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FOR A COMPLETIVE LOGIC

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Dedicated to Professor Grigore Călugăreanu on his 60th birthday

Abstract. We give some argument for a completive logic.

1. Starting from a slander case

Example 1.

Y: Nr. the president, I blame the subject X for slander to my address.

P: In what consist the slander

Y: He said that I am a blockhead.

P: It is true, X?

X: It is not true, Mr.!

Y: I registered on cassette. Look: "Hey Y, I believe that you are a blockhead."

P: Then X are not assert that you are a blockhead, but that he believes... Therefore, the case is cancelled.

2. The logical form

At first sigh, the problem seems to be formulated in the second order predicate logic. We though prefer to formulate it in the terms of a "completive-logic", in the following sense:

The above statement in Example 1 may be expressed by the proposition:

"X believes that Y are a blockhead" which may be also formulated by:

$$\text{"}C(X) \text{ that } \mathcal{M}(Y)\text{"}$$

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where $C(X) =$ "X believes" and

$$\mathcal{M}(Y) = \text{"}Y \text{ are a blockhead".}$$

Observe that $\mathcal{M}(Y)$ has the role of "complement" of the predicate $C(X)$, what justify the name "completive logic".

The conjuction "that" gives an affirmative aspect to the proposition above; as such, near the predicate $C(X)$ ("X believes") we consider also the predicates:

$$B(X) = \text{"}X \text{ guess" ("}X \text{ supposes") and}$$

$$S(X) = \text{"}X \text{ is sure" ("}X \text{ is convinced").}$$

3. **An implicative hierarchy**

Between the above predicates we establish the following implicative hierarchy:

$$S(X) \Rightarrow C(X) \Rightarrow B(X). \tag{1}$$

Justification. Besides the moral sense of the word "to believe" (see [2]), to be sure implies to believe (generally, inverse not). Also, to believe implies to suppose.

The predicates S, C, B have as term variables the human follows $X \in \mathcal{U}$ and as "complements", some predicates $\mathcal{P}$ of arbitrary individuals $Y \in M$. Evidently, the possibility $U \subseteq M$ is not excluded.

To formulate more concisely, let be $\mathcal{X} \in \{S, C, B\}$, $X \in \mathcal{U}$, $\mathcal{Y} \in \Pi_M$ (predicates on the set M) and $Y \in M$; thus, the proposition "$\mathcal{X}(X)$ that $\mathcal{Y}(Y)$" one transcribes simply: $(\mathcal{X}(X))(\mathcal{Y}(Y))$.

Particularly, the proposition from 2 becomes: $(C(X))(\mathcal{M}(Y))$.

Remark. The above proposition may be formulated also by a binary predicate: $C_{\mathcal{M}}(X, Y)$.

(See, for example, the Problem 12 in [1]).

4. **Some properties**

From the convention (1) it results that:

$$(S(X))(\mathcal{Y}(Y)) \Rightarrow (C(X))(\mathcal{Y}(Y)) \Rightarrow (B(X))(\mathcal{Y}(Y)). \tag{2}$$

Also, if $\mathcal{Y}_1(Y) \Rightarrow \mathcal{Y}_2(Y)$ then

$$(\mathcal{X}(X))(\mathcal{Y}_1(Y)) \Rightarrow (\mathcal{X}(X))(\mathcal{Y}_2(Y)). \tag{3}$$

By transposition, from (1) we obtain:

$$\overline{B(X)} \Rightarrow \overline{C(X)} \Rightarrow \overline{S(X)}, \tag{4}$$

and so (as in (2)), we have

$$(\overline{B}(X))(\mathcal{Y}(Y)) \Rightarrow (\overline{C}(X))(\mathcal{Y}(Y)) \Rightarrow (\overline{S}(X))(\mathcal{Y}(Y)). \tag{5}$$

Finally, we get some properties of simultaneous negations of the type

$$(\overline{\mathcal{X}}(X))(\overline{\mathcal{Y}(Y)}).$$

Besides the implications of the type (5), it seems to be justificated the following:

$$(\overline{S}(X))(\overline{\mathcal{Y}}(Y)) \Rightarrow (B(X))(\mathcal{Y}(Y)) \tag{6}$$

Justification. To be not sure that there is not, it means to guess that there is

$$(\overline{C}(X))(\overline{(Y)(Y)}) \Rightarrow (C(X))(\mathcal{Y}(Y)). \tag{7}$$

Justification. To don't believe that there is not, it means to believe that there is.

By syllogism from (7) (with (2)) we obtain:

$$(\overline{C}(X))(\overline{\mathcal{Y}}(Y)) \Rightarrow (B(X))(\mathcal{Y}(Y)). \tag{8}$$

References

[1] Both, N., *Algebra logicii cu aplicaţii*, Ed. Dacia, Cluj, 1984.

[2] Both, N., *Secvenţe de logică psiho-teologică*, în "Mozaic Teologic I", Ed. Viaţa Creştină Cluj, 2001.

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