|$, by subtracting (2.12) from (2.10), we get

$$2m\gamma(1+2\lambda m)\left[2a_{2m+1}-(m+1)a_{m+1}^2\right] = \alpha\left(p_{2m}-q_{2m}\right) + \frac{\alpha(\alpha-1)}{2}\left(p_m^2-q_m^2\right). \tag{2.16}$$

It follows from (2.13), (2.14) and (2.16) that

$$a_{2m+1} = \frac{\alpha^2(m+1)\left(p_m^2+q_m^2\right)}{4m^2\gamma^2\left(1+\lambda m\right)^2} + \frac{\alpha\left(p_{2m}-q_{2m}\right)}{4m\gamma(1+2\lambda m)}. \tag{2.17}$$

Taking the absolute value of (2.17) and applying Lemma 1.1 once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we obtain

$$|a_{2m+1}| \le \frac{2\alpha^2(m+1)}{m^2\gamma^2\left(1+\lambda m\right)^2} + \frac{\alpha}{m\gamma(1+2\lambda m)},$$

which completes the proof of Theorem 2.3. □

For one-fold symmetric bi-univalent functions, Theorem 2.3 reduce to the following corollary:

Corollary 2.4. *Let* $f \in \mathcal{AS}_\Sigma(\gamma,\lambda;\alpha)$ $(0<\alpha\le 1,\ 0\le\gamma\le 1,\ 0\le\lambda\le 1)$ *be given by (*1.1*). Then*

$$|a_2| \le \frac{2\alpha}{\sqrt{2\alpha\gamma(1+\lambda)+\gamma(\gamma-\alpha)\left(1+\lambda\right)^2}}$$

and

$$|a_3| \le \frac{4\alpha^2}{\gamma^2\left(1+\lambda\right)^2} + \frac{\alpha}{\gamma(1+2\lambda)}.$$

3. Coefficient bounds for the function class $\mathcal{AS}^*_{\Sigma_m}(\gamma,\lambda;\beta)$

Definition 3.1. A function $f \in \Sigma_m$ given by (1.3) is said to be in the class $\mathcal{AS}^*_{\Sigma_m}(\gamma,\lambda;\beta)$ if it satisfies the following conditions:

$$Re\left\{\left[(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^\gamma\right\} > \beta \tag{3.1}$$

and

$$Re\left\{\left[(1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^\gamma\right\} > \beta, \tag{3.2}$$

$$\left(z,w\in U, 0\le\beta<1,\ 0\le\gamma\le 1,\ 0\le\lambda\le 1,\ m\in\mathbb{N}\right),$$

where the function $g=f^{-1}$ is given by (1.4).

In particular, for one-fold symmetric bi-univalent functions, we denote the class

$$\mathcal{AS}^*_{\Sigma_1}(\gamma,\lambda;\beta) = \mathcal{AS}^*_{\Sigma}(\gamma,\lambda;\beta).$$

Remark 3.2. It should be remarked that the classes $\mathcal{AS}^*_{\Sigma_m}(\gamma,\lambda;\beta)$ and $\mathcal{AS}^*_{\Sigma}(\gamma,\lambda;\beta)$ are a generalization of well-known classes consider earlier. These classes are:
(1) For $\lambda=0$ and $\gamma=1$, the class $\mathcal{AS}^*_{\Sigma_m}(\gamma,\lambda;\beta)$ reduce to the class $S^\beta_{\Sigma_m}$ which was considered by Altinkaya and Yalçin [1];
(2) For $\gamma=1$, the class $\mathcal{AS}^*_{\Sigma}(\gamma,\lambda;\beta)$ reduce to the class $B_\Sigma(\beta,\tau)$ which was introduced by Liu and Wang [9];
(3) For $\lambda=0$ and $\gamma=1$, the class $\mathcal{AS}^*_{\Sigma}(\gamma,\lambda;\beta)$ reduce to the class $S^*_\Sigma(\beta)$ which was given by Brannan and Taha [3].

Theorem 3.3. *Let* $f \in \mathcal{AS}^*_{\Sigma_m}(\gamma,\lambda;\beta)$ $(0 \le \beta < 1,\ 0 \le \gamma \le 1,\ 0 \le \lambda \le 1,\ m \in \mathbb{N})$ *be given by* (1.3)*. Then*

$$|a_{m+1}| \le \frac{2}{m}\sqrt{\frac{1-\beta}{2\gamma(1+\lambda m)+\gamma(\gamma-1)(1+\lambda m)^2}} \tag{3.3}$$

and

$$|a_{2m+1}| \le \frac{2(m+1)(1-\beta)^2}{m^2\gamma^2(1+\lambda m)^2} + \frac{1-\beta}{m\gamma(1+2\lambda m)}. \tag{3.4}$$

Proof. It follows from conditions (3.1) and (3.2) that there exist $p, q \in \mathcal{P}$ such that

$$\left[(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right]^\gamma = \beta + (1-\beta)p(z) \tag{3.5}$$

and

$$\left[(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1+\frac{wg''(w)}{g'(w)}\right)\right]^\gamma = \beta + (1-\beta)q(w), \tag{3.6}$$

where $p(z)$ and $q(w)$ have the forms (2.7) and (2.8), respectively. Equating coefficients (3.5) and (3.6) yields

$$m\gamma(1+\lambda m)a_{m+1} = (1-\beta)p_m, \tag{3.7}$$

$$m\left[2\gamma(1+2\lambda m)a_{2m+1} - \gamma\left(\lambda m^2+2\lambda m+1\right)a_{m+1}^2\right] + \frac{m^2\gamma}{2}(1+\lambda m)(\gamma-1)(1+\lambda m)a_{m+1}^2 = (1-\beta)p_{2m}, \tag{3.8}$$

$$-m\gamma(1+\lambda m)a_{m+1} = (1-\beta)q_m \tag{3.9}$$

and

$$m\left[\gamma\left(3\lambda m^2+2(\lambda+1)m+1\right)a_{m+1}^2 - 2\gamma(1+2\lambda m)a_{2m+1}\right] + \frac{m^2\gamma}{2}(1+\lambda m)(\gamma-1)(1+\lambda m)a_{m+1}^2 = (1-\beta)q_{2m}. \tag{3.10}$$

From (3.7) and (3.9), we get

$$p_m = -q_m \tag{3.11}$$

and

$$2m^2\gamma^2(1+\lambda m)^2 a_{m+1}^2 = (1-\beta)^2(p_m^2+q_m^2). \tag{3.12}$$

Adding (3.8) and (3.10), we obtain

$$m^2\left[2\gamma(1+\lambda m)+\gamma(\gamma-1)(1+\lambda m)^2\right]a_{m+1}^2 = (1-\beta)(p_{2m}+q_{2m}). \tag{3.13}$$

Therefore, we have

$$a_{m+1}^2 = \frac{(1-\beta)(p_{2m}+q_{2m})}{m^2\left[2\gamma(1+\lambda m)+\gamma(\gamma-1)(1+\lambda m)^2\right]}.$$

Applying Lemma 1.1 for the coefficients p_{2m} and q_{2m}, we obtain

$$|a_{m+1}| \le \frac{2}{m}\sqrt{\frac{1-\beta}{2\gamma(1+\lambda m)+\gamma(\gamma-1)(1+\lambda m)^2}}.$$

This gives the desired estimate for $|a_{m+1}|$ as asserted in (3.3).
In order to find the bound on $|a_{2m+1}|$, by subtracting (3.10) from (3.8), we get

$$2m\gamma(1+2\lambda m)\left[2a_{2m+1}-(m+1)a_{m+1}^2\right]=(1-\beta)\left(p_{2m}-q_{2m}\right).$$

or equivalently

$$a_{2m+1}=\frac{m+1}{2}a_{m+1}^2+\frac{(1-\beta)\left(p_{2m}-q_{2m}\right)}{4m\gamma(1+2\lambda m)}.$$

Upon substituting the value of a_{m+1}^2 from (3.12), it follows that

$$a_{2m+1}=\frac{(m+1)\left(1-\beta\right)^2\left(p_m^2+q_m^2\right)}{4m^2\gamma^2\left(1+\lambda m\right)^2}+\frac{(1-\beta)\left(p_{2m}-q_{2m}\right)}{4m\gamma(1+2\lambda m)}.$$

Applying Lemma 1.1 once again for the coefficients p_m, p_{2m}, q_m and q_{2m}, we obtain

$$|a_{2m+1}|\leq\frac{2(m+1)\left(1-\beta\right)^2}{m^2\gamma^2\left(1+\lambda m\right)^2}+\frac{1-\beta}{m\gamma(1+2\lambda m)}.$$

which completes the proof of Theorem 3.3. □

For one-fold symmetric bi-univalent functions, Theorem 3.3 reduce to the following corollary:

Corollary 3.4. *Let* $f\in\mathcal{AS}_{\Sigma}^{*}(\gamma,\lambda;\beta)$ $(0\leq\beta<1,\ 0\leq\gamma\leq1,\ 0\leq\lambda\leq1)$ *be given by (*1.1*). Then*

$$|a_2|\leq2\sqrt{\frac{1-\beta}{2\gamma(1+\lambda)+\gamma(\gamma-1)\left(1+\lambda\right)^2}}$$

and

$$|a_3|\leq\frac{4\left(1-\beta\right)^2}{\gamma^2\left(1+\lambda\right)^2}+\frac{1-\beta}{\gamma(1+2\lambda)}.$$

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Abbas Kareem Wanas
Department of Mathematics, College of Science,
University of Al-Qadisiyah, Iraq
e-mail: abbas.kareem.w@qu.edu.iq

Ágnes Orsolya Páll-Szabó
Department of Mathematics,
Faculty of Mathematics and Computer Science,
Babeş-Bolyai University, Cluj-Napoca, Romania
e-mail: pallszaboagnes@math.ubbcluj.ro