

BOOK REVIEWS

Alberto Guzman, *Derivatives and Integrals of Multivariable Functions*, Birkhäuser Verlag, Boston-Basel-Berlin 2003, x+319 pp., ISBN: 0-8176-4274-9 and 3-7643-4274-9.

This is a text for a one-semester course in advanced calculus of several variables (differential and integral calculus). The continuity properties of functions as well as some topology questions are treated, within the framework of normed spaces, in a previous book by the same author " *Continuous functions of vector variable*", Birkhäuser 2002. Together they can be used for a one-year advanced course in multivariable calculus. The author indicates in the Preface how it can be used for shorter term courses by skipping some tedious parts (as, e.g., the proof of the implicit function theorem, the treatment of generalized integrals, etc).

Although the present volume contains some references to the previous one, it is fairly self-contained, the prerequisites being familiarity with the topology of the Euclidean space and some linear algebra.

The differentiability is treated in the first three chapters: 1. *Differentiability of multivariable functions*, 2. *Derivatives of scalar functions*, and 3. *Derivatives of vector functions*. They contain the basic definitions and properties, a careful treatment of higher order derivatives and their symmetry, implicit and inverse function theorems, local extrema and conditioned extrema (the rule of Lagrange multipliers), and some geometric applications (curves, surfaces, tangents and normals).

Multivariable integration is treated in chapters 4. *Integrability of multivariable functions* and 5. *Integrals of scalar functions*. Starting with integration on boxes, one defines then Jordan measurable sets (called Archimedean) and integrals over Archimedean domains. Improper integrals are also considered. This chapters contains also a proof of the change of variables formula and a treatment of line and surface integrals.

The last chapter of the book, 6. *Vector integrals and the vector-field theorems*, contains proofs of the fundamental theorems of Green, Stokes and Gauss. Some applications of these theorems to physics are discussed.

The book contains also exercises and their solutions are given at the end of the book –"For emergency use only", as the author says quoting Buck.

The book reflects the teaching experience of the author as well as his taste and scientific ideas. It fits excellently for a course in multivariable calculus for students in mathematics, physics or engineering, preparing them for more advanced courses in real analysis and differential geometry.

S. Cobzaş

Steven G. Krantz, *A Handbook of Real Variables*, Birkhäuser Verlag, Boston-Basel-Berlin 2004, xii+201 pp., ISBN: 0-8176-4329-X and 3-7643-4329-X.

The aim of this handbook is to provide the reader with a quick and accessible treatment of the main ideas of the theory of functions of a real variable (within the limits of Riemann integration), including elements of Fourier analysis and applications to the solving of differential and partial differential equations. It is devoted to scientists who need real analysis, but have no time nor patience to look into the details. So it contains a few proofs, the main idea being to give cute explanations of the notions and to present the basic results along with some examples of application. There are references to textbooks on real analysis for further reading, but these are optional, the book being self-contained.

The topics covered by the book are: basic set theory and real numbers; sequences of numbers and their limits (including Limsup and Liminf); series – convergence tests, operations with series, Riemann's theorem on the rearrangements of conditionally convergent series, the series of the number e ; the topology of the real line, Cantor set; limits and the continuity of functions, intermediate value property, monotonic functions and their discontinuities; the derivative and the main theorems of

differential calculus (mean value theorems), Weierstrass example of a nowhere differentiable function; the Riemann integral and its fundamental properties, the Riemann-Stieltjes integral; sequences and series of functions and Weierstrass approximation theorem; some special functions – the exponential function and the logarithm, the trigonometric functions and their inverses, the Gamma function; Fourier series; the topology of metric spaces and Ascoli-Arzelà theorem; Picard's iteration technique for solving differential equations; Fourier analytic methods for solving partial differential equations.

The book will be a very useful tool for physicists, engineers, economists, but also for students in mathematics, computer science, physics or engineering as a helping instrument when working exercises and problems.

S. Cobzaş

Jiri Matousek, *Using the Borsuc-Ulam Theorem – Lectures on Topological Methods in Combinatorics and Geometry*, Universitext, Springer Verlag, Berlin, 2003, ISBN: 3-540-00362-2.

The difficulty with the algebraic topology is the immense amount of preparatory knowledge, the sophisticated technical prelude preceding the proving the significant results of the field. But most part of these results are formulated in terms of the elementary calculus using only the topology and the geometry of the Euclidean space. There is a considerable gap between the geometric intuition and the mathematical machinery permitting its rigorous handling. Hence, last time the algebraic topology was driven out of an usual mathematical curriculum. This circumstance deprives the mathematical student of acquaintance with one of the most brilliant records which the human intelligence ever achieves. The various attempts to make the ideas of the field more accessible produce a sort of popularizing literature renouncing somewhere to rigor. By a difference, Matousek's book is focused not on the whole algebraic topology, but on a part of it, which can be gathered under the title "Borsuc-Ulam theorem, and its relation with the combinatorics". After an introductory chapter concerning the elementary theory of simplicial complexes, the author presents a lot

of equivalent versions of the Borsuc-Ulam theorem, proves their interrelations and presents various their proofs. The following sections concerns direct applications, as the well known Ham Sandwich Theorem, Coloring of Necklets, the Lovász-Kneser Theorem, Gales Lemma and Schrijver's Theorem. A next chapter is destined to introduce deeper topological concepts as k -connectedness and cell complexes in order to handle in the last two chapters nonembeddability results as the Van Kampen-Flores Theorem, Sarkaria's Inequality as well as problems concerning multiple points of coincidence with special emphasis on the Topological Tverberg Theorem and the problematic around it.

The book excels in presenting carefully the intuitive background of the material inserting amusing examples and pictures. The idea of presenting first the easier results and interconnections and only then the technically more difficult proofs makes the material more attractive for the beginner, and at the same time an agreeable lecture for the specialist.

A. B. Németh

Sampling, Wavelets, and Tomography, John J. Benedetto and Ahmed I. Zayed – Editors, Applied and Numerical Harmonic Analysis, Birkhäuser Verlag, Boston-Basel-Berlin 2004, xxi+344 pp., ISBN: 0-8176-4304-4.

The papers included in the present volume emerged from the materials presented at the biennial Sampling Theory and Applications (SamTA01) Conference, held in Orlando, Florida, in May 2001. The conference celebrated the accomplishments of Claude Elwood Shannon, born on April 30, 1916 and died on February 22, 2001, the creator of modern information theory. This was the third of SamTA conferences, the first one took place in 1997 in Jurmala, Latvia, the second in 1999 in Aveiro, Portugal, and the fourth in 2003 in Strobl, Austria.

Its aim, as it is presented in the first chapter of the book, "*A prelude to sampling, wavelets, and tomography*", by Ahmed I. Zayed, the organizer of the conference and co-editor of the volume, was to emphasize the connections between the three topics mentioned in the title of the volume and. This chapter gives also a

general introduction to the remaining chapters, which, written by leading experts in the fields, mathematicians and engineers, deal with mathematical topics as well as with applications. For many years, the research in the sampling has been carried by communication engineers, but, with the advent of new techniques in mathematical analysis, many mathematicians get involved into the matter, leading to new interesting results and interconnections.

A good idea on the contents of the book is given by the headings of its chapters: 2. *Sampling without input constraints: Consistent reconstruction in arbitrary spaces*, by Yoanina C. Eldar; 3. *An introduction to irregular Weyl-Heisenberg frames*, by Peter G. Casazza; 4. *Robustness of regular sampling in Sobolev algebras*, by Hans G. Feichtinger and Tobias Werther; 5. *Sampling theorems for nonbandlimited signals*, by P. P. Vaidyanathan; 6. *Polynomial matrix factorization, multidimensional filter banks, and wavelets*, by N. K. Bose and S. Lertrattanapanich; 7. *Function spaces based on wavelet expansions*, by Stéphane Jaffard; 8. *Generalized frame multiresolution analysis of abstract Hilbert spaces*, by Manos Papadakis; 9. *Sampling theory and parallel-beam tomography*, by Adel Faridani; 10. *Filtered backprojection algorithms for spiral cone beam CT*, by Alexander Katsevich and Guenter Lauritsch; 11. *Adaptive irregular sampling in meshfree flow simulation*, by Armin Iske; 12. *Thin-plate spline interpolation*, by David C. Wilson and Bernard A. Mair.

The book is addressed to mathematicians, scientists, and engineers working on signal and image processing and medical imaging. It is written by experts and for experts, but each chapter has an introductory part written for non-specialists, giving them the possibility to find what the chapter is dealing with.

The book contains important contributions to the areas mentioned in the title: sampling, wavelets and tomography.

S. Cobzaş