STUDIA UNIV. BABEŞ–BOLYAI, INFORMATICA, Volume **LXI**, Number 1, 2016

A TRAPEZOIDAL INTUITIONISTIC FUZZY MCDM METHOD BASED ON SOME AGGREGATION OPERATORS AND SEVERAL RANKING METHODS

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Abstract. Intuitionistic fuzzy numbers extend fuzzy numbers and they are characterized by two functions that express the degree of membership and respectively non-membership. Therefore, intuitionistic fuzzy numbers better quantify uncertain information that occurs in many real situations and can be successfully used in multicriteria decision making (MCDM) methods. MCDM is a process of problem identification, construction of preferences, evaluation of alternatives and determination of the best alternative. Intuitionistic fuzzy numbers aggregating and ranking are still open research topics. In this paper we propose a MCDM method based on trapezoidal intuitionistic fuzzy numbers ($TIFN$s). We use two aggregation operators and four ranking methods with $TIFN$s in order to obtain eight hierarchies of the given alternatives to assist in making a decision. An algorithm for ranking alternatives based on performance of alternatives versus criteria and weights of the given criteria, both represented by $TIFN$s is elaborated. The applicability of the proposed method is shown by a numerical example.

1. Introduction

MCDM methods are the main content of the decision theory research (see [19]). Specifically, a MCDM method is a procedure for ranking alternatives, according to several criteria, knowing the opinion of the decision-makers regarding the performance of alternatives and weights of criteria (see, e.g., [11]). MCDM has a wide range of applications such as personal evaluation, product evaluation, evaluation of employee performance, economic evaluation, assisting investment decisions, risk assessment etc. (see [21]). Classical MCDM supposes the existence of accurate data, but in practice it is almost impossible to

Received by the editors: October 19, 2015.

2010 *Mathematics Subject Classification.* 03E72, 62C86.

1998 *CR Categories and Descriptors.* H.4.2 [**Information Systems Applications**]: Types of Systems – *Decision support (e.g., MIS)*.

Key words and phrases. trapezoidal intuitionistic fuzzy number, aggregation operator, ranking method, intuitionistic fuzzy MCDM.

obtain exact information due to the uncertainty and imprecision of available data. Because of the complexity and uncertainty of decision making process, MCDM methods based on fuzzy environment has become in last years an area of research that has received more and more attention (see, e.g., [2], [5], [10], [11], [17], [18], [20], [22], [23], [25], [28]). In [1] and [2] was introduced, for the first time, the notion of intuitionistic fuzzy set, as a generalization of fuzzy sets, characterized by two functions that express the degree of membership and respectively the degree of non-membership. An intuitionistic fuzzy number is a particular intuitionistic fuzzy set and an extension of a fuzzy number as well. The degree of non-membership is different from the complement of the degree of membership. In many real situations (see [15]) the intuitionistic fuzzy numbers model better the uncertainty than fuzzy numbers.

The ranking of intuitionistic fuzzy numbers is still an important issue, although several methods have been proposed (see, e.g., [14], [16], [21], [27], [28]). Due to the simple form and easy computation, the $TIFN$s can be successfully used in the intuitionistic fuzzy MCDM methods. In order to develop the proposed method, there will be defined on $TIFN$s two aggregation operators and four ranking methods.

The paper is structured as follows.

In Section 2 we recall notions and operations related to intuitionistic fuzzy numbers and especially with $TIFN$s, we consider two aggregation operators of the $TIFN$s, namely the weighted arithmetic aggregation (WAA) operator and the weighted geometric aggregation (WGA) operator and we mention some numerical characteristics of $TIFN$s such as the index, the value, the ambiguity, the value-index and the ambiguity-index, the score, the accuracy and the expected value and four ranking methods on $TIFN$s based on these associated characteristics. In Section 3 we give a proposed MCDM method with $TIFN$s based on the aggregation operators and ranking methods described in Section 2. It is also given the algorithm for ranking alternatives versus criteria, knowing the performances of alternatives and weights of criteria, both given by $TIFN$s. An example is used to show the applicability of the proposed method in Section 4. Section 5 provides other intuitionistic fuzzy MCDM methods from the literature and the obtained results are compared. The paper ends with a conclusive section.

2. Definitions and notations

In this section we consider the basic definitions, notations and operations used in this paper.

Even if there are other definitions or representations of the notion of fuzzy number, the following definition is already accepted in the scientific community (see [7], [12]). This definition leads also to operations between fuzzy numbers taken from arithmetic interval by the Zadeh's extension principle.

Definition 1. *(see [13]) A fuzzy number A is a fuzzy set in $\mathbb{R}$, that is a mapping $A : \mathbb{R} \to [0, 1]$, which satisfies the following properties:*
(i) A is normal, i.e. $\exists x_0 \in \mathbb{R}$ such that $A(x_0) = 1$;
(ii) A is fuzzy convex, i.e. $A(\lambda x_1 + (1-\lambda)x_2) \geq min\{A(x_1), A(x_2)\}$, for every $\lambda \in [0, 1]$ and $x_1, x_2 \in \mathbb{R}$;
(iii) A is upper semicontinuous in $\mathbb{R}$, i.e. $\forall \epsilon > 0$ $\exists \delta > 0$ such that $A(x) - A(x_0) < \epsilon$, $|x - x_0| < \delta$;
(iv) A is compactly supported, i.e. $cl\{x \in \mathbb{R};\ A(x) > 0\}$ is compact, where $cl(M)$ denotes the closure of a set M.

Trapezoidal fuzzy numbers are particular fuzzy numbers often used in applications.

Definition 2. *(see [7]) A trapezoidal fuzzy number $A = (a, b, c, d)$, where $a \leq b \leq c \leq d$, is a fuzzy set in $\mathbb{R}$ with the membership function given by*

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } x \in [a, b) \\ 1, & \text{if } x \in [b, c] \\ \frac{d-x}{d-c}, & \text{if } x \in (c, d] \\ 0, & \text{otherwise.} \end{cases}$$

Definition 3. *(see [1] and [3]) An intuitionistic fuzzy set in $X \neq \emptyset$ is an object $\widetilde{A}$ given by $\widetilde{A} = \left\{\left\langle x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x)\right\rangle; x \in X\right\}$, where the membership function $\mu_{\widetilde{A}} : X \to [0, 1]$ and the non-membership function $\nu_{\widetilde{A}} : X \to [0, 1]$ satisfy the condition $0 \leq \mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x) \leq 1$, for every $x \in X$.*

$TIFN$s are used to represent an ill-known information in applications (see, e.g., [8], [9], [16], [26]).

Definition 4. *(see [16]) A $TIFN$ $\widetilde{A} = \langle(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2)\rangle$ is an intuitionistic fuzzy set in $\mathbb{R}$, with the membership function $\mu_{\widetilde{A}}$ and the non-membership function $\nu_{\widetilde{A}}$ defined as*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a_1}{b_1-a_1}, & \text{if } x \in [a_1, b_1) \\ 1, & \text{if } x \in [b_1, c_1] \\ \frac{d_1-x}{d_1-c_1}, & \text{if } x \in (c_1, d_1] \\ 0, & \text{otherwise} \end{cases} \quad \text{and } \nu_{\widetilde{A}}(x) = \begin{cases} \frac{b_2-x}{b_2-a_2}, & \text{if } x \in [a_2, b_2) \\ 0, & \text{if } x \in [b_2, c_2] \\ \frac{x-c_2}{d_2-c_2}, & \text{if } x \in (c_2, d_2] \\ 1, & \text{otherwise} \end{cases},$$

where $a_2 \leq a_1 \leq b_2 \leq b_1 \leq c_1 \leq c_2 \leq d_1 \leq d_2$.

Definition 5. *(see [14]) A $TIFN$ $\widetilde{A} = \langle(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2)\rangle$ is said to be non-negative $TIFN$ if and only if $a_2 \geq 0$.*

Remark 1. *Any trapezoidal fuzzy number $A = (a, b, c, d)$ can be considered as a $TIFN$ $\widetilde{A} = \langle (a, b, c, d), (a, b, c, d) \rangle$.*

We denote by $TIFN(\mathbb{R})$ the set of $TIFN$s.

In the following we recall the following basic operations on $TIFN$s based on Zadeh's extension principle.

Let $\widetilde{A} = \langle (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2) \rangle$ and $\widetilde{B} = \langle (a_3, b_3, c_3, d_3), (a_4, b_4, c_4, d_4) \rangle$ be two $TIFN$s and λ a real number. The sum of $\widetilde{A}$ and $\widetilde{B}$ is defined by (see [6])

$$\widetilde{A} + \widetilde{B} = \langle (a_1 + a_3, b_1 + b_3, c_1 + c_3, d_1 + d_3), (a_2 + a_4, b_2 + b_4, c_2 + c_4, d_2 + d_4) \rangle, \tag{1}$$

the scalar multiplication (see [6]), such as

$$\lambda \cdot \widetilde{A} = \langle (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1), (\lambda a_2, \lambda b_2, \lambda c_2, \lambda d_2) \rangle, \text{ for } \lambda \geq 0 \text{ and} \tag{2}$$

$$\lambda \cdot \widetilde{A} = \langle (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1), (\lambda d_2, \lambda c_2, \lambda b_2, \lambda a_2) \rangle, \text{ for } \lambda < 0, \tag{3}$$

the product on non-negative $TIFN$s (see [14]), which is an approximation of the product obtained by Zadeh's extension principle, such as

$$\widetilde{A} \otimes \widetilde{B} = \langle (a_1 a_3, b_1 b_3, c_1 c_3, d_1 d_3), (a_2 a_4, b_2 b_4, c_2 c_4, d_2 d_4) \rangle \tag{4}$$

and the rise to positive power of a non-negative $TIFN$, such as

$$\widetilde{A}^{\lambda} = \langle (a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}), (a_2^{\lambda}, b_2^{\lambda}, c_2^{\lambda}, d_2^{\lambda}) \rangle, \text{ for } \lambda \geq 0. \tag{5}$$

It is obvious that the neutral element for the sum is $\langle (0, 0, 0, 0), (0, 0, 0, 0) \rangle$ and for the product is $\langle (1, 1, 1, 1), (1, 1, 1, 1) \rangle$.

Suppose that $\widetilde{A_i}$, $i = \{1, \ldots, n\}$ is a set of non-negative $TIFN$s and $\widetilde{\omega_i}$ given by a non-negative $TIFN$ is the weight of A_i, for all $i = \{1, \ldots, n\}$. Then the WAA aggregation operator (see [29]) is $WAA_{\widetilde{\omega}} : TIFN^n(\mathbb{R}) \to TIFN(\mathbb{R})$,

$$WAA_{\widetilde{\omega}}(\widetilde{A_1}, \ldots, \widetilde{A_n}) = (1/n) \cdot (\widetilde{\omega_1} \otimes \widetilde{A_1} + \ldots + \widetilde{\omega_n} \otimes \widetilde{A_n}). \tag{6}$$

If ω_i, $i = \{1, \ldots, n\}$ are given by positive crisp numbers, then the WGA aggregation operator (see [24]) is $WGA_{\omega} : TIFN^n(\mathbb{R}) \to TIFN(\mathbb{R})$,

$$WGA_{\omega}(\widetilde{A_1}, \ldots, \widetilde{A_n}) = \widetilde{A_1}^{\omega_1} \otimes \ldots \otimes \widetilde{A_n}^{\omega_n}. \tag{7}$$

Among many ranking methods on $TIFN$s (see, e.g., [14], [16], [21], [27], [28]), in this section we consider four of them. For this purpose, we recall the definition of some numerical characteristics of the $TIFN$s, such as the index, the value, the ambiguity, the value-index and the ambiguity-index, the score, the accuracy and the expected value. The ranking methods based on these characteristics will be used in Section 3 for ranking the alternatives in an intuitionistic fuzzy frame.

We consider the $TIFN$ $\widetilde{A} = \langle (a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2) \rangle$.

Firstly, we consider a ranking method on $TIFN$s based on the index $M_\mu^{\beta,k}$ for membership function and index $M_\nu^{\beta,k}$ for non-membership function (see [14]). In the particular case when $\beta = \frac{1}{3}$ and $k = 0$, these indexes are:

$$(8) \quad M_\mu^{\frac{1}{3},0}(\widetilde{A}) = \frac{1}{6}(a_1 + 2b_1 + 2c_1 + d_1),\ M_\nu^{\frac{1}{3},0}(\widetilde{A}) = \frac{1}{6}(a_2 + 2b_2 + 2c_2 + d_2).$$

Further, for simplification, we denote $M_\mu(\widetilde{A}) = M_\mu^{\frac{1}{3},0}(\widetilde{A})$, $M_\nu(\widetilde{A}) = M_\nu^{\frac{1}{3},0}(\widetilde{A})$.

Definition 6. *(see [14]) Let $\widetilde{A}$ and $\widetilde{B}$ be two $TIFN$s. Then*

$$\widetilde{A} \prec_M \widetilde{B} \Leftrightarrow M_\mu(\widetilde{A}) < M_\mu(\widetilde{B}) \text{ or } (M_\mu(\widetilde{A}) = M_\mu(\widetilde{B}) \text{ and} - M_\nu(\widetilde{A}) < -M_\nu(\widetilde{B})),$$
$$\widetilde{A} \succ_M \widetilde{B} \Leftrightarrow M_\mu(\widetilde{A}) > M_\mu(\widetilde{B}) \text{ or } (M_\mu(\widetilde{A}) = M_\mu(\widetilde{B}) \text{ and} - M_\nu(\widetilde{A}) > -M_\nu(\widetilde{B})),$$
$$\widetilde{A} \sim_M \widetilde{B} \Leftrightarrow M_\mu(\widetilde{A}) = M_\mu(\widetilde{B}) \text{ and } M_\nu(\widetilde{A}) = M_\nu(\widetilde{B}).$$

The second ranking method is a ranking method on $TIFN$s based on the value-index V_λ and ambiguity-index A_λ (see [28]). The value of the membership function is given by $V_\mu(\widetilde{A}) = \frac{1}{6}(a_1 + 2b_1 + 2c_1 + d_1)$ and the value of the non-membership function is given by $V_\nu(\widetilde{A}) = \frac{1}{6}(a_2 + 2b_2 + 2c_2 + d_2)$. Analogously, the ambiguity of the membership function is given by $A_\mu(\widetilde{A}) = \frac{1}{6}(-a_1 - 2b_1 + 2c_1 + d_1)$ and the ambiguity of the non-membership function is given by $A_\nu(\widetilde{A}) = \frac{1}{6}(-a_2 - 2b_2 + 2c_2 + d_2)$. Then the value-index and the ambiguity-index of $\widetilde{A}$ are given by

$$(9) \quad V_\lambda(\widetilde{A}) = \lambda V_\mu(\widetilde{A}) + (1-\lambda)V_\nu(\widetilde{A}) \text{ and } A_\lambda(\widetilde{A}) = \lambda A_\mu(\widetilde{A}) + (1-\lambda)A_\nu(\widetilde{A}).$$

Here $\lambda \in [0, 1]$ is a weight which represents the decision-maker's preference information, namely $\lambda \in [0, 0.5)$ shows that the decision-maker prefers certainty, $\lambda \in (0.5, 1]$ shows that the decision-maker prefers uncertainty and $\lambda = 0.5$ shows that the decision-maker is indifferent between certainty and uncertainty.

Definition 7. *(see [28]) Let $\widetilde{A}$ and $\widetilde{B}$ be two $TIFN$s. Then*

$$\widetilde{A} \prec_{VA} \widetilde{B} \Leftrightarrow V_\lambda(\widetilde{A}) < V_\lambda(\widetilde{B}) \text{ or } (V_\lambda(\widetilde{A}) = V_\lambda(\widetilde{B}) \text{ and } A_\lambda(\widetilde{A}) > A_\lambda(\widetilde{B})),$$
$$\widetilde{A} \succ_{VA} \widetilde{B} \Leftrightarrow V_\lambda(\widetilde{A}) > V_\lambda(\widetilde{B}) \text{ or } (V_\lambda(\widetilde{A}) = V_\lambda(\widetilde{B}) \text{ and } A_\lambda(\widetilde{A}) < A_\lambda(\widetilde{B})),$$
$$\widetilde{A} \sim_{VA} \widetilde{B} \Leftrightarrow V_\lambda(\widetilde{A}) = V_\lambda(\widetilde{B}) \text{ and } A_\lambda(\widetilde{A}) = A_\lambda(\widetilde{B}).$$

For a third ranking method, introduced in [29], we recall the following definition of the score S and of the accuracy E of $\widetilde{A}$:

$$S(\widetilde{A}) = (a_1 - a_2 + b_1 - b_2 + c_1 - c_2 + d_1 - d_2)/4,$$
$$E(\widetilde{A}) = (a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2)/4. \tag{10}$$

If $a_i, b_i, c_i, d_i \in [0,1]$, for $i \in \{1,2\}$ then $S(\widetilde{A}) \in [-1,1]$ and $E(\widetilde{A}) \in [0,2]$.

Definition 8. *(see* [29]*) Let* $\widetilde{A}$ *and* $\widetilde{B}$ *be two* $TIFNs$*. Then*

$$\widetilde{A} \prec_{SE} \widetilde{B} \Leftrightarrow S(\widetilde{A}) < S(\widetilde{B}) \text{ or } (S(\widetilde{A}) = S(\widetilde{B}) \text{ and } E(\widetilde{A}) < E(\widetilde{B})),$$
$$\widetilde{A} \succ_{SE} \widetilde{B} \Leftrightarrow S(\widetilde{A}) > S(\widetilde{B}) \text{ or } (S(\widetilde{A}) = S(\widetilde{B}) \text{ and } E(\widetilde{A}) > E(\widetilde{B})),$$
$$\widetilde{A} \sim_{SE} \widetilde{B} \Leftrightarrow S(\widetilde{A}) = S(\widetilde{B}) \text{ and } E(\widetilde{A}) = E(\widetilde{B}).$$

Last ranking method, but not the least important, because it is simple and has suitable properties, is based on the expected value EV (see, e.g., [6]):

$$EV(\widetilde{A}) = (a_1 + b_1 + c_1 + d_1 + a_2 + b_2 + c_2 + d_2)/8. \tag{11}$$

Definition 9. *(see* [6]*) Let* $\widetilde{A}$ *and* $\widetilde{B}$ *be two* $TIFNs$*. Then*

$$\widetilde{A} \prec_{EV} \widetilde{B} \Leftrightarrow EV(\widetilde{A}) < EV(\widetilde{B}),$$
$$\widetilde{A} \succ_{EV} \widetilde{B} \Leftrightarrow EV(\widetilde{A}) > EV(\widetilde{B}),$$
$$\widetilde{A} \sim_{EV} \widetilde{B} \Leftrightarrow EV(\widetilde{A}) = EV(\widetilde{B}).$$

3. Proposed trapezoidal intuitionistic fuzzy MCDM method

A MCDM problem assumes the evaluation of m alternatives $A_1, \ldots, A_m$, under n criteria $C_1, \ldots, C_n$ by a committee of k decision-makers $D_1, \ldots, D_k$. We consider that all criteria are subjective criteria or objective criteria with respect to the benefit. The performances of alternatives versus criteria indicate the degree that the alternatives satisfy or do not satisfy the criteria and are given by decision-makers or experts according to the specified linguistic terms. In addition, we know the weight of each criterion, given by the decision-makers according to either the same linguistic terms or another. The problem is resumed to the evaluation of alternatives and choosing the best one.

The method described in this section follows the standard steps (see, e.g., [4]), but our goal is to compare the results when using different aggregation operators and/or ranking methods. The method can be summarized as follows. First we determine the average of performances, obtaining the decision matrix and the average of weights of criteria, obtaining a vector (see Algorithm 1, Steps 1-2). Then we normalize both of them (see Algorithm 1, Steps 3-4).

The value of each alternative is calculated using, one at a time, the aggregation operators from Section 2 (see Algorithm 1, Steps 5-7). The hierarchy of the alternative values is determined by using one of the ranking methods considered in Definitions 6 - 9, used one at a time, too (see Algorithm 1, Steps 8-12). We consider that the performance of an alternative A_i on a criterion C_j in the opinion of the decision-maker D_t is given by a non-negative $TIFN$ $\widetilde{r_{ijt}} = \langle(a_{1ijt}, b_{1ijt}, c_{1ijt}, d_{1ijt}), (a_{2ijt}, b_{2ijt}, c_{2ijt}, d_{2ijt})\rangle$ and the weight of the criterion C_j in the opinion of the decision-maker D_t is also given also by a non-negative $TIFN$ $\widetilde{w_{jt}} = \langle(e_{1jt}, f_{1jt}, g_{1jt}, h_{1jt}), (e_{2jt}, f_{2jt}, g_{2jt}, h_{2jt})\rangle$.

For the first step of the proposed method we calculate the average rating $\widetilde{r_{ij}}$ of A_i versus C_j, $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$, in order to obtain the decision matrix, as follows:

$$\widetilde{r_{ij}} = (1/k) \cdot (\widetilde{r_{ij1}} + \ldots + \widetilde{r_{ijk}}), \text{ using (1) and (2).} \tag{12}$$

Next step is the calculation of the average weight $\widetilde{w_j}$ of the criterion C_j, $j \in \{1, \ldots, n\}$, as follows:

$$\widetilde{w_j} = (1/k) \cdot (\widetilde{w_{j1}} + \ldots + \widetilde{w_{jk}}), \text{ using (1) and (2) too.} \tag{13}$$

For the next step we have to normalize the values of average performances with respect to criteria and the values of averaged weights of criteria. This is only necessary if the maximum value of the performances and/or respectively the maximum value of the weights are greater than 1. We normalize as follows: if $\widetilde{r_{ij}} = \langle(a_{1ij}, b_{1ij}, c_{1ij}, d_{1ij}), (a_{2ij}, b_{2ij}, c_{2ij}, d_{2ij})\rangle$, $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$ and we find that $\alpha = \max\limits_{\substack{1 \le i \le m \\ 1 \le j \le n}} d_{2ij} > 1$, then

$$\widetilde{r_{ij}} = (1/\alpha) \cdot \widetilde{r_{ij}}, \text{ using (2),} \tag{14}$$

where, for simplicity, we used the same notation $\widetilde{r_{ij}}$ for the normalized values in decision matrix. In the same way, if $\widetilde{w_j} = \langle(e_{1j}, f_{1j}, g_{1j}, h_{1j}), (e_{2j}, f_{2j}, g_{2j}, h_{2j})\rangle$, $j \in \{1, \ldots, n\}$ and we find that $\beta = \max\limits_{1 \le j \le n} h_{2j} > 1$, then

$$\widetilde{w_j} = (1/\beta) \cdot \widetilde{w_j}, \text{ using (2).} \tag{15}$$

We also used the same notation $\widetilde{w_j}$ for the normalized values of the weights of the criteria. Next step is to evaluate the alternatives A_i, $i \in \{1, \ldots, m\}$ by the aggregation of the performances with weights using the $WAA_{\widetilde{\omega}}$ operator, developed as

$$\widetilde{G_i} = (1/n) \cdot (\widetilde{r_{i1}} \otimes \widetilde{w_1} + \ldots + \widetilde{r_{in}} \otimes \widetilde{w_n}), \text{ using (1), (2) and (4).} \tag{16}$$

If we use the WGA_ω operator, for the beginning, the weights must be defuzzified using the expected value (see (11)), namely $w_j = EV(\widetilde{w_j})$, for $j =$

$\{1, \ldots, n\}$, then

$$\widetilde{H_i} = \widetilde{r_{i1}}^{w_1} \otimes \ldots \otimes \widetilde{r_{in}}^{w_n}, \text{ for } i \in \{1, \ldots, m\}, \text{ using (4) and (5).} \tag{17}$$

In order to obtain the ranking of alternatives, we used, one at a time, all four criteria from Definitions 6 - 9, separately for $\widetilde{G_i}$ and $\widetilde{H_i}$.

The above method can be summarized in the following procedure.

Algorithm 1.

IN: m - alternatives
n - criteria
k - decision-makers
$\widetilde{r_{ijt}} = \langle(a_{1ijt}, b_{1ijt}, c_{1ijt}, d_{1ijt}), (a_{2ijt}, b_{2ijt}, c_{2ijt}, d_{2ijt})\rangle$ - performance of the alternative A_i on criterion C_j in the opinion of the decision-maker D_t, given by a non-negative $TIFN$, for all $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$, $t \in \{1, \ldots, k\}$
$\widetilde{w_{jt}} = \langle(e_{1jt}, f_{1jt}, g_{1jt}, h_{1jt}),\ (e_{2jt}, f_{2jt}, g_{2jt}, h_{2jt})\rangle$ - weight of the criterion C_j in the opinion of the decision-maker D_t, given by a non-negative $TIFN$, for all $j \in \{1, \ldots, n\}$, $t \in \{1, \ldots, k\}$

Step 1. Compute $\widetilde{r_{ij}}$ for $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$ as follows:

$$\begin{aligned}\widetilde{r_{ij}} = \langle(&\frac{1}{k} \cdot \sum_{t=1}^{k} a_{1ijt}, \frac{1}{k} \cdot \sum_{t=1}^{k} b_{1ijt}, \frac{1}{k} \cdot \sum_{t=1}^{k} c_{1ijt}, \frac{1}{k} \cdot \sum_{t=1}^{k} d_{1ijt}),\\ (&\frac{1}{k} \cdot \sum_{t=1}^{k} a_{2ijt}, \frac{1}{k} \cdot \sum_{t=1}^{k} b_{2ijt}, \frac{1}{k} \cdot \sum_{t=1}^{k} c_{2ijt}, \frac{1}{k} \cdot \sum_{t=1}^{k} d_{2ijt})\rangle.\end{aligned}$$

Step 2. Compute $\widetilde{w_j}$ for $j \in \{1, \ldots, n\}$ as follows:

$$\begin{aligned}\widetilde{w_j} = \langle(&\frac{1}{k} \cdot \sum_{t=1}^{k} e_{1jt}, \frac{1}{k} \cdot \sum_{t=1}^{k} f_{1jt}, \frac{1}{k} \cdot \sum_{t=1}^{k} g_{1jt}, \frac{1}{k} \cdot \sum_{t=1}^{k} h_{1jt}),\\ (&\frac{1}{k} \cdot \sum_{t=1}^{k} e_{2jt}, \frac{1}{k} \cdot \sum_{t=1}^{k} f_{2jt}, \frac{1}{k} \cdot \sum_{t=1}^{k} g_{2jt}, \frac{1}{k} \cdot \sum_{t=1}^{k} h_{2jt})\rangle.\end{aligned}$$

Step 3. If $\alpha = \max\limits_{\substack{1 \le i \le m \\ 1 \le j \le n}} d_{2ij} > 1$, then for $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, n\}$

$$\widetilde{r_{ij}} = \langle(\frac{a_{1ij}}{\alpha}, \frac{b_{1ij}}{\alpha}, \frac{c_{1ij}}{\alpha}, \frac{d_{1ij}}{\alpha}), (\frac{a_{2ij}}{\alpha}, \frac{b_{2ij}}{\alpha}, \frac{c_{2ij}}{\alpha}, \frac{d_{2ij}}{\alpha})\rangle.$$

Step 4. If $\beta = \max_{1 \le j \le n} h_{2j} > 1$, then for $j \in \{1, \dots, n\}$

$$\widetilde{w_j} = \langle (\frac{e_{1j}}{\beta}, \frac{f_{1j}}{\beta}, \frac{g_{1j}}{\beta}, \frac{h_{1j}}{\beta}), (\frac{e_{2j}}{\beta}, \frac{f_{2j}}{\beta}, \frac{g_{2j}}{\beta}, \frac{h_{2j}}{\beta}) \rangle.$$

Step 5. Compute $\widetilde{G_i}$ for $i \in \{1, \dots, m\}$ as follows:

$$\widetilde{G_i} = \langle (\frac{1}{n} \sum_{j=1}^{n} (a_{1ij} \cdot e_{1j}), \frac{1}{n} \sum_{j=1}^{n} (b_{1ij} \cdot f_{1j}), \frac{1}{n} \sum_{j=1}^{n} (c_{1ij} \cdot g_{1j}), \frac{1}{n} \sum_{j=1}^{n} (d_{1ij} \cdot h_{1j})),$$

$$(\frac{1}{n} \sum_{j=1}^{n} (a_{2ij} \cdot e_{2j}), \frac{1}{n} \sum_{j=1}^{n} (b_{2ij} \cdot f_{2j}), \frac{1}{n} \sum_{j=1}^{n} (c_{2ij} \cdot g_{2j}), \frac{1}{n} \sum_{j=1}^{n} (d_{2ij} \cdot h_{2j})) \rangle. \tag{18}$$

Step 6. Compute $w_j = EV(\widetilde{w_j})$, for $j \in \{1, \dots, n\}$, using (11).

Step 7. Compute $\widetilde{H_i}$ for $i \in \{1, \dots, m\}$ as follows:

$$\widetilde{H_i} = \langle (\prod_{j=1}^{n} {a_{1ij}}^{w_j}, \prod_{j=1}^{n} {b_{1ij}}^{w_j}, \prod_{j=1}^{n} {c_{1ij}}^{w_j}, \prod_{j=1}^{n} {d_{1ij}}^{w_j}), \tag{19}$$

$$(\prod_{j=1}^{n} {a_{2ij}}^{w_j}, \prod_{j=1}^{n} {b_{2ij}}^{w_j}, \prod_{j=1}^{n} {c_{2ij}}^{w_j}, \prod_{j=1}^{n} {d_{2ij}}^{w_j}) \rangle.$$

Step 8. Compute $M_\mu(\widetilde{G_i})$, $M_\nu(\widetilde{G_i})$, $M_\mu(\widetilde{H_i})$ and $M_\nu(\widetilde{H_i})$ for $i \in \{1, \dots, m\}$, using (8).

Step 9. If $G_{i_1} \succ_M G_{i_2} \succ_M \dots \succ_M G_{i_m}$ then the first descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}, that is A_{i_1} is better than A_{i_2} and so on, A_{i_m} is the worst alternative.

Step 10. If $H_{i_1} \succ_M H_{i_2} \succ_M \dots \succ_M H_{i_m}$ then the second descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}.

Step 11. Compute $V_\lambda(\widetilde{G_i})$, $A_\lambda(\widetilde{G_i})$, $V_\lambda(\widetilde{H_i})$ and $A_\lambda(\widetilde{H_i})$ for $i \in \{1, \dots, m\}$, using (9).

Step 12. If $G_{i_1} \succ_{VA} G_{i_2} \succ_{VA} \dots \succ_{VA} G_{i_m}$ then the third descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}.

Step 13. If $H_{i_1} \succ_{VA} H_{i_2} \succ_{VA} \dots \succ_{VA} H_{i_m}$ then the fourth descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}.

Step 14. Compute $S(\widetilde{G_i})$, $E(\widetilde{G_i})$, $S(\widetilde{H_i})$ and $E(\widetilde{H_i})$ for $i \in \{1, \dots, m\}$ using (10).

Step 15. If $G_{i_1} \succ_{SE} G_{i_2} \succ_{SE} \dots \succ_{SE} G_{i_m}$ then the fifth descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}.

Step 16. If $H_{i_1} \succ_{SE} H_{i_2} \succ_{SE} \dots \succ_{SE} H_{i_m}$ then the sixth descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}.

Step 17. Compute $EV(\widetilde{G_i})$ and $EV(\widetilde{H_i})$ for $i \in \{1, \dots, m\}$ using (11).

Step 18. If $G_{i_1} \succ_{EV} G_{i_2} \succ_{EV} \ldots \succ_{EV} G_{i_m}$ then the seventh descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}.

Step 19. If $H_{i_1} \succ_{EV} H_{i_2} \succ_{EV} \ldots \succ_{EV} H_{i_m}$ then the eighth descending order of alternatives is A_{i_1}, A_{i_2}, ..., A_{i_m}.

OUT: eight descending orders of alternatives.

The proposed algorithm was implemented obtaining a C# program that returns all of these results for numerical examples.

4. Numerical examples

The linguistic variables are used to describe situations where the classical quantitative values can not be used. For example, if we consider a survey and a five-level Likert scale, the values given by a customer to the performance of the alternatives can be in the set {*very poor, poor, fair, good, very good*} and respectively to the weights of the criteria in the set {*very low, low, medium, high, very high*}. Their representations by $TIFN$s can be, for example, those from Table 1.

Table 1. Ratings in a five-level Likert scale

Perform. of alt.	Weight of criteria	$TIFN$s
Very poor (VP)	Very low (VL)	$\langle(0.0, 0.1, 0.2, 0.3), (0.0, 0.1, 0.2, 0.3)\rangle$
Poor (P)	Low (L)	$\langle(0.1, 0.2, 0.3, 0.4), (0.0, 0.2, 0.3, 0.5)\rangle$
Fair (F)	Medium (M)	$\langle(0.3, 0.4, 0.5, 0.6), (0.2, 0.4, 0.5, 0.7)\rangle$
Good (G)	High (H)	$\langle(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9)\rangle$
Very good (VG)	Very high (VH)	$\langle(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0)\rangle$

In this section we give a numerical example, in order to illustrate the proposed method in Section 3. The problem is taken from [27].

Example 1. (see [27]). An investment company must take a decision from four possible alternatives to invest the money, namely, A_1 - a car company, A_2 - food company, A_3 - computer company and A_4 - television company. The decision must be taken according to the following three criteria: C_1 - risk analysis, C_2 - growth analysis and C_3 - environmental impact analysis. The four possible alternatives are to be evaluated under the above three criteria using the corresponding $TIFN$s for linguistic terms, as shown in Table 1. The ratings of the alternatives with respect to criteria and the ratings of the weights of criteria are given in Table 2.

TABLE 2. Ratings of alternatives and weights.

	Criteria / alternatives														
	C_1				C_2				C_3				Criteria		
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	C_1	C_2	C_3
D_1	F	F	P	G	G	F	F	G	F	VG	G	P	M	L	H
D_2	P	F	VP	G	VG	F	F	G	P	VG	G	P	M	M	M
D_3	F	G	P	F	G	F	G	G	F	VG	VG	F	H	L	H
D_4	G	G	F	G	F	G	G	G	F	G	VG	F	H	M	H
D_5	P	P	P	VG	G	F	F	VG	F	VG	VG	P	M	M	H

Using the proposed method, we obtain the normalized averaged ratings of alternatives versus criteria (Steps 1 and 3 of Algorithm 1), as follows:

$$\widetilde{r_{11}} = \langle(0.260, 0.360, 0.460, 0.560), (0.160, 0.360, 0.460, 0.660)\rangle,$$
$$\widetilde{r_{21}} = \langle(0.500, 0.600, 0.700, 0.800), (0.420, 0.600, 0.700, 0.880)\rangle,$$
$$\widetilde{r_{31}} = \langle(0.380, 0.480, 0.580, 0.680), (0.280, 0.480, 0.580, 0.780)\rangle,$$
$$\widetilde{r_{41}} = \langle(0.660, 0.760, 0.860, 0.960), (0.640, 0.760, 0.860, 0.980)\rangle,$$
$$\widetilde{r_{12}} = \langle(0.340, 0.440, 0.540, 0.640), (0.240, 0.440, 0.540, 0.740)\rangle,$$
$$\widetilde{r_{22}} = \langle(0.500, 0.600, 0.700, 0.800), (0.420, 0.600, 0.700, 0.880)\rangle,$$
$$\widetilde{r_{32}} = \langle(0.540, 0.640, 0.740, 0.840), (0.460, 0.640, 0.740, 0.920)\rangle,$$
$$\widetilde{r_{42}} = \langle(0.620, 0.720, 0.820, 0.920), (0.580, 0.720, 0.820, 0.960)\rangle,$$
$$\widetilde{r_{13}} = \langle(0.120, 0.220, 0.320, 0.420), (0.040, 0.220, 0.320, 0.500)\rangle,$$
$$\widetilde{r_{23}} = \langle(0.340, 0.440, 0.540, 0.640), (0.240, 0.440, 0.540, 0.740)\rangle,$$
$$\widetilde{r_{33}} = \langle(0.260, 0.360, 0.460, 0.560), (0.160, 0.360, 0.460, 0.660)\rangle,$$
$$\widetilde{r_{43}} = \langle(0.180, 0.280, 0.380, 0.480), (0.080, 0.280, 0.380, 0.580)\rangle$$

and respectively the normalized averaged ratings of weights of criteria (Steps 2 and 4 of Algorithm 1), as follows:

$$\widetilde{w_1} = \langle(0.380, 0.480, 0.580, 0.680), (0.280, 0.480, 0.580, 0.780)\rangle,$$
$$\widetilde{w_2} = \langle(0.220, 0.320, 0.420, 0.520), (0.120, 0.320, 0.420, 0.620)\rangle,$$
$$\widetilde{w_3} = \langle(0.460, 0.560, 0.660, 0.760), (0.360, 0.560, 0.660, 0.860)\rangle.$$

Obviously, the values $\widetilde{r_{ij}}$ and respectively $\widetilde{w_j}$ are obtained after running the C# program that implements the Algorithm 1 described in Section 3. The aggregated values (Steps 5 and 7 of Algorithm 1) are:

$$\widetilde{G_1} = \langle(0.076, 0.146, 0.235, 0.344), (0.029, 0.146, 0.235, 0.468)\rangle,$$

$$\widetilde{G_2} = \langle(0.152, 0.242, 0.352, 0.482), (0.085, 0.242, 0.352, 0.623)\rangle,$$
$$\widetilde{G_3} = \langle(0.128, 0.212, 0.317, 0.442), (0.064, 0.212, 0.317, 0.582)\rangle,$$
$$\widetilde{G_4} = \langle(0.157, 0.251, 0.365, 0.499), (0.093, 0.251, 0.365, 0.619)\rangle,$$
$$\widetilde{H_1} = \langle(0.090, 0.171, 0.263, 0.367), (0.031, 0.171, 0.263, 0.470)\rangle,$$
$$\widetilde{H_2} = \langle(0.278, 0.383, 0.498, 0.623), (0.192, 0.383, 0.498, 0.742)\rangle,$$
$$\widetilde{H_3} = \langle(0.210, 0.308, 0.417, 0.537), (0.125, 0.308, 0.417, 0.660)\rangle,$$
$$\widetilde{H_4} = \langle(0.236, 0.352, 0.475, 0.606), (0.138, 0.352, 0.475, 0.699)\rangle,$$

where the defuzzified weights are $w_1 = 0.53$, $w_2 = 0.37$, $w_3 = 0.61$.

Therefore, for the first ranking method we obtain for $M_\mu(\widetilde{G_i})$, $i \in \{1, \ldots, m\}$ the values in the second column of the Table 3, the first four rows. Then, using Definition 6, the ranking order is $A_1 \prec_M A_3 \prec_M A_2 \prec_M A_4$, which means that the best alternative is A_4 and the worst A_1. In order to compare the results, using the same ranking method, we obtain for $M_\mu(\widetilde{H_i})$, $i \in \{1, \ldots, m\}$ the values in the second column of the Table 3, the last four rows and using Definition 6, the ranking order is $A_1 \prec_M A_3 \prec_M A_4 \prec_M A_2$. The difference between these two hierarchies is not very significant, namely $M_\mu(\widetilde{G_2}) \sim M_\mu(\widetilde{G_4})$ and $M_\mu(\widetilde{H_2}) \sim M_\mu(\widetilde{H_4})$.

Using the second ranking method, for $\lambda = 0.76$ we obtain for $V_\lambda(\widetilde{G_i})$, $i \in \{1, \ldots, m\}$ the values in the third column of the Table 3, the first four rows. Then, using Definition 7, the ranking order is $A_1 \prec_{VA} A_3 \prec_{VA} A_2 \prec_{VA} A_4$. Analogously, for $V_\lambda(\widetilde{H_i})$, $i \in \{1, \ldots, m\}$ we obtain the values in the third column of the Table 3, the last four rows and the ranking order $A_1 \prec_{VA} A_3 \prec_{VA} A_4 \prec_{VA} A_2$. The difference between these two hierarchies, in this case, is also not very significant given the defuzzified values, namely $V_\lambda(\widetilde{G_2}) \sim V_\lambda(\widetilde{G_4})$ and respectively $V_\lambda(\widetilde{H_2}) \sim V_\lambda(\widetilde{H_4})$.

For the third ranking method, if we calculate the score and the accuracy according to (10), we obtain for $S(\widetilde{G_i})$ and respectively for $E(\widetilde{G_i})$, $i \in \{1, \ldots, m\}$ the values in the fourth column of the Table 3, the first four rows and using Definition 8, the ranking order is $A_1 \prec_{SE} A_3 \prec_{SE} A_2 \prec_{SE} A_4$. Then, we obtain for $S(\widetilde{H_i})$ and respectively for $E(\widetilde{H_i})$, $i \in \{1, \ldots, m\}$ the values in the fourth column of the Table 3, the last four rows and the ranking order $A_1 \prec_{SE} A_3 \prec_{SE} A_2 \prec_{SE} A_4$, therefore the same hierarchy.

Finally, for the last ranking method, we obtain for $EV(\widetilde{G_i})$, $i \in \{1, \ldots, m\}$ the values in the fifth column of the Table 3, the first four rows and using Definition 9, the ranking order is $A_1 \prec_{EV} A_3 \prec_{EV} A_2 \sim_{EV} A_4$. Analogously, for $EV(\widetilde{H_i})$, $i \in \{1, \ldots, m\}$ we obtain the values in the fifth column of the

Table 3, the last four rows and the ranking order $A_1 \prec_{EV} A_3 \prec_{EV} A_4 \prec_{EV} A_2$, therefore the same hierarchy.

TABLE 3. Comparing hierarchies.

Agg.	Rank 1 (M_μ)	Rank 2 ($V_{0.76}$)	Rank 3 (S/E)	Rank 4 (EV)
$WAA_{\widetilde{\omega}}$	A_4: 0.31	A_4: 0.32	A_4: $-0.01/0.65$	A_4: 0.32
	A_2: 0.30	A_2: 0.31	A_2: $-0.02/0.63$	A_2: 0.32
	A_3: 0.27	A_3: 0.28	A_3: $-0.02/0.57$	A_3: 0.28
	A_1: 0.20	A_1: 0.21	A_1: $-0.02/0.42$	A_1: 0.21
WGA_ω	A_2: 0.44	A_2: 0.45	A_4: $0.00/0.83$	A_2: 0.45
	A_4: 0.42	A_4: 0.42	A_2: $-0.01/0.90$	A_4: 0.42
	A_3: 0.37	A_3: 0.37	A_3: $-0.01/0.75$	A_3: 0.37
	A_1: 0.22	A_1: 0.23	A_1: $-0.01/0.46$	A_1: 0.23

Therefore, if we use the $WAA_{\widetilde{\omega}}$ operator, we get the same hierarchy for every ranking method. If we use the WGA_ω operator, we also obtain almost the same hierarchy, for every ranking method, with one exception, probably due to very small differences between the aggregated values of alternatives A_2 and A_4. However, the two hierarchies obtained by different aggregation operators differ. Specifically, regarding the worst alternative, this is definitely A_1. Instead, regarding the best alternative, what matters actually most, can not be predicted accurately, because using the $WAA_{\widetilde{\omega}}$ operator we obtain the best alternative A_4 and using WGA_ω operator, A_2 seems to be the best alternative. In this case it requires further study. But, in our opinion, this is due to very similar values obtained for A_2 and respectively A_4. Indeed, this assumption is confirmed by Example 2.

Example 2. Using the same problem from Example 1, we change only two linguistic variables in Table 2, namely for alternative A_4 versus criterion C_3 we assume that the decision-makers D_3 and D_4 choose *"very good"* instead of *"fair"* as it appears in Example 1.

By running the application that implements Algorithm 1, we obtain the following results. The values $\widetilde{r_{ij}}$ remain the same, except for the A_4 versus C_3, for which we obtain $\widetilde{r_{43}} = \langle(0.340, 0.440, 0.540, 0.640), (0.280, 0.440, 0.540, 0.700)\rangle$. Obviously, the values $\widetilde{w_j}$ remain the same and therefore the deffuzified values of the weights of the criteria are the same. The aggregated value of A_4 using $WAA_{\widetilde{\omega}}$ operator is $\widetilde{G_4} = \langle(0.181,\ 0.281,\ 0.400,\ 0.539), (0.117,\ 0.281,\ 0.400,\ 0.654)\rangle$ and the aggregated value of A_4 using WGA_ω operator is $\widetilde{H_4} = \langle(0.348,\ 0.464,\ 0.589,\ 0.723), (0.297,\ 0.464,\ 0.589,\ 0.784)\rangle$. In this case, the obtained hierarchies coincide for all aggregation operators and for all ranking methods, as shown in Table 4 and certainly, the best alternative is A_4.

TABLE 4. Comparing the new hierarchies.

Agg.	Rank 1 (M_μ)	Rank 2 ($V_{0.84}$)	Rank 3 (S/E)	Rank 4 (EV)
$WAA_{\tilde{\omega}}$	A_4: 0.35	A_4: 0.35	A_4: $-0.01/0.71$	A_4: 0.36
	A_2: 0.30	A_2: 0.31	A_2: $-0.02/0.63$	A_2: 0.32
	A_3: 0.27	A_3: 0.28	A_3: $-0.02/0.57$	A_3: 0.28
	A_1: 0.20	A_1: 0.21	A_1: $-0.02/0.42$	A_1: 0.21
WGA_ω	A_4: 0.53	A_4: 0.53	A_4: $0.00/1.06$	A_4: 0.53
	A_2: 0.44	A_2: 0.45	A_2: $-0.01/0.90$	A_2: 0.45
	A_3: 0.37	A_3: 0.37	A_3: $-0.01/0.75$	A_3: 0.37
	A_1: 0.22	A_1: 0.23	A_1: $-0.01/0.46$	A_1: 0.23

5. Related work and comparison analysis of the results obtained

Firstly, in this section we present other relevant fuzzy MCDM approaches from the recent literature.

In [29] it was proposed a fuzzy MCDM method that uses triangular intuitionistic fuzzy numbers, two aggregation operators, namely the arithmetic and geometric aggregation operators and a ranking method based on score and accuracy. Thus, the method returns two hierarchies of alternatives relative to the given criteria. Both aggregation operators and also the ranking method have been integrated in our method, using trapezoidal intuitionistic fuzzy numbers. We can not do a comparison with the method from [29] for the following reason: in [29] there are used other operations with intuitionistic fuzzy numbers than those used by us and in addition triangular intuitionistic fuzzy numbers considered in [29] are not actually triangular intuitionistic fuzzy numbers in our acceptance, because it does not verify the conditions from Definition 4. From our point of view not even the input data considered in Table 1 from Section 6 in [29] are not triangular intuitionistic fuzzy numbers, therefore this is why it is not relevant to do a comparison of the results.

In [16] it was proposed a new ranking method for triangular intuitionistic fuzzy numbers based on value and ambiguity. The advantage of this method is that it reflects the subjective attitude of the decision makers by using a parameter $\lambda \in [0, 1]$. The proposed ranking method is exemplified in a fuzzy MCDM method that uses a comprehensive aggregation operator. Neither this time we do not compare the obtained results because in [16] it was used another notation for triangular intuitionistic fuzzy numbers which has no counterpart in our notation.

In [28] it was proposed a fuzzy MCDM method based on arithmetic aggregation operator and the ranking method based on value and ambiguity proposed in [16] uses trapezoidal intuitionistic fuzzy numbers. To show the effectiveness of our method, we intend to further analyze the results from the application in [28] compared to the results for the same problem using our method.

In the following we consider the example from [28], Section 5.1. For this example, they were obtained by the proposed method in [28] the hierarchies $x_4 \succ_{VA} x_2 \succ_{VA} x_3 \succ_{VA} x_1$ for $\lambda \in [0, 0.354)$, $x_4 \succ_{VA} x_3 \succ_{VA} x_2 \succ_{VA} x_1$ for $\lambda \in [0.354, 0.947]$ and respectively $x_3 \succ_{VA} x_4 \succ_{VA} x_2 \succ_{VA} x_1$ for $\lambda \in (0.947, 1]$. In [28] the proposed method was compared to three other methods from the literature (see [28], Table 2), getting for the same example, in the case of all three methods the hierarchy $x_4 \succ x_2 \succ x_3 \succ x_1$. By considering the parameter λ which reflects the attitude of the decision makers about the preference for the risk, in [28] it was obtained for higher values of λ (indicating a decision makers preference for the risk), a different hierarchy in which the alternative x_3 easily outpaced the alternative x_4. Besides, in [28] it stated out that *"a risk-taking decision maker may prefer x_3, whereas a risk-averse decision maker may prefer x_4"*.

If we consider the same example and treat it by the method proposed in this paper, we get the following hierarchies, lined up in the same order as in the examples from Section 4, namely: $x_4 \succ_M x_3 \succ_M x_2 \succ_M x_1$, $x_4 \succ_{VA} x_3 \succ_{VA} x_2 \succ_{VA} x_1$, for $\lambda = 0.97$, $x_3 \succ_{SE} x_1 \succ_{SE} x_4 \succ_{SE} x_2$, $x_4 \succ_{EV} x_3 \succ_{EV} x_2 \succ_{EV} x_1$, $x_3 \succ_M x_4 \succ_M x_2 \succ_M x_1$, $x_2 \succ_{VA} x_4 \succ_{VA} x_3 \succ_{VA} x_1$, for $\lambda = 0.97$, $x_3 \succ_{SE} x_4 \succ_{SE} x_1 \succ_{SE} x_2$ and $x_3 \succ_{EV} x_4 \succ_{EV} x_2 \succ_{EV} x_1$. The second hierarchy from the previous list was obtained with our method using the same aggregation operator and the same ranking method as those used in the method from [28]. But the obtained hierarchies are different. Deeper analyzing, by our method are obtained the values $V_\lambda(\widetilde{S}_3) = 0.19$ and $V_\lambda(\widetilde{S}_4) = 0.20$, for $\lambda = 0.97$, therefore very close values, but yet different. If we replace $\lambda = 0.97$ in (42) from [28], we obtain $V_\lambda(\widetilde{S}_1) = 0.38$, $V_\lambda(\widetilde{S}_2) = 0.58$, $V_\lambda(\widetilde{S}_3) = 0.59$ and $V_\lambda(\widetilde{S}_4) = 0.59$, therefore the hierarchy $x_3 \sim_{VA} x_4 \succ_{VA} x_2 \succ_{VA} x_1$, which is not in contradiction with our result. Moreover, if we use in the example from [28] the geometric aggregation operator and the same ranking method based on value and ambiguity, we get for $\lambda = 0.97$ the values $V_\lambda(\widetilde{S}_3) = 0.57 = V_\lambda(\widetilde{S}_4)$, therefore another proof that x_3 and x_4 "competing" together for the position of the best alternative.

In conclusion, as we have seen, in the example from [28] it was obtained, somewhat at the limit of, that x_3 is the best alternative in the case when the decision makers prefer the risk. Using our method, the alternative x_3 it was also obtained as the best alternative in the four of the eight cases.

6. Conclusion

In this paper we used $TIFN$s for modelling real problems in relationship with the MCDM. The proposed method is based on two aggregation operators, namely, the WAA operator and the WGA operator and on four ranking methods, based on the index, value, ambiguity, value-index, ambiguity-index, score, accuracy and expected value. The method is suitable for MCDM because it is well known that $TIFN$s works well with the uncertainty. We elaborated an algorithm for the proposed method and we compared the eight hierarchies of alternatives obtained by using each aggregation operator and each ranking method. In the other papers it was also tried to use several aggregation operators and/or several ranking methods in the same MCDM method, in order to obtain more than one hierarchy of alternatives, which could be compared and analyzed later. For example, in [29] there were used within a proposed MCDM method the arithmetic and the geometric aggregation operators and a ranking method based on score and accuracy, thus obtaining two hierarchies of alternatives. In the example given in [29] the two obtained hierarchies coincided.

The proposed method is better than other existing methods in the literature (see, e.g., [21], [23]) because it preserves more information. As example, the method proposed in [21] transforms the values of decision matrix from $TIFN$s in interval numbers, uses the interval density aggregation operators and the ranking of alternatives is based on sorting the interval numbers. Therefore, it does not use operations with $TIFN$s, but there is a prior defuzzification before the application of the method. Instead, our method is operating with $TIFN$s throughout the method, only at the end the results being defuzzified for easy interpretation of the results.

In [6] it was demonstrated a bijection between the set of $TIFN$s and the set of interval-valued trapezoidal fuzzy numbers and the corresponding properties, therefore the proposed method from this paper can be applied for interval-valued trapezoidal fuzzy numbers too. In addition, taking into account Remark 1, it is obvious that our method generalizes other similar methods developed for trapezoidal fuzzy numbers, for example the method given in [4].

As future research directions, it can be seen that the proposed method can be easily extended to other types of intuitionistic fuzzy numbers. Also, because of the fact that trapezoidal intuitionistic fuzzy numbers are a generalization of trapezoidal fuzzy numbers, other existing fuzzy MCDM methods that use fuzzy numbers can be extended to intuitionistic fuzzy numbers. Last but not least, we intend to search for other effective aggregation operators and/or ranking methods for the trapezoidal intuitionistic fuzzy numbers which will be integrated in our method.

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