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New subclasses of bi-univalent functions defined by multiplier transformation

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Abstract. In the present paper, we introduce new subclasses of the function class $\sum$ of bi-univalent functions by using multiplier transformation. Furthermore, we obtain estimates on the coefficients $|a_2|, |a_3|$ and $|a_4|$ for functions of this class. Relevant connections of the results presented here with various well-known results are briefly indicated.

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1. Introduction

Let $\mathcal{A}$ denote the class of functions f of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

which are analytic in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ and satisfy the normalization condition $f(0) = f'(0) - 1 = 0$. Let $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of functions of the form (1.1) which are also univalent in U.

It is well known that every $f \in \mathcal{S}$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z, \ (z \in U)$$

and

$$f\left(f^{-1}(\omega)\right) = \omega, \ \left(|\omega| < r_0(f), \ r_0(f) \geq \frac{1}{4}\right),$$

where

$$f^{-1}(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)\omega^4 + \dots$$

A function $f(z) \in \mathcal{A}$ is said to be bi-univalent in U if both $f(z)$ and $f^{-1}(z)$ are univalent in U. Let Σ denote the class of bi-univalent functions in U given by the Taylor-Maclaurin series expansion (1.1). Examples of functions in the class Σ are $\frac{z}{1-z}$, $-\log(1-z)$, $\frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$ and so on.

However, the familiar Koebe function is not a member of Σ. Other common examples of functions in $\mathcal{S}$ such as $z-\frac{z^2}{2}$ and $\frac{z}{1-z^2}$ are also not members of Σ.

Lewin [7] first investigated the bi-univalent function class Σ showed that

$$|a_2| < 1.5.$$

Subsequently, Brannan and Clunie [2] conjectured that $|a_2| \leq \sqrt{2}$.

Netanyahu [9], on the other hand, showed that $max_{f\in\Sigma}|a_2| = \frac{4}{3}$.

The coeffiecient estimate problems for bi-univalent function class Σ is an interesting problem of Geometric Function Theory. Several researchers e.g. (see [1], [6], [11], [12], [14], [16], [17]), introduced the various new subclasses of the bi-univalent function class Σ and obtained non-sharp bounds on the first two coefficients $|a_2|$ and $|a_3|$. Recently, Mishra and Soren [8] obtain coefficient bounds for bi-starlike analytic functions and improve results of [3].

In order to prove our main results, we shall require the following lemma due to [10].

Lemma 1.1. *If $h \in P$ then $|c_k| \leq 2$ for each k, where P is the family of all functions h analytic in U for which $\Re\{h(z)\} > 0$,*

$$h(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \dots \text{ for } z \in U.$$

2. Coefficient bounds for the function class $B_\Sigma(n, \beta, \lambda, \mu)$

Cho and Srivastava [5], (see also [4]), introduced the operator $I_\lambda^n : \mathcal{A} \to \mathcal{A}$ defined for the function $f(z)$ of the form (1.1) as

$$I_\lambda^n f(z) = z + \sum_{k=2}^{\infty} \left(\frac{k+\lambda}{1+\lambda}\right)^n a_k z^k,$$

where $n \in N_0 = N \cup \{0\}$ and $-1 < \lambda \leq 1$. For $\lambda = 1$, the operator $I_\lambda^n \equiv I^n$ was studied by Uralegaddi and Somanatha [15] and for $\lambda = 0$ the operator I_λ^n reduces to well-known Sălăgean operator introduced by Sălăgean [13].

Definition 2.1. A function $f(z)$ given by (1.1) is said to be in the class $B_\Sigma(n, \beta, \lambda, \mu)$, $(n \in N_0, 0 < \beta \leq 1, \mu \geq 1, -1 < \lambda < 1)$, if the following conditions are satisfied:

$$f \in \Sigma \text{ and } \left|\arg\left\{\frac{(1-\mu)I_\lambda^n f(z) + \mu I_\lambda^{n+1} f(z)}{z}\right\}\right| < \frac{\beta\pi}{2}, \ (z \in U) \tag{2.1}$$

and

$$\left|\arg\left\{\frac{(1-\mu)I_\lambda^n g(\omega) + \mu I_\lambda^{n+1} g(\omega)}{\omega}\right\}\right| < \frac{\beta\pi}{2}, \ (\omega \in U), \tag{2.2}$$

where the function g is given by

$$g(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)\omega^3 - (5a_2^3 - 5a_2 a_3 + a_4)\omega^4 + \dots. \tag{2.3}$$

Throught this paper, we shall frequently use the notation

$$M_k = \left[(1-\mu) \left(\frac{k+\lambda}{1+\lambda} \right)^n + \mu \left(\frac{k+\lambda}{1+\lambda} \right)^{n+1} \right], \; k = 2, 3, \ldots .$$

By specializing the parameters in the class $B_\Sigma(n, \beta, \lambda, \mu)$, we obtain the following known subclasses studied earlier by various researchers.

1. $B_\Sigma(n, \beta, 0, \mu) \equiv B_\Sigma(n, \beta, \mu)$ studied by Porwal and Darus [11].
2. $B_\Sigma(0, \beta, 0, \mu) \equiv B_\Sigma(\beta, \mu)$ studied by Frasin and Aouf [6].
3. $B_\Sigma(0, \beta, 0, 1) \equiv B_\Sigma(\beta)$ studied by Srivastava *et al.* [14].

Theorem 2.2. *Let the function $f(z)$ given by (1.1) be in the class $B_\Sigma(n, \beta, \lambda, \mu)$, $n \in N_0, 0 < \beta \leq 1$ and $\mu \geq 1, -1 < \lambda \leq 1$. Then*

$$|a_2| \leq \frac{2\beta}{\sqrt{2M_3\beta + M_2^2(1-\beta)}}, \tag{2.4}$$

$$|a_3| \leq \frac{2\beta}{M_3}, \tag{2.5}$$

and

$$|a_4| \leq \begin{cases} \frac{\beta}{M_4}\left[2 + \frac{4(1-\beta)}{3}\left\{\frac{2M_3\beta + (1-2\beta){M_2}^2}{2M_3\beta + {M_2}^2(1-\beta)}\right\}\right], & \left(0 < \beta \leq \frac{1}{2}\right) \\ \frac{\beta}{M_4}\left[2 + \frac{4(1-\beta)}{3}\left\{\frac{2M_3\beta + (2\beta-1){M_2}^2}{2M_3\beta + {M_2}^2(1-\beta)}\right\}\right], & \left(\frac{1}{2} \leq \beta \leq 1\right) \end{cases}. \tag{2.6}$$

Proof. It follows from (2.1) and (2.2) that

$$\frac{(1-\mu)I_\lambda^n f(z) + \mu I_\lambda^{n+1} f(z)}{z} = [p(z)]^\beta, \tag{2.7}$$

and

$$\frac{(1-\mu)I_\lambda^n g(\omega) + \mu I_\lambda^{n+1} g(\omega)}{\omega} = [q(\omega)]^\beta, \tag{2.8}$$

where $p(z)$ and $q(\omega)$ in P and have the forms

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots \tag{2.9}$$

and

$$q(\omega) = 1 + q_1\omega + q_2\omega^2 + q_3\omega^3 + \ldots \tag{2.10}$$

Now, equating the coefficients in (2.7) and (2.8), we obtain

$$M_2 a_2 = \beta p_1 \tag{2.11}$$

$$M_3 a_3 = \beta p_2 + \frac{\beta(\beta-1)}{2} p_1^2, \tag{2.12}$$

$$M_4 a_4 = \beta p_3 + \beta(\beta-1) p_1 p_2 + \frac{\beta(\beta-1)(\beta-2)}{6} p_1^3, \tag{2.13}$$

$$-M_2 a_2 = \beta q_1, \tag{2.14}$$

$$M_3(2a_2^2 - a_3) = \beta q_2 + \frac{\beta(\beta-1)}{2} q_1^2, \tag{2.15}$$

$$-M_4(5a_2^3 - 5a_2a_3 + a_4) = \beta q_3 + \beta(\beta-1) q_1 q_2 + \frac{\beta(\beta-1)(\beta-2)}{6} q_1^3, \tag{2.16}$$

where

$$M_k = \left[(1-\mu)\left(\frac{k+\lambda}{1+\lambda}\right)^n + \mu\left(\frac{k+\lambda}{1+\lambda}\right)^{n+1}\right].$$

From (2.11) and (2.14), we obtain

$$p_1 = -q_1. \tag{2.17}$$

We shall obtain a estimates on $|p_1|$ for use in the estimates of $|a_2|$, $|a_3|$ and $|a_4|$. For this purpose we first add (2.12) and (2.15), we get

$$2M_3 a_2^2 = \beta(p_2+q_2) + \frac{\beta(\beta-1)}{2}(p_1^2+q_1^2).$$

Using (2.17) in last equation, we have

$$2M_3 a_2^2 = \beta(p_2+q_2) + \beta(\beta-1)p_1^2.$$

Putting $a_2 = \frac{\beta p_1}{M_2}$ from (2.11), we have after simplification

$$p_1^2 = \frac{(p_2+q_2)\,M_2^2}{2M_3\beta + M_2^2(1-\beta)}. \tag{2.18}$$

By applying $|p_2| \le 2$ and $|q_2| \le 2$, we get

$$|p_1| \le \frac{2M_2}{\sqrt{2M_3\beta + M_2^2(1-\beta)}}.$$

Therefore

$$|a_2| \le \frac{2\beta}{\sqrt{2M_3\beta + M_2^2(1-\beta)}}.$$

To find a bound on $|a_3|$ we may express a_3 in terms of p_1, p_2, q_1 and q_2. For this purpose we first subtract (2.15) from (2.12), we get

$$2M_3(a_3 - a_2^2) = \beta(p_2-q_2) + \frac{\beta(\beta-1)}{2}(p_1^2 - q_1^2).$$

Using (2.17) in last equations, we have

$$2M_3 a_3 = 2M_3 a_2^2 + \beta(p_2-q_2). \tag{2.19}$$

Putting $a_2 = \frac{\beta p_1}{M_2}$ from (2.11) and using (2.18), we get

$$\begin{aligned}
2M_3 a_3 &= 2M_3\left(\frac{\beta p_1}{M_2}\right)^2 + \beta(p_2-q_2)\\
&= \frac{2M_3\beta^2\,(p_2+q_2)}{2M_3\beta + M_2^2(1-\beta)} + \beta(p_2-q_2)\\
&= \beta\left[\frac{\left(4M_3\beta + M_2^2(1-\beta)\right)p_2 - M_2^2(1-\beta)q_2}{2M_3\beta + M_2^2(1-\beta)}\right].
\end{aligned}$$

Using the inequalities $|p_2| \le 2$ and $|q_2| \le 2$ and after simple calculation, we have

$$|a_3| \le \frac{2\beta}{M_3}.$$

We shall next find an estimates on $|a_4|$. At first we shall derive a relation connecting p_1, p_2, p_3, q_1, q_2 and q_3. To this end, first we add the equations (2.13) and (2.16), we get

$$-M_4\left(5a_2^3 - 5a_2a_3\right) = \beta(p_3+q_3) + \beta(\beta-1)(p_1p_2+q_1q_2) + \frac{\beta(\beta-1)(\beta-2)}{6}(p_1^3+q_1^3).$$

Using (2.17) and (2.19), we have

$$\frac{5M_4\beta(p_2-q_2)a_2}{2M_3} = \beta(p_3+q_3) + \beta(\beta-1)p_1(p_2-q_2).$$

Using (2.11) and after simple calculation we have

$$p_1(p_2-q_2) = \frac{2M_2M_3(p_3+q_3)}{5M_4\beta + 2M_2M_3(1-\beta)}. \tag{2.20}$$

We wish to express $|a_4|$ in terms of p_1, p_2, p_3, q_1, q_2 and q_3. For this purpose subtracting equation (2.16) from (2.13), we get

$$\begin{aligned} M_4\left(2a_4 + 5a_2^3 - 5a_2a_3\right) &= \beta(p_3-q_3) + \beta(\beta-1)p_1(p_2+q_2) \\ &\quad + \frac{\beta(\beta-1)(\beta-2)}{6}(p_1^3 - q_1^3). \end{aligned}$$

Using (2.17), (2.19) and after simple calculation we have

$$2M_4a_4 - \frac{5M_4a_2\beta}{2M_3}(p_2-q_2) = \beta(p_3-q_3) + \beta(\beta-1)p_1(p_2+q_2) + \frac{\beta(\beta-1)(\beta-2)}{3}p_1^3.$$

Again using equations (2.11) and (2.18), we get

$$\begin{aligned} 2M_4a_4 &= \frac{5M_4\beta}{2M_3}\frac{\beta p_1}{M_2}(p_2-q_2) + \beta(p_3-q_3) + \beta(\beta-1)p_1(p_2+q_2) \\ &\quad + \frac{\beta(\beta-1)(\beta-2)}{3}p_1\frac{(p_2+q_2)\,M_2^2}{2M_3\beta + M_2^2(1-\beta)}. \end{aligned}$$

Using (2.20) we have

$$\begin{aligned} 2M_4a_4 &= \beta\left[\frac{5M_4\beta(p_3+q_3)}{5M_4\beta + 2M_2M_3(1-\beta)} + p_3 - q_3 + (\beta-1)p_1(p_2+q_2)\right. \\ &\quad \left. + \frac{(\beta-1)(\beta-2)}{3}p_1\frac{(p_2+q_2)\,M_2^2}{2M_3\beta + M_2^2(1-\beta)}\right] \\ &= \beta\left[\frac{\left(10M_4\beta + 2M_2M_3(1-\beta)\right)p_3 - 2M_2M_3(1-\beta)q_3}{5M_4\beta + 2M_2M_3(1-\beta)}\right. \\ &\quad \left. + (1-\beta)p_1(p_2+q_2)\left\{\frac{(2-\beta)M_2^2}{3(2M_3\beta + M_2^2(1-\beta))} - 1\right\}\right] \end{aligned}$$

Using Lemma 1.1 and after simple calculation, we have

$$|a_4| \le \begin{cases} \frac{\beta}{M_4}\left[2 + \frac{4(1-\beta)}{3}\left\{\frac{2M_3\beta + (1-2\beta)\,{M_2}^2}{2M_3\beta + {M_2}^2\,(1-\beta)}\right\}\right], & \left(0 < \beta \le \frac{1}{2}\right) \\ \frac{\beta}{M_4}\left[2 + \frac{4(1-\beta)}{3}\left\{\frac{2M_3\beta + (2\beta-1)\,{M_2}^2}{2M_3\beta + {M_2}^2\,(1-\beta)}\right\}\right], & \left(\frac{1}{2} \le \beta \le 1\right). \end{cases} \qquad \square$$

3. Coefficient bounds for the function class $H_\Sigma(n,\gamma,\lambda,\mu)$

Definition 3.1. A function $f(z)$ given by (1.1) is said to be in the class $H_\Sigma(n,\gamma,\lambda,\mu)$, $(n \in N_0, 0 \le \gamma < 1, \mu \ge 1, -1 < \lambda \le 1)$, if the following conditions are satisfied:

$$f \in \Sigma \text{ and } \Re\left\{\frac{(1-\mu)I_\lambda^n f(z) + \mu I_\lambda^{n+1} f(z)}{z}\right\} > \gamma,\ (z \in U) \tag{3.1}$$

and

$$\Re\left\{\frac{(1-\mu)I_\lambda^n g(\omega) + \lambda I_\lambda^{n+1} g(\omega)}{\omega}\right\} > \gamma,\ (\omega \in U), \tag{3.2}$$

where the function g is defined by (2.3).

By specializing the parameters in the class $H_\Sigma(n,\gamma,\lambda,\mu)$, we obtain the following known subclasses studied earlier by various researchers

1. $H_\Sigma(n,\gamma,0,\mu) \equiv H_\Sigma(n,\gamma,\mu)$ studied by Porwal and Darus [11].
2. $H_\Sigma(0,\gamma,0,\mu) \equiv H_\Sigma(\gamma,\mu)$ studied by Frasin and Aouf [6].
3. $H_\Sigma(0,\gamma,0,1) \equiv H_\Sigma(\gamma)$ studied by Srivastava *et al.* [14].

Theorem 3.2. *Let the function $f(z)$ given by (1.1) be in the class $H_\Sigma(n,\gamma,\lambda,\mu)$, $n \in N_0, 0 \le \gamma < 1, \mu \ge 1, -1 < \lambda \le 1$. Then*

$$|a_2| \le \sqrt{\frac{2(1-\gamma)}{M_3}}, \tag{3.3}$$

$$|a_3| \le \frac{2(1-\gamma)}{M_3}, \tag{3.4}$$

and

$$|a_4| \le \frac{2(1-\gamma)}{M_4}. \tag{3.5}$$

Proof. It follows from (3.1) and (3.2) that there exists $p(z) \in P$ and $q(\omega) \in P$ such that

$$\frac{(1-\mu)I_\lambda^n f(z) + \mu I_\lambda^{n+1} f(z)}{z} = \gamma + (1-\gamma)p(z) \tag{3.6}$$

and

$$\frac{(1-\mu)I_\lambda^n g(\omega) + \mu I_\lambda^{n+1} g(\omega)}{\omega} = \gamma + (1-\gamma)q(\omega), \tag{3.7}$$

where $p(z)$ and $q(\omega)$ have the forms (2.9) and (2.10), respectively. Equating coefficients in (3.6) and (3.7) yields

$$M_2 a_2 = (1-\gamma)p_1, \tag{3.8}$$

$$M_3 a_3 = (1-\gamma)p_2, \tag{3.9}$$

$$M_4 a_4 = (1-\gamma)p_3, \tag{3.10}$$

$$-M_2 a_2 = (1-\gamma)q_1, \tag{3.11}$$

$$M_3(2a_2^2 - a_3) = (1-\gamma)q_2 \tag{3.12}$$

and

$$-M_4(5a_2^3 - 5a_2a_3 + a_4) = (1-\gamma)q_3. \tag{3.13}$$

From (3.8) and (3.11), we have

$$p_1 = -q_1. \tag{3.14}$$

Adding equation (3.9) and (3.14), we get

$$2M_3 a_2^2 = (1-\gamma)(p_2 + q_2).$$

Putting $a_2 = \frac{(1-\gamma)p_1}{M_2}$ from (3.8), we have

$$p_1^2 = \frac{(p_2 + q_2)M_2^2}{2M_3(1-\gamma)}. \tag{3.15}$$

By applying the inequalities $|p_2| \leq 2$ and $|q_2| \leq 2$, we get

$$|p_1| \leq \sqrt{\frac{2}{M_3(1-\gamma)}} M_2.$$

Therefore

$$|a_2| \leq \sqrt{\frac{2(1-\gamma)}{M_3}}.$$

To obtain a bound on $|a_3|$ we wish express in terms of p_1, p_2, q_1 and q_2. For this purpose subtracting (3.12) from (3.9), we get

$$2M_3(a_3 - a_2^2) = (1-\gamma)(p_2 - q_2). \tag{3.16}$$

Using (3.8), (3.15) and after simple calculation, we have

$$2M_3 a_3 = 2(1-\gamma)p_2.$$

Using $|p_2| \leq 2$ we have

$$|a_3| \leq \frac{2(1-\gamma)}{M_3}.$$

We shall next find an estimate on $|a_4|$. At first we shall derive a relation connecting p_1, p_2, p_3, q_1, q_2 and q_3. To this end, we first add the equations (3.10) and (3.13), we get

$$-M_4(5a_2^3 - 5a_2 a_3) = (1-\gamma)(p_3 + q_3).$$

Using (3.8), (3.16) and after simple calculation, we get

$$p_1(p_2 - q_2) = \frac{2M_2 M_3 (p_3 + q_3)}{5M_4(1-\gamma)}. \tag{3.17}$$

We wish to express a_4 in terms of p_1, p_2, p_3, q_1, q_2 and q_3. Now subtracting (3.13) from (3.10), we get

$$M_4(2a_4 + 5a_2^3 - 5a_2 a_3) = (1-\gamma)(p_3 - q_3).$$

Using (3.8), (3.16), (3.17) and after simple calculation

$$2M_4 a_4 = 2(1-\gamma)p_3.$$

Using inequality $|p_3| \leq 2$, we have

$$|a_4| \leq \frac{2(1-\gamma)}{M_4}. \qquad \square$$

Remark 3.3. If we put $\lambda = 0$ in Theorems 2.2 and 3.2, then our estimate on $|a_3|$ improves the corresponding results of Porwal and Darus [11].

Remark 3.4. If we put $n = 0$, $\lambda = 0$ in Theorems 2.2 and 3.2, then our estimate on $|a_3|$ improves the corresponding results due to Frasin and Aouf [6].

Remark 3.5. If we put $n = 0$, $\lambda = 0$, $\mu = 1$ in Theorems 2.2 and 3.2, then our estimate on $|a_3|$ improves the corresponding results due to Srivastava *et al.* [14].

Remark 3.6. Sharp estimates for the coefficients $|a_2|$, $|a_3|$ and other coefficients of functions belonging to the classes investigated in this paper are yet open problems. Indeed it would be of interest even to find estimates (not necessarily sharp) for $|a_n|,\ n \geq 5$.

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