HYBRID UPDATE STRATEGIES RULES IN SOCIAL HONESTY GAME

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Abstract. Update strategies are essential for the outcome of evolutionary games. Different problems often require different update strategies of approach. When individuals interact they choose update strategies according to the situations they face or their past encounters. In the following paper we discuss three different update strategies involved in the outcome of the Social Honesty game: *Best, Fermi and Myopic.* The purpose of this study is to analyze the impact of combining different update strategies on the punishment probability in the Social Honesty game. Our results may help to explain how the presence of hybrid update strategies influences the mechanisms based on individual punishment.

1. Introduction

In the context of today's technological advancement, digital data is becoming available of all aspects of human life. Designing computational theories of human behavior has become a goal for many researchers hoping to create better social systems. Interactions between people in social and economic environment are studied intensively in Evolutionary Game theory.

The emergence of cooperation in a population of individuals became the subject of study in a Darwinian framework. Dynamics of honest/dishonest behavior translates into cooperation/defection in social dilemma games. Individuals change their standpoint based on the update strategies.

Update strategies are essential as algorithms in evolutionary games, by telling every player what to do for every possible situation throughout the game. In real world, just like in evolutionary games, individuals can choose a specific update strategy for some time, from a finite set of pure update strategies (for example: rock, scissors, paper, best, myopic etc.). The concept

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of update strategy should not be confused with that of a move (for example: honest, dishonest, cooperator, defector, moralist, immoralist etc.).

In real situations people do not keep their strategy unchanged, but rather choosing the right strategy for a given situation. History proved that there is a clear difference between what is right to do and what is the best thing to do. Moral values, religion, education and imitation [9] also play a crucial role in people's decisions and the mechanisms used to take those decisions. *The universe provides us with a great and sometimes undesired diversity.*

Imitation is an advanced behavior whereby an individual observes and replicates another's behavior. Imitation is an effective mechanism of spreading social behavior.

We define hybrid update strategies as strategies that combine two or more pure update strategies. In Social Honesty game a finite set of three update strategies was used: Best, Myopic and Fermi. Both honest and dishonest players may choose from a finite set of update strategies. [13]

The previous studies based on the Social Honesty game [13] have used the same update strategy for all players over the entire duration of the game and different values for the punishment probability. An important issue about punishment is that in terms of social connections, the punishment probability was found to be more important than the punishment severity in promoting a honest behavior. Research and direct observation in human interactions reveal that individuals choose more than one update strategy. In social interactions players can choose to change their update strategy, therefore, we propose in this paper several new experiments based on hybrid update strategy. Since people have the opportunity to choose alternative solutions we will let every player change his strategy with a certain probability.

In this paper we study the effects of hybrid update strategies on the "*p-transition intervals*", using different combinations of pure update strategies. A "p-transition interval" [13] was defined as an interval of values for punishment probabilities where the percentage rate of honest players changes from 0% to 100%.

The following paper is organized as follows: Section 2 describes the presence of hybrid update strategies in biological systems, the spatial form of the Social Honesty game is presented in Section 3, the hybrid update mechanisms in Section 4, experimental results in Section 5 and conclusions and directions for further research in Section 6.

2. Hybrid update strategies in real world complex systems

In psychology, decision-making is regarded as the cognitive process resulting in the selection of a belief or a course of action among several alternative possibilities. In other words, decision-making is the process of making choices by setting goals, gathering information and assessing alternative occupations. Update strategies can be seen as forms of human behavior, part of the decision-making process. The complexity of human behavior makes it impossible to designing computational systems that include all the mechanisms involved in the decision-making process of the human intellect. Research reveals that hybrid update strategies are present all around us. From traditions, cultures and economy to biological systems in nature, hybrid update strategies are present.

Let us consider, for example, the rock-scissors-paper game. Studies proved that it is almost impossible to have an advantage over players who are using random update strategies. Since the purpose of this game is to gain advantage over the opponent, playing the same strategy over and over again, will result in failure of beating the opponent. For one player to gain advantage over the opponent, he or she needs to change his or her update strategy. If we consider our player to use the same update strategy with several other players, for a limited number of rounds, and later change his or her update strategy, then we have hybrid update strategies, as part of the decision-making process.

A different approach is proposed by Nowak [2]. Cooperation takes into account the degree of relationship and the nature of groups. Five different mechanisms are defined in order for cooperation to evolve: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. Without a mechanism needed for cooperation to evolve, natural selection promotes defection. No punishment is proposed, instead a cost-to-benefit ratio is used as a reference probability for promoting cooperation.

In Social Honesty game we use a punishment probability p as a dishonest behavior deterrence mechanism instead of a cost-to-benefit ratio. Honest behavior is promoted by increasing the punishment probability. Compared to existing results [13], in Social Honesty game the presence of hybrid update strategies leads to higher values of the limits of the p-transition intervals, thus the honest behavior becomes harder to promote.

Studies made thus far [11] reveal a relation between the number of offenses, the probability of conviction, the punishment per offense and a portfolio of other influences. A change in the probability of conviction proves to be more important than the severity of the punishment. However the increase of the probability of conviction or the number of offenses, leads to the increase of economical costs associated to crimes and fighting felonies. More importantly, Becker states that the cost of crime has an important role for the evolution or regression of criminal behavior. As a consequence, an increase in rewarding legal activities or of obedience to the law, would reduce the intent to commit a crime.

Of course every individual follows his or her economical interests [10]. However models proposed thus far, by Adam Smith, fail to take into account non-economical motivations, which Akerlof and Shiller name "animal spirits". In [10] the authors describe five different aspects of "animal spirits": confidence, fairness, corruption and antisocial behavior, money illusion and stories, and how they affect economic decisions. On things through economic perspective, corruption and antisocial behavior bring a negative impact on the economy. Decisions are made because people have the confidence they are right, not as a result of rational calculations.

Public and lab experiments made by Ariely [5] reveal that getting caught for unethical behavior matters less than we think. In the context of cognitive science and behavioral economics, dishonesty proved to be discouraged by moral reminders, supervision and signs or pledges to confirm honesty. In contrast with Becker, who proposed a rational cost-benefit analysis of crime, Ariely demonstrates that it is the irrational forces that we do not take into account often motivate individuals to behave ethically or not. Moreover Akerlof and Shiller state that animal spirits lie behind our subjectively defined interests and decisions, leaving us with a restless and inconsistent element in the economy. It refers to our peculiar relationship with ambiguity or uncertainty [10].

3. The spatial form of the Social Honesty game

Our study is based on Cellular Automata and Evolutionary Game Theory. Cellular automata (CA) are discrete, abstract computational systems, which proved useful in various areas of scientific study of mathematics, physics, social phenomena, biology. Cellular automata are composed of a finite set of cells, or atoms, each with a finite number of states, such as *on* and *off*. For each cell, the neighborhood was defined as a finite number of cells, relative to the specified cell. The most popular approach to cellular automata, represents Conway's game of life, as seen in Fig. 1.

An Evolutionary algorithm (EA) is a set of operations and symbols using techniques inspired by mechanisms of organic evolution, such as reproduction, mutation, recombination and natural selection, in order to obtain optimum configurations for a specific system within specific constraints. To solve a certain problem, we create an environment where potential solutions can evolve over many generations (iterations). A known use of evolutionary algorithms lies in finding solutions to the Queens Problem. Starting from an initial population and using a fitness function, that evaluates each candidate according to how close it is to the solution, the quality of the solutions in the population should improve. Although the queen problem has 4,426,165,368 possible

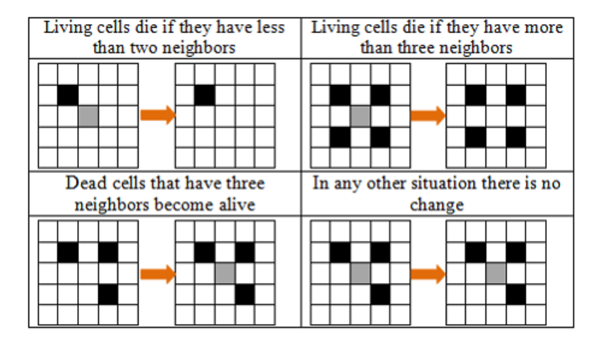

Fig. 1. Basic rules of Conway's Game of Life.

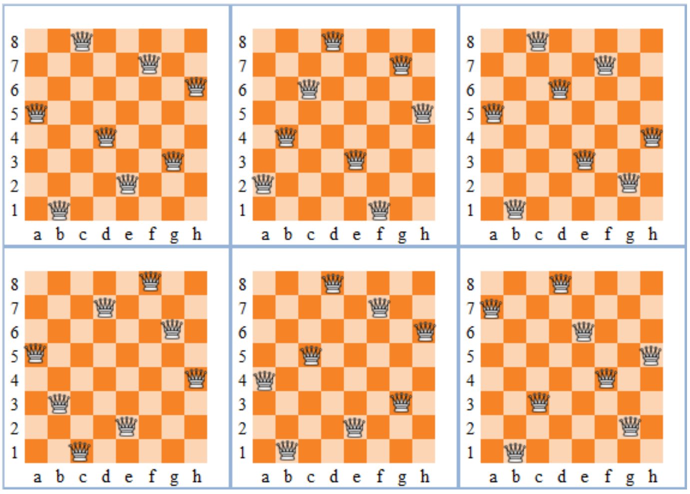

Fig. 2. The eight Queen problem.

arrangements of eight queens on an 8×8 board, it does have only 92 viable solutions, among which 12 are fundamental solutions. Fig. 2 depicts 6 viable solutions for the eight queen problem.

Evolutionary Game models have been extensively used in order to explain social and biological phenomena [12, 7, 16, 15].

The Social Honesty (SH) game was introduced to study the dynamics of honest/dishonest behavior in social environment. The SH game analyses human interaction based on imitation. The probabilistic payoff determines what move will achieve a player for next round [13]. The '*Best*' update strategy is used in the majority of the experiments: each player imitates the move of the neighbor with the highest payoff.

For convenience we call an H-player a player who chooses an honest strategy and a D-player is a player who chooses a dishonest strategy [13].

In the Social Honesty game, players are arranged on a regular $N \times N$ lattice with joint boundaries of a cellular automaton [15, 1]. Each cell of the lattice represents a player. The state of a cell is the move of the corresponding player (H-honest or D-dishonest). Every player plays the Social Honesty game with the nearest 8 neighbors (the Moore neighborhood [6]) and compares the payoffs. According to the update strategy mechanisms and the player's gain, each player may choose to become honest or dishonest at every game round.

Players in the Social Honesty game get certain payoffs as follows:

- If two honest players interact, the payoffs for each player are c>0;

- If an H-player interacts with a D-player, the H-player gets a payoff equal to 0 and the player is punished with the probability p1. If not punished the D-player gets the payoff a (a>c). The D-player's payoff can be described as a random variable A:

$$A = \begin{pmatrix} -S & a \\ p_1 & 1-p_1 \end{pmatrix}$$

Interactions between two D-players lead to the following:

- each one of them may be punished with probability p_2;
- if none of them is punished then only one gets a positive b and the other gets 0.

The payoffs for each D-player can be described as the next variables B and B':

$$B = \begin{pmatrix} -S & 0 & b \\ p_2 & \dfrac{1-p_2}{2} & \dfrac{1-p_2}{2} \end{pmatrix}$$

and

$$B' = \begin{pmatrix} -S & 0 & b \\ p_2 & \dfrac{1-p_2}{2} & \dfrac{1-p_2}{2} \end{pmatrix}$$

The following table describes the Social Honesty game:

4. Hybrid update strategies mechanisms

In order to implement the different strategies adopted by players we used combinations of three possible strategies: '*Best*', '*Fermi*' and '*Myopic*' [18, 3].

Player 1/Player 2	*Honest (H)*	*Dishonest (D)*
Honest (H)	*c, c*	*0, A*
Dishonest (D)	*A, 0*	*B, B'*

Fig. 3. The matrix of the Social Honesty game

In game theory, the '*Best* update strategy' is the strategy which leads to the most favorable payoff for an individual [6, 17]. In our game the player who chooses the *Best* update strategy imitates the neighbor with the highest payoff.

'Myopic' play describes a situation in which the players take the future strategies of their opponents as given, irrespective of the actual history of the game [4]. In essence, myopic means missing information. In our simulations we use 'Best-Myopic update strategy': players imitate the player with the highest payoff with probability of 0.75 and a randomly chosen neighbor with probability of 0.25 [13].

In our experiments the *'Fermi'* update strategy is defined as follows: a player is imitated with a probability (*p'*) given by the Fermi function [8]:

$$p = \frac{1}{1 + e^{w(U_x - U_y)}}$$

where U_x is the payoff for the player x and is evaluated identically for player y.

For a better understanding, the following denominations are used: *'Myopic'* & *'Best'*, *'Fermi'* & *'Best'*, *'Fermi'* & *'Myopic'* hybrid update strategies, as combinations between *'Myopic'* and *'Best'*, *'Fermi'* and *'Best'*, respectively *'Fermi'* and *'Myopic'* update strategies. For each hybrid update strategy we increase the percentage rate of players activating according to a pure update strategy with 5%, while we decrease the percentage rate of players activating according to the other pure update strategy with 5%.

5. Experiments

5.1. **The Transition Intervals for Hybrid Strategy Updating Mechanisms.** We study the effects of hybrid update strategies on an initial population of 100 × 100 players, with 50% honest players distributed randomly. We let S=2, w=0.1, without a loss of generality, and use the Moore neighborhood [14] and different punishment probabilities (p). The use of hybrid update strategies reveals changes of the transition intervals, in size and limits.

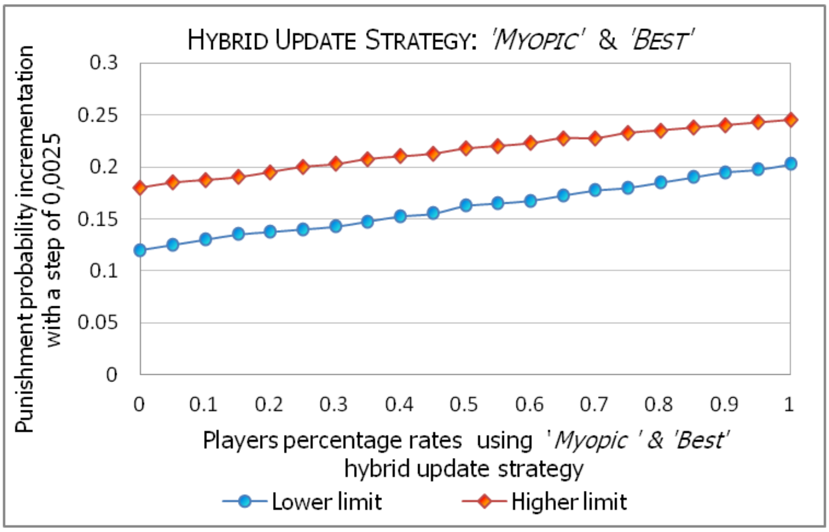

Fig. 4. Upper and lower limits of the transition interval with different percentage rates of players using the *'Myopic'* & *'Best'* hybrid update strategy.

It can be observed that the size of the transition intervals is greater when using *'Fermi'* & *'Best'* hybrid update strategy as shown in Fig. 10, than using any of the other two hybrid update strategies available. The fluctuation in size of the transition interval is also greater when using *'Fermi'* & *'Best'* hybrid update strategy(see Fig. 10).

The sizes of p-transition intervals, in our simulations using *'Myopic'* & *'Best'* hybrid update strategy, follow a downward trajectory, in contrast with simulations using *'Fermi'* & *'Myopic'* hybrid update strategy, which follow an upward trajectory (see Fig.5, 7, 9 and 11).

A comparison between the three different hybrid strategies reveals that the lower and upper limits of p-transition intervals are greater when using *'Fermi'* & *'Myopic'* hybrid update strategy, than in the simulations using *'Myopic'* & *'Best'* or *'Fermi'* & *'Best'* hybrid update strategies (see Fig. 4, 6, 8, 12 and 13). Also the use of *'Myopic'* & *'Best'* hybrid update strategies leads to the lowest values of the limits in our simulations (see Fig. 12 and 13). The values of the limits follow a sharper upward trajectory when using *'Myopic'* & *'Best'* and *'Fermi'* & *'Best'* hybrid update strategies, than in simulations using *'Fermi'* & *'Myopic'* hybrid update strategy.

5.2. **The Transition Intervals for *'Myopic'* & *'Best'* Hybrid Update Strategies.** Simulations of the game start with 50% honest players distributed randomly on the lattice. Fig. 4 shows the evolution of the limits of p-transition

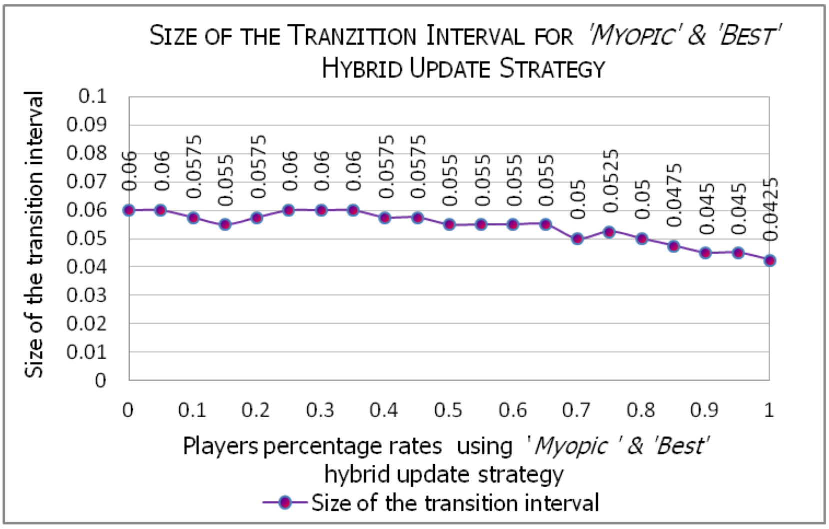

Fig. 5. The size of transition intervals with different percentage rates of players using the *'Myopic'* & *'Best'* hybrid update strategy.

intervals using different percentage rates of players activating according to *'Myopic'* & *'Best'* hybrid update strategies. Results were obtained over an average of 100 repetitions, each repetition with 1000 rounds, with a punishment severity of S=2. Players using the *'Myopic'* update strategy are chosen randomly every round on a 100×100 players lattice. Increasing the value of p, with a step of 0.0025, and the rate of players using the 'Myopic' update strategy, with a step of 5%, we notice that the limits of the transition interval increase in value, from 0.12 up to 0.2025 for the lower limit and from 0.18 up to 0.245 for the upper limit of the transition interval (see Fig. 4).

The sizes of the *p-transition intervals* follow a downward trajectory from 0.06 down to 0.0425 compared to *'Fermi'* & *'Myopic'* hybrid update strategy where the sizes of *p-transition intervals* follow a upward trajectory from 0.045 up to 0.06 (see Fig. 5, 9 and 11).

5.3. **The Transition Intervals for *'Fermi'* & *'Best'* Hybrid Update Strategies.** Simulations of the game start with 50% honest players distributed randomly on the lattice. Fig. 6 shows the evolution of the limits of *p-transition intervals* using different percentage rates of players activating according to *'Fermi'* & *'Best'* hybrid update strategies. Results were obtained over an average of 100 repetitions, each repetition with 1000 rounds, with a punishment

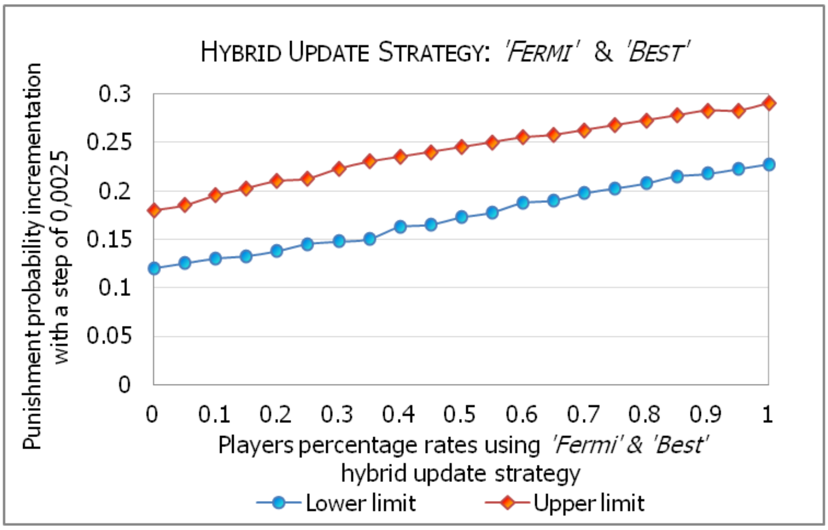

Fig. 6. Upper and lower limits of the transition interval with different percentage rates of players using the *'Fermi'* & *'Best'* update strategy.

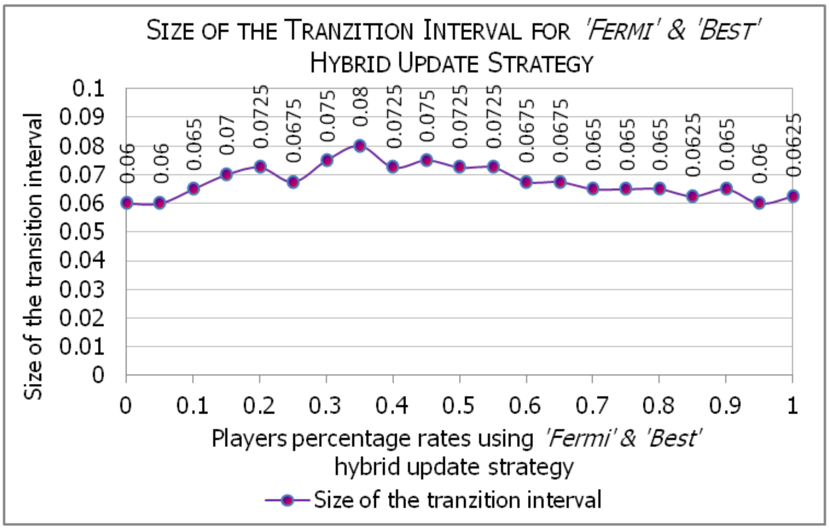

Fig. 7. The size of the transition intervals with different percentage rates of players using the *Fermi'* & *'Best* update strategy.

severity of S=2. Players using the *'Fermi'* update strategy are chosen randomly every round on a 100×100 players lattice. Increasing the value of p, with a step of 0.0025, and the rate of players using the *'Fermi'* update strategy,

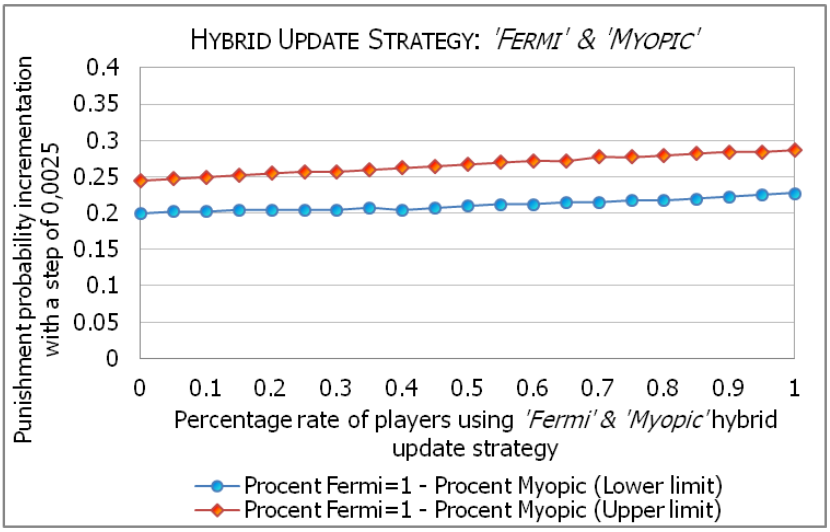

Fig. 8. Upper and lower limits of the transition interval with different percentage rates of players using the *'Fermi'* & *'Myopic'* update strategy.

with a step of 5%, we notice that the limits of the transition interval increase in value, from 0.12 up to 0.2275 for the lower limit and from 0.18 up to 0.29 for the upper limit of the transition interval (see Fig. 6). The values for the limits are greater than those resulted in simulations using *'Myopic'* & *'Best'* hybrid update strategies (see Fig. 4, 6, 12 and 13).

The sizes of p-transition intervals have greater fluctuations when using *'Fermi'* & *'Best'* hybrid update strategies than using any of the other hybrid update strategies available (see Fig. 7 and 11).

5.4. **The Transition Intervals for *'Fermi'* & *'Myopic'* Hybrid Update Strategies.** Simulations of the game, using *'Fermi'* & *'Myopic'* hybrid update strategies, start with 50% honest players distributed randomly on the lattice. Fig. 8 shows the evolution of the limits of *p-transition intervals* using different percentage rates of players activating according to *'Fermi'* & *'Myopic'* hybrid update strategies. Results were obtained over an average of 100 repetitions, each repetition with 1000 rounds, with a punishment severity of S=2. Players using the *'Fermi'* update strategy and *'Myopic'* update strategy, are chosen randomly every round on a 100×100 players lattice. Increasing the value of p, with a step of 0.0025, and the rate of players using the 'Fermi' update strategy, with a step of 5%, while decreasing the rate of players using the *'Myopic'* update strategy, with a step of 5%, we notice that the limits of

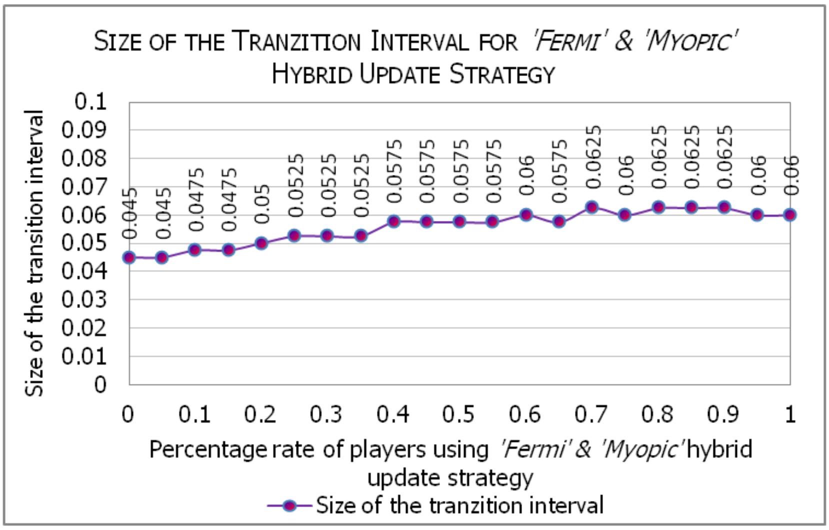

Fig. 9. The size of the transition interval with different percentage rates of players using the *'Fermi'* & *'Myopic'* update strategy.

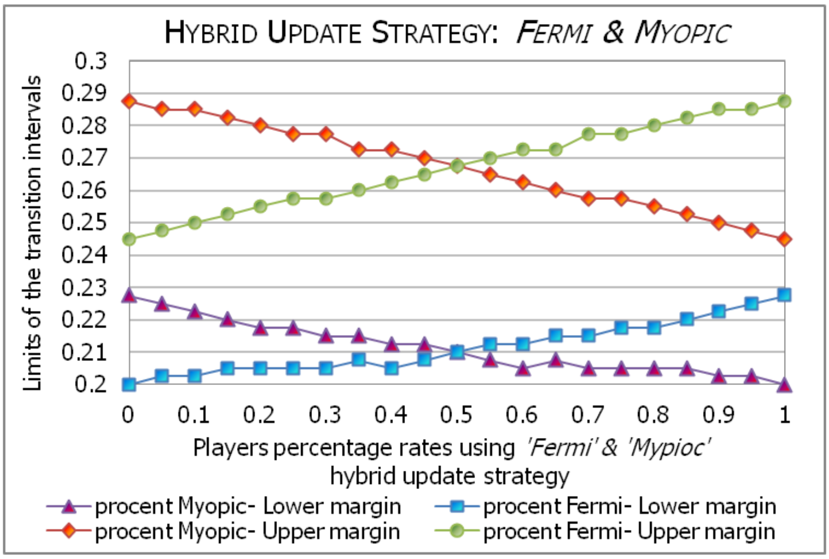

Fig. 10. *'Fermi'* & *'Myopic'* hybrid update Strategies. The evolution of the lower and upper limits of the transition intervals.

the transition interval increase in value, from 0.2 up to 0.2275 for the lower limit and from 0.245 up to 0.2875 for the upper limit of the transition interval (see Fig. 8 and 10).

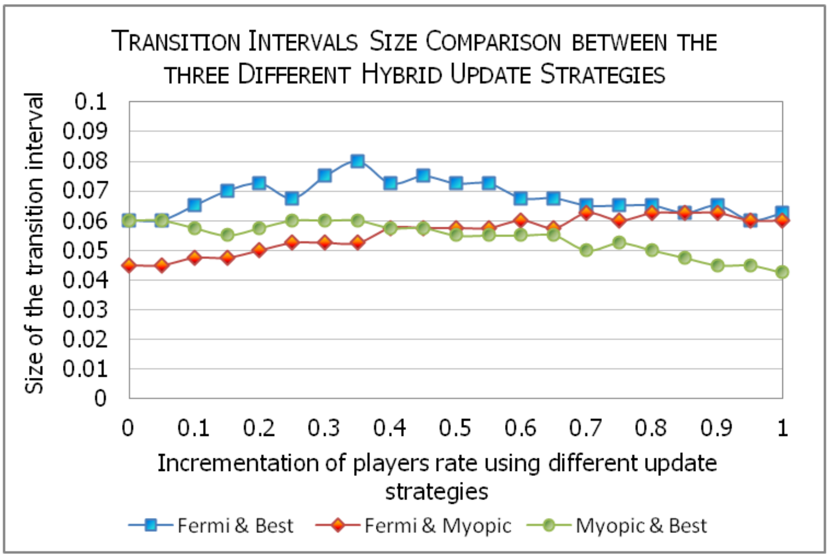

Fig. 11. Size comparison of transition intervals, between the three different Hybrid Update Strategies: *'Fermi'* & *'Best'* hybrid update strategy, *'Fermi'* & *'Myopic'* hybrid update strategy and *'Myopic'* & *'Best'* hybrid update strategy.

The sizes of the *p-transition intervals* follow an upward trajectory from 0.045 up to 0.06 (see Fig. 9), with fewer fluctuations than using *'Fermi'* & *'Best'* hybrid update strategies.

5.5. **Comparison Between the Three Hybrid Update Strategies in Social Honesty Game.** Fig. 11 depicts the sizes of *p-transition intervals* for hybrid updating strategies. Simulations reveal that using *'Fermi'* & *'Best'* hybrid update strategy leads to higher values for the sizes of *p-transition intervals*, which leads us to the assumption that honest behavior becomes harder to promote (see also Fig. 6). We also notice that fluctuations appear when using *'Fermi'* & *'Best'* hybrid update strategies.

Fig. 12 and Fig. 13 reveal the evolution of the *p-transition intervals* for all three hybrid update strategies used in our simulations. The limits of *p-transition intervals* follow an upward trajectory for all three hybrid update strategies, with the highest values for *'Fermi'* & *'Myopic'* hybrid update strategy and the lowest values for *'Myopic'* & *'Best'* hybrid update strategies. When *'Myopic'* & *'Best'* hybrid update strategies are used, honest behavior becomes easier to promote.

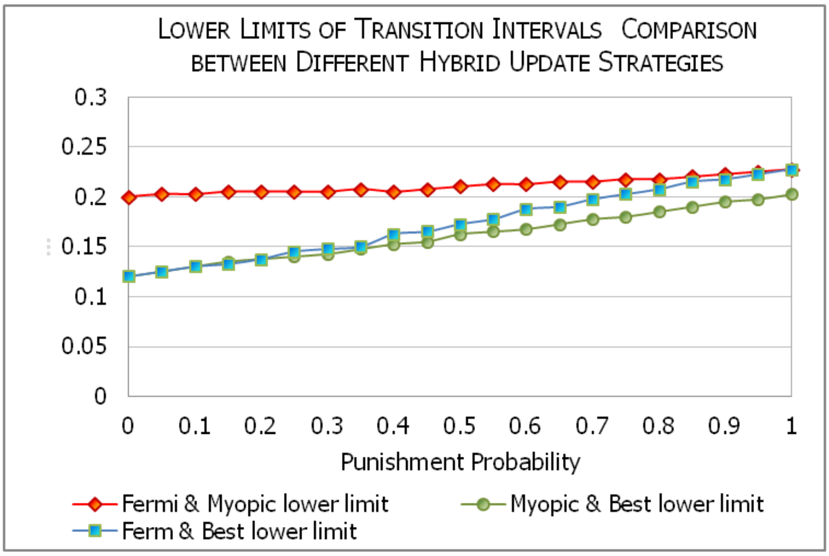

Fig. 12. Lower limits of transition intervals, using the three different Hybrid Update Strategies: *'Fermi'* & *'Best'* hybrid update strategy, *'Fermi'* & *'Myopic'* hybrid update strategy and *'Myopic'* & *'Best'* hybrid update strategy.

When players no longer activate according to a single update strategy, the size and limits of *p-transition intervals* are redefined. Hybrid update strategies cause unexpected changes in the games' outcome. People tend to make decisions in accordance with the successes and failures they encounter in life. Decision-making process takes into account the different experiences and potential obstacles that people encounter (i.e. exams, fines, prizes etc.). Pure update strategies are rare, since individuals interact with other people, taking into account the nature of relations (brother, sister, parent, friend, stranger etc.).

6. Conclusions

Previous studies based on Social Honesty game [13] have considered only one update strategy mechanism for all players. Compared to hybrid update strategies, using a single update strategy leads to greater ease in promoting a honest behavior.

In the present study different players may use different update strategies (hybrid update strategies). Numerical experiments indicate that when players use *'Fermi'* & *'Best'* hybrid update strategy the honest behavior becomes

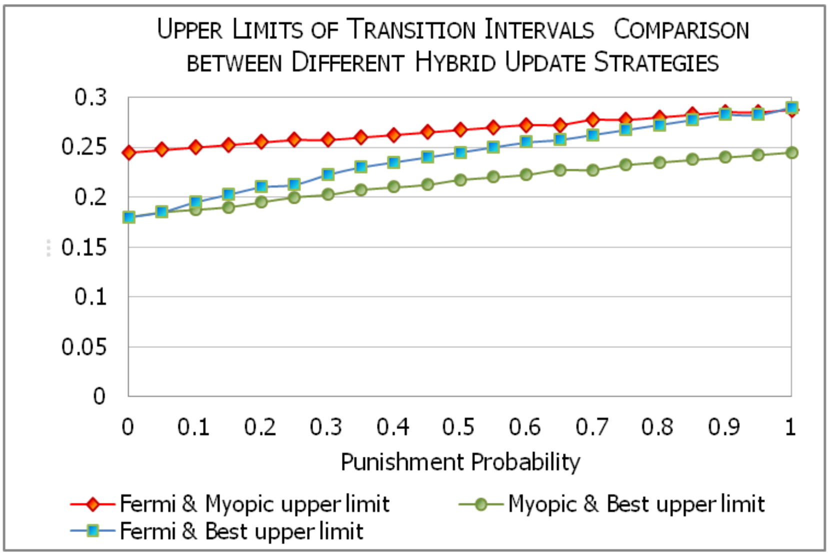

Fig. 13. Upper limits of transition intervals, using the three different Hybrid Update Strategies: *'Fermi'* & *'Best'* hybrid update strategy, *'Fermi'* & *'Myopic'* hybrid update strategy and *'Myopic'and'Best'* hybrid update strategy.

harder to promote than using the other two hybrid update strategies. The limits and sizes of *p-transition intervals* are redefined. Compared to the *'Myopic'* & *'Best'* hybrid update strategy the use of *'Fermi'* & *'Best'* hybrid update strategy leads to higher values of the limits of the *p-transition intervals* (see Fig. 4, 6, 12 and 13).

Compared to the other two hybrid update strategies *'Fermi'* & *'Best'* provided the most sharper increase in value for the limits of *p-transition intervals* (Fig. 4, 10 and 11). Transition intervals have the biggest fluctuations in size, and their width exceeds them on the other two hybrid update strategies (Fig. 11).

When players use *'Myopic'* & *'Best'* hybrid update strategies the limits of *p-transition intervals* are lower in value (see Fig. 4, 12 and 13). Transition intervals have fewer fluctuations in size (see Fig. 5 and 11).

From all three hybrid update strategies *'Fermi'* & *'Myopic'* proves to be the most intriguing. The limits of transition intervals are higher than any of the other two hybrid update strategies (see Fig. 8, 12 and 13). Also the evolution of the limits is somewhat more lightly. The sizes of transition intervals are lower in value and have fewer fluctuations (Fig. 9 and 11).

When it comes to 'Myopic' strategy, whether it is combined with *'Best'* or *'Fermi'* update strategies, the sizes of *p-transition intervals* are lower and have fewer fluctuations which leads us to the assumption that when people are missing information they become easier to control and an honest behavior is easier to promote. As future work we intend to experiment new scenarios that combine tit-for-tat update strategy with synchronous and asynchronous activation in Social Honesty game. Another direction for study is examining how a mixed strategy influences the game's outcome in combination with asynchronous updating.

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