Stud. Univ. Babeş-Bolyai Math. 63(2018), No. 4, 561–562

Book reviews

Boris S. Mordukhovich; Variational Analysis and Applications,
Springer Monographs in Mathematics. Springer, Cham, 2018. xix+622 p.
ISBN: 978-3-319-92773-2/hbk; 978-3-319-92775-6/ebook.

Although variational principles in mathematical physics and mechanics were known since the 18th century, variational analysis, in its current acceptance, is a relatively new discipline. Its aim is to treat optimization and control problems via perturbations, approximations and generalized differentiation of nonsmooth or set-valued maps. As the author mentions in Preface, the first monograph dedicated to variational analysis in finite dimensions is that by R. T. Rockafellar and R. J.B. Wets, Springer 1998, where this very name was coined. The infinite-dimensional case is treated at large in the impressive two-volume monograph of the author, *Variational analysis and generalized differentiation.* I: *Basic theory* (579 p), II: *Applications* (610 p), Springer 2006 (a review of these volumes is published in vol. 52 (2007), no. 1, of the present journal).

The present book can be viewed as a companion to the two-volume monograph mentioned above. The first 6 chapters of the book are dedicated to a presentation of variational analysis in finite-dimensional spaces. This restriction allows to present simplified proofs (*ad usum Delphini*) of the main results, being accessible to graduate students in mathematics as well as to those in applied sciences and engineering. Each chapter is completed by a consistent section of exercises containing further results, including infinite dimensional ones. The most difficult of them are accompanied by hints or references.

The contents of this part is well illustrated by the headings of its chapters: 1. *Constructions of generalized differentiation*; 2. *Fundamental principles of variational analysis*; 3. *Well-posedness and coderivative calculus*; 4. *First-order subdifferential calculus*; 5. *Coderivatives of maximal monotone operators*; 6. *Nondifferentiable and bilevel optimization.*

The second part of the book, Chapters 7 to 10, is dedicated to applications of variational analysis to optimization and economics (in Ch. 10. *Set-valued optimization and economics*), and to other domains. Here the topics are treated in full infinite-dimensional generality, being addressed to researchers, graduate students and practitioners. As the author mention in Preface:

> The results obtained demonstrate the strength of variational analysis and dual-space constructions in solving concrete problems that may not even be of a variational nature.

Again, each chapter ends with a large number of exercises. As the author mentions, the exercise sections (containing some open problems and conjectures as well) play a crucial role in the organization of the book, providing the reader with a handy reference source to the enormous material available in first-order variational analysis, as well as with ideas for further research and developments.

Besides exercises, each chapter ends with a consistent section of *Commentaries*, containing references for the results included in the chapter or to other related results.

The book is very well organized – besides the *Subject Index*, it contains a *List of Statements* and a *Glossary of Notations and Acronyms*. The rich bibliography counts 790 items.

In conclusion, written by an expert in the areas of variational analysis and optimization and based on his didactic experience, this is an excellent textbook. By the wealth of information contained in the second part of the book and in exercises, it can be also used by researchers in optimization theory and its applications as a reference text.

S. Cobzaş