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BOOK REVIEWS

K. Burns and M. Gidea, ***Differential Geometry and Topology With a View to Dynamical Systems***, Chapman & Hall/CRC (Studies in Advanced Mathematics), 2005, ISBN 1-58488-253-0, 978-1-58488-253-4, Hardcover, IX+389 pp.

This is a graduate course on the topology and differential geometry of smooth manifolds, introducing, in parallel, the basic notions of smooth dynamical systems.

The first two chapters of the book introduce the basics of differential topology (manifolds and maps, the tangent bundle, immersions, submersions, embeddings, submanifolds, critical points, the Sard's theorem) and vector fields and the associated dynamical systems. The following three chapters make up a concise introduction to Riemannian geometry, covering most of the standard material (Riemannian metrics, connections, geodesics, the exponential map, minimal geodesics, the Riemannian distance, Riemannian curvature, Riemannian submanifolds, sectional and Ricci curvature, Jacobi fields and conjugate points,manifolds of constant curvature). Chapter 6, *Tensors and Differential Forms*, is devoted, essentially, to integration theory of manifolds, as well to the de Rham cohomology. It is also, introduced the singular homology and it is given a proof of the de Rham theorem. Chapter 7 is concerned with some global results in the theory of smooth manifolds and Riemannian geometry (the Brouwer degree, the intersection number, the fixed point index, the Lefschetz number, the Euler characteristic and the Gauss-Bonnet theorem), while the chapter 8 covers the basic notions and results of Morse theory. Finally, the chapter 9 provides a short introduction to the theory of hyperbolic dynamical systems.

There are plenty of worked examples in the book and each chapter ends with a comprehensive list of exercises. Another feature that has to be remarked is the presence of a great number of very suggestive and well realized graphical illustration.

The book is very well written, in a very pedagogical manner and it covers a lot of material in a very clear way. I think this is an ideal introduction to differential geometry and topology for beginning graduate students or advanced undergraduate students in mathematics, but it will be, also, useful to physicists or other scientists with an interests in differential geometry and dynamical systems.

Paul Blaga

Donal O'Regan, Yeol Je Cho and Yu-Qing Chen, *Topological Degree Theory and Applications*, Series in Mathematical Analysis and Applications (R.P. Agarwal and D. O'Regan eds.), Vol. 10, Chapman & Hall/CRC, Taylor & Francis Group, Boca Raton, 2006, 221 pp, ISBN 1-58488-648-X.

The degree theory for continuous maps on finite dimensional spaces was created by Brouwer in 1910-1912, and later, for compact maps on infinite dimensional spaces, by Leray and Schauder in 1934, and it has become one of the most useful tool in nonlinear analysis. Since the 1960s, several extensions have been done for various classes of non-compact type maps. The present book focuses on topological degree theory in normed spaces and its applications to integral, ordinary differential and partial differential equations.

The Contents are as follows: Chapter 1: *Brouwer degree theory*, presents the construction of Brouwer degree, the degree for vanishing mean oscillation functions, and in particular for Sobolev maps, of Brezis and Nirenberg (1995), and applications to periodic and anti-periodic problems for ordinary differential equations in $\mathbf{R}^n$.

Chapter 2: *Leray-Schauder degree theory*, starts with the basic result on the approximation of a compact map by finite dimensional maps and presents the Leray-Schauder extension of Brouwer degree to compact maps in Banach spaces. Then a degree theory is described for upper semicontinuous compact maps with closed convex values. Applications are given to bifurcations and to the existence of solutions of the Cauchy problem, the Dirichlet problem for a second order partial differential equation, and anti-periodic problems in Hilbert spaces.

Chapter 3: *Degree theory for set contractive maps*, presents the degree theory for k-set contraction maps and condensing maps and some applications to the initial value problem and anti-periodic problems for ordinary differential equations in Banach spaces.

Chapter 4: *Generalized degree theory for A-proper maps*, is devoted to Petryshyn's generalized degree theory and some typical applications to periodic problem for second order differential equations and semilinear wave equation.

Chapter 5: *Coincidence degree theory*, introduces Mawhin's degree for L-compact maps and gives application to periodic ordinary differential equations.

Chapter 6: *Degree theory for monotone-type maps*, presents basic contributions of Skrypnik, Browder, Berkovitz, Mustonen, Kartsatos and others, to the construction of the degree for monotone-type maps. Applications to evolution equations are included.

Chapter 7: *Fixed point index theory*, is in connection with the problem of the existence of non-negative solutions (in a cone) to operator equations. After defining the fixed point index, the authors present a variety of fixed point theorems of compression-expansion type in cones of Banach spaces and give applications to integral and differential equations.

The book is very well written, presents essential ideas and results with typical applications, being extremely useful to the beginners in nonlinear analysis. Each chapter of the book is concluded by a section of exercises and the bibliography contains 314 titles.

This is really a valuable text for self-study and special courses in nonlinear analysis and also a good reference for anyone applying topological methods to integral, ordinary and partial differential equations.

Radu Precup

Rajendra Bhatia, ***Positive Definite matrices,*** Princeton Series in Applied Mathematics, Princeton University Press, Princeton and Oxford 2007, ix + 254 pp., ISBN 0-691-12918-5.

Denote by $\mathcal{H}$ be the n-dimensional Hilbert space $\mathbb{C}^n$ with inner product $\langle x, y\rangle$. Let $\mathcal{L}(\mathcal{H})$ be the space of all linear operators on $\mathcal{H}$ and $\mathbb{M}_n = \mathbb{M}_n(\mathbb{C})$ the space of $n \times n$-matrices over $\mathbb{C}$. An operator $A \in \mathcal{L}(\mathcal{H})$ is identified with the associated matrix, denoted also by A, with respect to the standard basis $\{e_j\}$ of $\mathbb{C}^n$. A matrix A in $\mathbb{M}_n$ is called positive if $\langle x, Ax\rangle \geq 0$ for all $x \in \mathbb{C}^n$, and positive definite (or strictly positive) if $\langle x, Ax\rangle > 0$ for all $x \neq 0$.

The present book is devoted to the study of positive matrices, positive linear maps, and positive definite functions. This is a rich field with numerous interesting results and with deep and far reaching applications. One of the domains of application, of great interest in the last time, is quantum information theory, where the quantum communication channels are thought as completely positive trace preserving linear maps. The author presents in the fourth chapter two fundamental results in quantum entropy - the inequalities of Lieb-Ruskai and Furuta's inequality. Recall that the quantum entropy of a positive definite matrix A was defined in 1927 by J. von Neumann by the formula $S(A) = -\mathrm{tr}(A\log A)$.

The first chapter of the book, 1. *Positive matrices,* presents the basic notions and results: characterizations of positivity, the Schur product, block matrices. Note that, as it was shown by T. Ando and M.-D. Choi in 1986, the $2\times$ 2-block matrices play a crucial role in the proofs of many results on positive matrices, a point made very clear by the author of the book too.

Although many results in Chapters 2. *Positive linear maps,* and 3. *Completely positive maps,* hold in the more general framework of C^*-algebras, the presentation is restricted to their finite dimensional versions (called "toy versions" by the author), which are sufficient for matrix theory and for the applications as well.

As we did mention, in Chapter 4. *Matrix means,* some spectacular applications of various means for matrices to convex matrices and to quantum information theory are presented.

Chpater 5. *Positive definite functions,* is devoted to the study of positive definite functions from $\mathbb{R}$ to $\mathbb{C}$. There are proved the fundamental theorems of Herglotz and Bochner and applications to various matrix inequalities and to the study of Loewner matrices are given.

In the last chapter of the book, 6. *Geometry of positive matrices,* the set of positive matrices is studied as a Riemannian manifold of nonpositive curvature, a domain of very active current research, mainly due to the results of M. Gromov. This is a promising area of investigation, of great interest to analysts and geometers as well.

Written by an expert in the area, the book presents in an accessible manner a lot of important results from the realm of positive matrices and of their applications. Although, in some places, references to the author's book, *Matrix Analysis* (MA), Springer 1997, are made, the present one is practically self-contained and can be read independently of MA.

The book can be used for graduate courses in linear algebra, or as supplementary material for courses in operator theory, and as a reference book by engineers and researchers working in the applied field of quantum information.

S. Cobzaş

M. de Gosson, *Symplectic Geometry and Quantum Mechanics*, Birkhäuser (Operator Theory and Applications, vol. 166), 2006, Hardback, 367 pp., ISBN-10: 3-7643-7574-4, ISBN-13: 978-3-7643-7574-4.

Hamiltonian formalism lies at the very heart of quantum mechanics. In the recent decades, Hamiltonian mechanics "happily married" differential geometry, giving birth to one of the most beautiful parts of geometry, symplectic geometry. This kind of geometry was quite successfully applied to quantum mechanics in the so-called geometric quantization approach. Several monographs on geometric quantization are available by now, but the focus mainly on the geometrical formalism without doing justice to quantum mechanics. It is the aim of this book to correct this deficiency.

The book has three parts. The first one is a detailed exposition of the basic notions of symplectic geometry, as well as of those of an extension of it, the so-called multiply-oriented symplectic geometry, or q-symplectic geometry. In particular, there are studied a series of indices, essential in this field (e.g. the Arnold-Leray-Maslov and the Conley-Zehnder indices).

The second part is dedicated to the Heisenberg group, the Weyl calculus and metaplectic group, while the final part is more physically-oriented. It begins with a geometrical approach to the uncertainty principle from quantum mechanics and its connections to the symplectic capacity. It follows a rigorous treatment of the density matrix, by using the Hilbert-Schmidt and trace-class operators and, finally, the Weyl pseudo-differential calculus is extended to the phase space, via the Stone-von Neumann theorem on the irreducible representation of the Heisenberg group. Several appendices review some standard mathematical material (classical Lie groups, covering spaces, pseudo-differential operators, elementary probability theory).

The book is very clearly written, by one of the most active researchers in the field, and, in my opinion, it successfully manages to fill a gap in the mathematical physics literature. It will be very useful for graduate students and researchers both in theoretical physics and geometry.

Paul A. Blaga

A. Mallios, *Modern Differential Geometry in Gauge Theories, Maxwell Fields*, volume I, Birkhäuser, 2006, 293 pp., Paperback, ISBN-10: 0-8176-4378-8, ISBN-13: 978-0-8176-4378-2.

Much of the differential geometry of a smooth manifold X can be built staring from a small number of objects: the sheaf of smooth functions $\mathcal{C}^\infty_X$, the sheaf of differential forms and the differential associating to functions 1-forms. For instance, vector bundles are just projective, locally free $\mathcal{C}^\infty_X$-modules, the connections can also be constructed easily out of the three mentioned objects.

This book starts with a more general framework: a space X (which is not necessarily a manifold), two sheaves $\mathcal{A}$ and $\mathcal{E}$ on X and a sheaf morphism $\partial\mathcal{A} \to \mathcal{E}$, having similar properties to those of the ordinary differential of functions (linearity and a kind of Leibniz property). Such a triple is called a *triad* on the space X. There are introduced, then, some generalizations of vector bundles, through the so-called *vector sheaves*, which are just locally-free sheaves of $\mathcal{A}$-modules.

All the constructions from differential geometry (connections, metrics, curvature and torsion tensors) can be carried out in this generalized context. The theory obtained is called *abstract differential geometry* (ADG).

The book under review is the first volume of a two-volume work dedicated to the applications of ADG to gauge theories. This first volume focuses only on electromagnetic fields (Maxwell theory). It first gives a review of ADG, then it recasts the classification of elementary particles by the spin structure in terms of sheaves. The next two chapters are devoted to electromagnetic fields and their classification in terms of sheaf cohomology. The final chapter is dedicated to the reformulation of geometric quantization in the language of abstract differential geometry.

Many of the results from this monographs belong to the author or to his collaborators. He, is, in fact, one of the founders of ADG. The book is very well written and it brings a fresh approach to gauge theories, that will probably be of a great help both to theoretical physicists and geometers.

Paul Blaga