STUDIA UNIV. "BABEŞ–BOLYAI", MATHEMATICA, Volume **LII**, Number 2, June 2007

ON THE COMPLETENESS OF THE SEMIHYPERGROUPS ASSOCIATED TO BINARY RELATIONS

HORAŢIU OLARU, COSMIN PELEA, AND IOAN PURDEA

Dedicated to Professor Grigore Călugăreanu on his 60th birthday

Abstract. We will determine the complete hypergroupoids, semihypergroups or hypergroups determined by binary relations (more exacly, by monounary multialgebras) an we will study some closure properties of their classes with respect to the products of some categories where they are contained.

1. Introduction

On the basis of [1, 7], in [4, 5] C. Pelea and I. Purdea started an investigation on some constructions of hypergroupoids associated to binary relations. This paper continues the investigation of C. Pelea and I. Purdea from the point of view of the completeness of the multialgebras involved in this discussion, which is another problem studied by Pelea and Purdea (for general multialgebras) in [3]. So, we will give a characterization for the complete hypergroupoids, complete semihypergroups and complete hypergroups associated to monounary multialgebras (hence with binary relations). Even if the subcategory of the hypergroupoids determined by monounary multialgebras is not closed under direct product (i.e. under the product from the category of hypergroupoids) the completeness condition seems to fix this problem. This is not surprising since we will see that the completeness condition on a hypergroupoid determined by a monounary multialgebra is very restrictive. We will see that the

Received by the editors: 11.03.2007.

2000 *Mathematics Subject Classification.* 20N99, 20N20.

Key words and phrases. semihypergroup associated to a binary relation, semihypergroup associated to a monounary multialgebra, complete semihypergroup, complete hypergroup.

complete hypergroupoids determined by monounary multialgebras coincide with the complete semihypergroups determined by monounary multialgebras, so, we deal most of the time with semihypergroups determined by monounary multialgebras. We will also be able to adapt the results we obtain to hypergroups determined by monounary multialgebras. We mention that the categorical notions are not complicated and they can be found in [6].

2. **Preliminaries**

Let H be a set and let R be a binary relation on H. Denote the inverse of the relation R by $\overset{-1}{R}$. For $x_1, \ldots, x_n \in H$, $X \subseteq H$ we denote

$$R(X) = \{y \in H \mid \exists x \in X : xRy\} \text{ and } R(x_1, \ldots, x_n) = R(\{x_1, \ldots, x_n\}).$$

As in [7], one can associate to R the partial hypergroupoid $H_R = (H, \circ)$ defined by

$$x \circ y = R(x, y).$$

It is obvious that $x^2 = x \circ x = R(x) = \{y \in H \mid xRy\}$ and

$$x \circ y = x^2 \cup y^2, \ \forall x, y \in H. \tag{1}$$

Lemma 1. [7, Lemma 1] *Let H be a set and let R be a binary relation on H. The partial hypergroupoid $H_R = (H, \circ)$ is a hypergroupoid if and only if $\overset{-1}{R}(H) = H$.*

An element $x \in H$ is an *outer element* of (the relation) R if there exists $h \in H$ such that $(h, x) \notin R^2$.

Proposition 1. [7, Proposition 2] *If H be a set and R is a binary relation on H with $\overset{-1}{R}(H) = H$ then H_R is a semihypergroup if and only if $R \subseteq R^2$ and*

$$(a, x) \in R^2 \Rightarrow (a, x) \in R$$

whenever x is an outer element of R.

Proposition 2. [7] *Let $H \neq \emptyset$ and let R be a binary relation on H. The hypergroupoid H_R is a hypergroup if and only if the following conditions hold:*

1) $\overset{-1}{R}(H) = H$;

2) $R(H) = H$;

3) $R \subseteq R^2$;

4) *whenever x is an outer element of R we have*

$$(a, x) \in R^2 \Rightarrow (a, x) \in R.$$

Let (H, R), (H', R') be relational systems with binary relations and $h : H \to H'$. One says that h is a *homomorphism* of relational systems if

$$xRy \Rightarrow h(x)R'h(y).$$

Let $(H, \circ)$, $(H', \circ')$ be hypergroupoids. A mapping $h : H \to H'$ is called *homomorphism* (of hypergroupoids) if

$$h(x \circ y) \subseteq h(x) \circ' h(y), \ \forall x, y \in H.$$

Remark 1. If R is a binary relation on H with $\overset{-1}{R}(H) = H$, we can see (H, R) as the multialgebra (H, f) with one unary multioperation $f : H \to P^*(H)$ defined by

$$xRy \Leftrightarrow y \in f(x). \tag{2}$$

An element $x \in H$ is *outer element* (of (H, f)) if there exists $h \in H$ such that $x \notin f(f(h))$. An element $x \in H$ is an *inner element* if it is not an outer element.

Remark 2. If (H', R') is also a relational system for which $\overset{-1}{R'}(H') = H'$ and (H', f') is the corresponding monounary multialgebra then h is a relational homomorphism between (H, R) and (H', R') if and only if h is a homomorphism between the multialgebras (H, f) and (H', f'). If $\mathcal{R}_2$ denotes the category of the relational systems with one binary relation (having as morphisms the homomorphisms of relational systems and as product the usual composition of homomorphisms) and $\mathcal{R}_2'$ its (full) subcategory consisting in the relational systems (H, R) for which $\overset{-1}{R}(H) = H$. The identification we made in the previous remark gives a categorical isomorphism between $\mathcal{R}_2'$ and the category **Malg**(1) of the monounary multialgebras (i.e. the multialgebras of type (1)), where the morphisms are the multialgebra homomorphisms and the product of two morphisms is the usual composition of homomorphisms.

The hypergroupoids (or semihypergroups, or hypergroups) associated to binary relations can be seen as hypergroupoids (or semihypergroups, or hypergroups) associated to monounary multialgebras (H, f) using the translation of (1) in the terms of the unary multioperation f. Thus we have

$$f(X) = \bigcup_{x \in X} f(x) \text{ and } x \circ y = f(\{x, y\}) = f(x) \cup f(y) (= x^2 \cup y^2)$$

for any $X \subseteq H,\ X \neq \emptyset$, $x, y \in H$ and Lema 1 can be rewritten as below:

Lemma 2. *For any multialgebra (H, f) with one unary multioperation, the equality*

$$x \circ y = f(\{x, y\})$$

defines a hypergroupoid $H_f = (H, \circ)$.

Propositions 1 and 2 can be restated as follows:

Proposition 3. *Let (H, f) be a multialgebra with one unary multioperation. The hypergroupoid H_f is a semihypergroup if and only if*

$$f(x) \subseteq f(f(x)),\ \forall x \in H$$

and for any outer element $x \in H$,

$$x \in f(f(a)) \Rightarrow x \in f(a).$$

Proposition 4. *Let $H \neq \emptyset$ and let (H, f) be a multialgebra with one unary multioperation. The hypergroupoid H_f is a hypergroup if and only if the following conditions hold:*

i) $f(H) = H$;
ii) $f(x) \subseteq f(f(x)),\ \forall x \in H$;
iii) *whenever x is an outer element we have*

$$x \in f(f(a)) \Rightarrow x \in f(a).$$

In [7, Proposition 3], Rosenberg determines the semihypergroups which can be obtained from a binary relations using (1). We restated the result of Rosenberg as follows:

Proposition 5. *Let $(H,*)$ be a hypergroupoid. There exists a binary relation R on H such that $(H,*) = H_R$ if and only if* (1) *holds. A hypergroupoid $(H,*)$ which satisfies the condition* (1) *is a semihypergroup if and only if it verifies the conditions:*

$$x^2 \subseteq (x^2)^2 \text{ and } (x^2)^2 \cap (H \setminus (y^2)^2) \subseteq x^2, \ \forall x, y \in H. \tag{3}$$

A hypergroupoid $(H,*)$ which satisfies the conditions (1) *and* (3) *is a hypergroup if and only if $\bigcup_{x\in H} x^2 = H$.*

The binary relation $R \subseteq H \times H$ from Rosenberg's proof is defined by

$$xRy \Leftrightarrow y \in x * x,$$

$\overset{-1}{R}(H) = H$ and the corresponding unary multioperation is

$$f_* : H \to P^*(H), \ f_*(x) = x * x.$$

Remark 3. The conditions (3) are the (equivalent) translation of the conditions 3) and 4) from Proposition 2 in the terms of the hyperoperation $*$. So, a hypergroupoid (or semihypergroup, or hypergroup) $(H,*)$ is determined by a unary multioperation f on H if and only if $(H,*)$ satisfies the condition (1).

In the category **Malg**(2) of hypergroupoids – the morphisms are the hypergroupoid homomorphisms and the product of two morphisms is the usual composition of homomorphisms – we consider the following subcatgories: the subcategory **Malg**$'$(2) of the hypergroupoids satisfying (1), the subcategory **SHG**$'$ whose objects are the semihypergroups which satisfy (1) and the subcategory **HG**$'$ whose objects are the hypergroups which satisfy (1). We also denote by **Malg**$'$(1) the full subcategory of **Malg**(1) whose objects are the monounary multialgebras (H,f) which satisfy the conditions ii),iii) from Proposition 4 and by **Malg**$''$(1) the full subcategory of **Malg**(1) whose objects are the monounary multialgebras (H,f), with $H \neq \emptyset$, which satisfy the conditions i), ii), iii) from Proposition 4.

Remark 4. [4, Corollaries 3, 4] The correspondences

$$(H,f) \mapsto H_f, \ h \mapsto h$$

define three covariant functors

$$F : \mathbf{Malg}(1) \to \mathbf{Malg}'(2),\ F' : \mathbf{Malg}'(1) \to \mathbf{SHG}',\ \text{and } F'' : \mathbf{Malg}''(1) \to \mathbf{HG}'.$$

These functors are isomorphisms of categories and their inverses are the functors

$$G : \mathbf{Malg}'(2) \to \mathbf{Malg}(1),\ G' : \mathbf{SHG}' \to \mathbf{Malg}'(1),\ \text{and } G'' : \mathbf{HG}' \to \mathbf{Malg}''(1),$$

respectively, given by

$$(H, *) \mapsto (H, f_*),\ h \mapsto h.$$

3. Complete semihypergroups associated to monounary multialgebras

In this section we will determine those monounary multialgebras which determine complete semihypergroups and complete hypergroups.

First, remember that a *multialgebra* $\mathfrak{A} = (A, (f_\gamma)_{\gamma<o(\tau)})$ of type τ is *complete* if for any $m, n \in \mathbb{N}$, any

$$\mathbf{q} \in P^{(m)}(\tau) \setminus \{\mathbf{x}_i \mid i \in \{0, \dots, m-1\}\},\ \mathbf{r} \in P^{(n)}(\tau) \setminus \{\mathbf{x}_i \mid i \in \{0, \dots, n-1\}\},$$

and any $a_0, \dots, a_{m-1}, b_0, \dots, b_{n-1} \in A$,

$$q(a_0, \dots, a_{m-1}) \cap r(b_0, \dots, b_{n-1}) \neq \emptyset \ \Rightarrow\ q(a_0, \dots, a_{m-1}) = r(b_0, \dots, b_{n-1}). \tag{4}$$

Remark 5. For a monounary multialgebra (H, f), the images of the term functions involved in (4) are the nonempty subsets

$$f^n(x) = \underbrace{f(f(\dots(f}_{n}(x))\dots)),$$

with $n \in \mathbb{N}^*$ and $x \in H$, hence the monounary multialgebra (H, f) is complete if and only if for any $m, n \in \mathbb{N}$, and any $x, y \in H$,

$$f^m(x) \cap f^n(y) \neq \emptyset \Rightarrow f^m(x) = f^n(y).$$

Lemma 3. *Let $(H, \circ)$ be a semihypergroup which satisfy* (1). *The semihypergroup $(H, \circ)$ is complete only if the multialgebra $(H, f_\circ)$ satisfies the identity*

$$\mathbf{f}(\mathbf{f}(\mathbf{x})) = \mathbf{f}(\mathbf{x}) \tag{5}$$

(**f** *denotes the symbol of the multioperation corresponding to the monounary multialgebra type*).

Proof. According to Proposition 5, in the semihypergroup $(H, \circ) = H_{f_\circ}$ we have

$$x^2 \subseteq (x^2)^2, \ \forall x \in H,$$

and the completeness of $(H, \circ)$ leads us to the equalities

$$x^2 = (x^2)^2, \ \forall x \in H.$$

Let us remember that $x^2 = f_\circ(x)$ and

$$\begin{aligned}(x^2)^2 &= (x \circ x) \circ (x \circ x) = \bigcup\{y \circ z \mid y, z \in x \circ x\} = \bigcup\{y^2 \cup z^2 \mid y, z \in f_\circ(x)\} \\ &= \bigcup\{y^2 \mid y \in f_\circ(x)\} = \bigcup\{f_\circ(y) \mid y \in f_\circ(x)\} = f_\circ(f_\circ(x)).\end{aligned}$$

Thus we have

$$f_\circ(f_\circ(x)) = f_\circ(x), \ \forall x \in H,$$

hence the identity (5) is satisfied on the multialgebra $(H, f_\circ)$. □

Remark 6. From Remark 5 one deduce easily that a monounary multialgebra (H, f) satisfying (5) is complete if and only if $\{f(x) \mid x \in H\}$ is a partition of $f(H)$. In the terms of binary relations, if R is the binary relation from (2), this happens when the restriction of R to $R(H)$ is an equivalence relation on $R(H)$.

The condition that the monounary multialgebra (H, f) satisfies the identity (5) is stronger than the conditions from Proposition 3, but it is not sufficient for obtaining a complete semihypergroup H_f.

Example 1. Let $H = \{1, 2, 3\}$ and $f : H \to P^*(H), \ f(x) = x$. Clearly, (H, f) satisfies (5). Yet, the corresponding hypergroupoid $H_f = (H, \circ)$ is a hypergroup which is not complete since

$$1 \circ 2 = \{1, 2\} \neq \{2, 3\} = 2 \circ 3, \text{ even if } (1 \circ 2) \cap (2 \circ 3) = \{2\} \neq \emptyset.$$

Lemma 4. *Let $(H, \circ)$ be a hypergroupoid determined by a monounary multialgebra (H, f). If the hypergroupoid $(H, \circ)$ is complete then:*

a) $x \circ x = x \circ y,\ \forall x, y \in H$;

b) $f(x) = f(H),\ \forall x \in H$;

c) (H, f) *is a complete multialgebra;*

d) $(H, \circ)$ *is a commutative* (*complete*) *semihypergroup.*

Proof. a) Since $(H, \circ)$ is a complete hypergroupoid and

$$(x \circ x) \cap (x \circ y) = f(x) \cap (f(x) \cup f(y)) = f(x) \neq \emptyset,$$

for all $x, y \in H$, we have

$$x \circ x = x \circ y,\ \forall x, y \in H.$$

b) From a) follows that $f(x) = f(\{x, y\})$ for all $x, y \in H$, thus for each $x \in H$,

$$f(x) = \bigcup_{y \in H} f(\{x, y\}) = f\left(\bigcup_{y \in H} \{x, y\}\right) = f(H).$$

c) We apply Remark 5 and from b) follows that

$$f^n(x) = f(H),\ \forall n \in \mathbb{N}^*,\ \forall x \in H.$$

d) It is clear that any hypergroupoid determined by a monounary multialgebra is commutative. For any $x, y, z \in H$, using b) we have

$$(x \circ y) \circ z = f(\{x, y\}) \circ z = f(H) \circ z = f(H).$$

Analogously, $x \circ (y \circ z) = f(H)$. □

Lemma 5. *Any commutative hypergroupoid* $(H, \circ)$ *which satisfies the identity*

$$\mathbf{x} \circ \mathbf{x} = \mathbf{x} \circ \mathbf{y} \tag{6}$$

is a complete semihypergroup determined by a monounary multialgebra.

Proof. If a commutative hypergroupoid $(H, \circ)$ satisfies the identity (6) then

$$x \circ y = x \circ x = y \circ y = x^2 \cup y^2,\ \forall x, y \in H,$$

so $(H, \circ)$ satisfies (1). This means that $(H, \circ)$ is determined by (H, f), where

$$f : H \to P^*(H),\ f(x) = x \circ x.$$

Under these circumstances, (6) leads us, as in the proof of the previous lemma, to

$$x \circ y = f(\{x, y\}) = f(x) = f(H),\ \forall x, y \in H$$

and to the fact that the hypergroupoid $(H, \circ)$ is a semihypergroup. The completeness of this semihypergroup follows from the form of its hyperproducts: it is easy to prove by induction on $n \in \mathbb{N}^*$, $n \geq 2$ that

$$x_1 \circ \cdots \circ x_n = f(H),$$

for any $x_1, \ldots, x_n \in H$. □

Theorem 1. *Let $(H, \circ)$ be a hypergroupoid determined by a monounary multialgebra (H, f). The following conditions are equivalent:*

1) *$(H, \circ)$ is a complete hypergroupoid;*
2) *$(H, \circ)$ satisfies the identity (6);*
3) *$f(x) = f(H),\ \forall x \in H$;*
4) *$(H, \circ)$ is a complete semihypergroup.*

Proof. 1) ⇒ 2) ⇒ 3) follows as in the proof of Lemma 4.
3) ⇒ 4) follows as in the final part of the proof of Lemma 5.
4) ⇒ 1) is obvious. □

Corollary 1. *A hypergroupoid $(H, \circ)$ determined by a nonempty monounary multialgebra (H, f) is a complete hypergroup if and only if*

$$f(x) = H,\ \forall x \in H$$

and this happens exactly when the $(H, \circ)$ is the total hypergroup on H.

Indeed, if $(H, \circ)$ is a hypergroup then $f(H) = H$, hence its completeness implies $f(x) = f(H) = H$ for each $x \in H$. Conversely, if $f(x) = H$ for each $x \in H$ then $f(H) = H$, thus $(H, \circ)$ is a complete semihypergroup which is a hypergroup. The hyperproduct of this hypergroup is defined by

$$x \circ y = f(\{x, y\}) = f(H) = H,\ \forall x, y \in H$$

(i.e. $(H, \circ)$ is *the total hypergroup* on H).

Remark 7. The condition on (H, f) to be complete is not equivalent with the conditions from the Theorem 1. For instance, the monounary multialgebra (H, f) from Example 1 is complete and determines a (semi)hypergroup which is not complete since (H, f) does not satisfy the condition 3) from Theorem 1.

The following corollary is the translation of the above results in the terms of binary relations.

Corollary 2. *Let $H_R = (H, \circ)$ be the hypergroupoid determined by the binary relation R. Then H_R is a complete semihypergroup if and only if*

$$R(x) = R(H), \ \forall x \in H.$$

If $H \neq \emptyset$ then H_R is a complete hypergroup if and only if

$$R(x) = H, \ \forall x \in H.$$

4. Products of complete semihypergroups associated to monounary multialgebras

Let $((H_i, f_i) \mid i \in I)$ be a family of monounary multialgebras. The direct product of the multialgebras (H_i, f_i) is the multialgebra $\left(\prod_{i \in I} H_i, f\right)$ with

$$f((x_i)_{i \in I}) = \prod_{i \in I} f_i(x_i).$$

This multialgebra, with the projections $e_i^I : \prod_{j \in I} H_j \to H_i$, $e_i^I((x_j)_{j \in I}) = x_i$ $(i \in I)$ is the product of the multialgebras (H_i, f_i) in the category $\mathbf{Malg}(1)$.

Proposition 6. [2, Proposition 4] *If $n \in \mathbb{N}$, $\mathbf{q}, \mathbf{r} \in \mathbf{P}^{(n)}(\tau)$, and $(\mathfrak{A}_i \mid i \in I)$ is a family of multialgebras of type τ such that $\mathbf{q} = \mathbf{r}$ is satisfied on each multialgebra $\mathfrak{A}_i$ then $\mathbf{q} = \mathbf{r}$ is also satisfied on the multialgebra $\prod_{i \in I} \mathfrak{A}_i$.*

Corollary 3. *If $((H_i, f_i) \mid i \in I)$ is a family of monounary multialgebras satisfying the identity* (5) *then the direct product $\left(\prod_{i \in I} H_i, f\right)$ also satisfies* (5).

Remark 8. [5, Remark 9] If K_2' is the subclass of $\mathbf{Malg}(1)$ which consists in multialgebras which satisfies (5) then K_2' is a subclass of $\mathbf{Malg}'(1)$ closed under the formation

of the direct products. The subclass K_2'' of $\mathbf{Malg}(1)$ which consists in nonempty multialgebras (H, f) which satisfy (5) and $f(H) = H$ is a subclass of $\mathbf{Malg}''(1)$ closed under the formation of the direct products.

Theorem 2. *The monounary multialgebras which determine complete hypergroupoids form a subclass of K_2' closed under the formation of direct products. Also, the monounary multialgebras which determine complete hypergroups form a subclass of K_2'' closed under the formation of direct products.*

Proof. From Lemma 3 it follows that a monounary multialgebra (H, f) which determines a complete hypergroupoid H_f satisfies (5), hence (H, f) is in K_2'. According to Theorem 1, the complete hypergroupoid H_f is a semihypergroup. It is immediate that if H_f is a hypergroup then (H, f) is in K_2''. Let I be a set and for each $i \in I$, let (H_i, f_i) be a monounary multialgebra for which

$$f_i(x_i) = f_i(H_i),\ \forall x_i \in H_i.$$

If $\left(\prod_{i\in I} H_i, f\right)$ is the direct product of the multialgebras (H_i, f_i) then

$$f((x_i)_{i\in I}) = \prod_{i\in I} f_i(x_i) = \prod_{i\in I} f_i(H_i) = f\left(\prod_{i\in I} H_i\right),$$

for any $(x_i)_{i\in I} \in \prod_{i\in I} H_i$, hence $\left(\prod_{i\in I} H_i, f\right)$ determines a complete semihypergroup. If, in addition, for any $i \in I$ we have $H_i \neq \emptyset$ and $f_i(H_i) = H_i$ then

$$f((x_i)_{i\in I}) = \prod_{i\in I} f_i(H_i) = \prod_{i\in I} H_i,$$

for any $(x_i)_{i\in I} \in \prod_{i\in I} H_i$, so the multialgebra $\left(\prod_{i\in I} H_i, f\right)$ determines a complete hypergroup. □

Let us denote by $\mathbf{SHG_c}'$ the subcategory of $\mathbf{SHG}'$ whose objects are the complete semihypergroups determined by monounary multialgebras and by $\mathbf{HG_c}'$ the subcategory of $\mathbf{HG}'$ whose objects are the complete hypergroups determined by monounary multialgebras. Since the direct product of monounary multialgebras is their product in $\mathbf{Malg}(1)$ from the above theorem, using Remark 4 and Remark 8 we obtain:

Corollary 4. *The subcategory* $\mathbf{SHG_c}'$ *of* $\mathbf{SHG}'$ *is closed under products. Moreover, if* I *is a set and for each* $i \in I$, (H_i, f_i) *is a monounary multialgebra which determines a complete semihypergroup* $(H_i)_{f_i} = (H_i, \circ_i)$ *then the product of* $((H_i, \circ_i) \mid i \in I)$ *in* $\mathbf{SHG_c}'$ *is the (complete) semihypergroup determined by the direct product* $\left(\prod_{i\in I} H_i, f\right)$.

Corollary 5. *The subcategory* $\mathbf{HG_c}'$ *of* $\mathbf{HG}'$ *is closed under products. Moreover, if* I *is a set and for each* $i \in I$, (H_i, f_i) *is a monounary multialgebra which determines a complete hypergroup* $(H_i)_{f_i} = (H_i, \circ_i)$ *then the product of* $((H_i, \circ_i) \mid i \in I)$ *in* $\mathbf{HG_c}'$ *is the (complete semi)hypergroup determined by the direct product* $\left(\prod_{i\in I} H_i, f\right)$.

If $((H_i, f_i) \mid i \in I)$ is a family of monounary multialgebras which determine the complete semihypergroups (hypergroups) $((H_i)_{f_i} = (H_i, \circ_i) \mid i \in I)$ then

$$x_i \circ y_i = f_i(x_i) = f_i(y_i) = f_i(H_i),\ \forall x_i, y_i \in H_i, \forall i \in I.$$

The product of $((H_i)_{f_i} \mid i \in I)$ in $\mathbf{SHG_c}'$ ($\mathbf{HG_c}'$) is the (complete) semihypergroup (hypergroup) $\left(\prod_{i\in I} H_i, \circ\right)$ determined by the direct product $\left(\prod_{i\in I} H_i, f\right)$. The hyperproduct $\circ$ is defined as follows: if $(x_i)_{i\in I}, (y_i)_{i\in I} \in \prod_{i\in I} H_i$ then

$$\begin{aligned}(x_i)_{i\in I} \circ (y_i)_{i\in I} &= f((x_i)_{i\in I}) \cup f((y_i)_{i\in I}) = \prod_{i\in I} f_i(x_i) \cup \prod_{i\in I} f_i(y_i)\\ &= \prod_{i\in I} f_i(H_i) = \prod_{i\in I} (x_i \circ y_i),\end{aligned}$$

hence $\left(\prod_{i\in I} H_i, \circ\right)$ is the direct product of $((H_i, \circ_i) \mid i \in I)$, i.e. the product of $((H_i, \circ_i) \mid i \in I)$ in $\mathbf{Malg}(2)$. Thus we have proved the following result:

Corollary 6. *The categories* $\mathbf{SHG_c}'$ *and* $\mathbf{HG_c}'$ *are subcategories of* $\mathbf{Malg}(2)$ *which are closed under products.*

References

[1] Corsini, P., On the hypergroups associated with binary relations, *Multiple Valued Logic* **5** (2000) 407-419.

[2] Pelea, C., On the direct product of multialgebras, *Studia Univ. Babeş-Bolyai Math.*, **48/2** (2003) 93-98.

[3] Pelea, C., Purdea, I., A characterization theorem for complete multialgebras, *Mathematica (Cluj)*, **46(69)**, 2, 2004, 205-211.

[4] Pelea, C., Purdea, I., Direct limits of direct systems of hypergroupoids associated to binary relations, *Advances in Mathematics of Uncertainty*, Proc. of the Symposium "Mathematics of Uncertainty", ECIT 2006, Iaşi, Romania, September 20-23, 2006.

[5] Pelea, C., Purdea, I., Products of hypergroupoids associated to binary relations, *Set-Valued Mathematics and Applications*, to appear.

[6] Purdea, I., Tratat de algebră moderna. Vol. II. (Romanian) [Treatise on modern algebra. Vol. II] With an English summary. *Editura Academiei Republicii Socialiste România, Bucharest* 1982.

[7] Rosenberg, I. G., Hypergroups and join spaces determined by relations, *Riv. Math. Pura e Appl.* **4** (1998) 93-101.

"Grigore Moisil" High School,
Str. Rodnei 23, 420141 Bistriţa, Romania
E-mail address: horiolaru@yahoo.com

Babeş-Bolyai University,
Faculty of Mathematics and Computer Science,
Str. M. Kogălniceanu 1, 400084 Cluj-Napoca, Romania
E-mail address: cpelea@math.ubbcluj.ro

Babeş-Bolyai University,
Faculty of Mathematics and Computer Science,
Str. Mihail Kogălniceanu 1, 400084 Cluj-Napoca, Romania
E-mail address: purdea@math.ubbcluj.ro