## **Book reviews**

Boris Zacharowitsch Wulich, Geometrie der Kegel in normierten Räumen, (Herausgegeben von Martin R. Weber) De Gruyter Studium, De Gruyter Berlin, 2017, xvi+223 p., ISBN: 978-3-11-047884-6/pbk; 978-3-11-047888-4/ebook

Boris Zakharovich Vulikh (1913-1978) was a distinguished Russian mathematician with outstanding contributions to various domains of functional analysis, mainly to the theory of ordered vector spaces. For almost thirty years he was the head of Chairs of Mathematical Analysis at Leningrad Higher Educational Institutions, from 1957 to 1963 at the Leningrad A.I. Herzen Pedagogical Institute (now A.I. Herzen State Pedagogical University of Russia, Sankt Peterburg) and from 1963 to 1978 at the Mathematics and Mechanics Faculty of the Leningrad State University (now Sankt Peterburg State University).

Besides the research papers, he wrote several well known books on analysis and functional analysis, including two on ordered vector spaces - one in 1950, jointly with L. V. Kantorovich and A. G. Pinsker, and one alone, Introduction to the theory of partially ordered spaces (in Russian), Leningrad 1961, an English translation being published with Wolters-Noordhoff in 1967. Less known are two booklets, The geometry of cones in normed spaces (72 p.), and Special questions of the geometry of cones in normed spaces (73 p.), published at the Kalinin (now Tver) State University in 1976 and 1977, respectively. These two booklets contain, in a condensed but complete and clear form, the basic results on cones in normed spaces, in particular, duality properties of a cone and its dual cone, and properties of the cone of positive operators between ordered normed spaces as well. Prof. Martin Weber from the Technical University of Dresden took the charge to translate into German and update them, being published as Chapters I and II in this book named The geometry of cones in normed spaces. This was not a simple translation, a lot of edifying footnotes are included in the text of translation. Also some interesting examples and counterexamples going back to I.I. Chuchaev (N.P. Ogarev Mordovia State University, Russia) and being only announced in the original text are included in detail into the German issue. Besides these, a consistent chapter, Some afterthoughts by the editor of the German *edition*, accompanied by a list of updated references, presents some developments in the theory of ordered vector spaces and their applications done since the publications of the Russian edition of the booklets.

It is worth to mention that Prof. Weber studied at the Leningrad State University (1963-1968) and earned a Ph.D. (Kandidat physiko-matematicheskih nauk -

Candidate in physical-mathematical sciences) in 1974 at the same University (with Prof. B. M. Makarov as supervisor). He was and remained in contact with the strong group of researchers in ordered vector spaces from Leningrad-St Petersburg University, so we have the privilege of a first hand information on the topics, people and events.

In spite of the years passed since their publication, these books by B. Z. Vulikh are still a valuable source of information for mathematicians, professionals and students as well, interested in the theory of ordered normed spaces and its applications. By translating and updating this masterpiece of mathematical exposition Prof. Martin Weber has done a wonderful (and hard) job and, at the same time, rendered a great service to the mathematical community.

S. Cobzaş

René L. Schilling; Wahrscheinlichkeit – Eine Einführung für Bachelor-Studenten. De Gruyter Studium, Walter de Gruyter GmbH, Berlin/Boston 2017, x+232 p., ISBN: 978-3-11-035065-4. Language: German; translated title: Probability – An Introduction for Bachelor Students.

Professor René L. Schilling from the Technical University in Dresden (Germany) is a well-known expert in the field of stochastic processes. This book continues the course of the author about measure and integration theory ( $Ma\beta$  und Integral, published in 2015 with De Gruyter, Berlin). It is addressed to students of mathematics, natural sciences (especially physics), economics, and engineering, but also to any researcher interested in the field of probability theory and its applications.

This textbook provides a modern access to the most important results of mathematical probability theory. Prerequisites for understanding the present book are basic notions of measure and integration theory. The main topics of this book are: models of probability theory, elementary combinatorics, conditional probabilities, random variables and their independence, characteristic functions, classic limit theorems, convergence of random variables. These topics are then supplemented by the study of sums of independent random variables, laws of large numbers, zero-one laws, random walks, the central limit theorem. Conditional expectations, applications of characteristic functions, and an introduction to the theory of infinitely divisible distributions and large deviations round off the book. Lastly, the author has included an appendix at the end of the book containing a summary of the main results that are used throughout the present book, as well as, a list of discrete and continuous distributions.

The material in this book consists of definitions, properties (with proofs or with references to the literature, where the proof can be found), many examples and counterexamples, exercises, explanatory comments and helpful hints, tables and suggestive figures.

The book is clearly written and well structured. It brings together theory, practice and research topics, and can be recommended as a German textbook for probability theory courses and seminars.

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**Palle Jorgensen and Feng Tian, Non-commutative Analysis.** World Scientific 2017, xxviii+533 p., ISBN: 978-981-3202-11-5 (hardcover); 978-981-3202-12-2 (softcover); 978-981-3202-14-6 (ebook)

As the authors mention in the Preface, the central themes of the book are: (i) Operators in Hilbert space; (ii) Multivariable spectral theory; (iii) Non-commutative analysis; (iv) Probability theory; (v) Unitary representations. The term "non-commutative analysis" is interpreted as including representations of non-Abelian groups, and non-Abelian algebras, with emphasis on Lie groups and operator algebras ( $C^*$  algebras and von Neumann algebras).

The book is oriented to applications, mainly in physics (quantum mechanics), from where the main motivation for the development of non-commutative analysis comes. According to a quotation from S. Doplicher and R. Longo (page viii), the novelty of physics of the XX century can be characterized with "a single magic word - non-commutativity". These applications are treated in two steps - in outline first and then, after developing the theoretic tools, with full details. The book is devoted to students with different backgrounds in mathematics, some of them coming from neighboring fields, so the authors tries to keep the prerequisites at a minimum. The general framework is that on Hilbert spaces, operators acting on them (with emphasis on unbounded operators) and spectral theory.

The book is divided into five parts: I. Introduction and motivation; II. Topics form functional analysis and operators in Hilbert space; III. Applications; IV. Extension of operators; V. Appendix.

The applications concern  $C^*$  algebras and their representations, completely positive maps, Brownian motion, Lie groups and their unitary representations. One discusses also the famous Kadison-Singer problem - Does every pure state on the von Neumann algebra of bounded diagonal operators on  $\ell^2$  have a unique extension to a (pure) state on the algebra  $\mathcal{B}(\ell^2)$  of all bounded linear operators on  $\ell^2$ ? The authors present only in outline this problem (dating from 1959), its recent difficult solution, by N. Srivastava, A. Marcus, and D. Spielman (2013, published in Annals of Mathematics, 2015) requiring a separate book (good presentations of Kadison-Singer problem are given in the papers by P. G. Casazza et al., arXiv:math/0510024, D. Timotin, arXiv:1501.00464, M. Bownik, arXiv:1702.04578).

The book is very well written and organized. All the notions and results are motivated by examples, the Appendix contains a list of significant books in functional analysis (with telegraphic reviews) and short biographies of some relevant mathematicians and physicists who essentially contributed to the field. A lot of suggestive (and amazing) quotations are spread throughout the book.

Based on two-semester courses on functional analysis taught over the years by the first-named author, the book is highly recommended to teachers in applied functional analysis, for students in mathematics and related areas, as well as for self-study by students needing a quick access to some top research tools in mathematics and physics, paving the way to more advanced and specialized texts on non-commutative analysis, non-commutative geometry and applications.