STUDIA UNIV. BABEŞ–BOLYAI, INFORMATICA, Volume **LIX**, Special Issue 1, 2014
10th Joint Conference on Mathematics and Computer Science, Cluj-Napoca, May 21-25, 2014

# AN INTERVAL FUZZY MULTICRITERIA DECISION MAKING METHOD BASED ON THE EXPECTED VALUE

DELIA A. TUŞE

Abstract. In the present paper we extend fuzzy multicriteria decision making methods elaborated in some recent articles. We use intervals of trapezoidal fuzzy numbers instead of trapezoidal fuzzy numbers, which allows two choices or even a intermediate responses in a survey. The expected value is used for the ranking of intervals of trapezoidal fuzzy numbers. We elaborate an algorithm of rankings the alternatives versus criteria and weights of criteria given by intervals of trapezoidal fuzzy numbers. Theoretical considerations are illustrated by a practical example.

## 1. Introduction

The large number of proposed methods in fuzzy multicriteria decision making proves that Fuzzy Set Theory works well in this topic.

The method presented in [1] is dedicated to the case of importance weights of criteria/ratings of alternatives expressed by real numbers and ratings of alternatives/importance weights of criteria expressed by fuzzy numbers. It optimizes and extends the method from [5] and its importance in practice is illustrated by examples.

The aim of this paper is to generalize the method to situations in which people surveyed want to choose two answers or an intermediate answer from the given response options. These situations are found in practice due to the fact that most often surveys are based on 5-levels responses. Introducing 9 or 11-levels scales are avoided because one survey should not be too long, but enjoyable, quick and easy to complete. To give freedom of choices in response, the possibility of multiple responses, which is generally avoided, should be considered. In Section 3 we introduce the expected value of an interval of

Received by the editors: May 01, 2014.

2010 *Mathematics Subject Classification.* 03E72, 62C86.

1998 *CR Categories and Descriptors.* H.4.2 [**Information Systems Applications**]: Types of Systems – *Decision support (e.g., MIS)*.

*Key words and phrases.* trapezoidal fuzzy number, interval of fuzzy numbers, expected value, fuzzy MCDM.

trapezoidal fuzzy numbers, following the idea from [3] and we compute it for the product of intervals of trapezoidal fuzzy numbers. In Section 4 we extend the method from [1] using intervals of trapezoidal fuzzy numbers and we elaborate an algorithm for ranking the alternatives versus criteria and weights of criteria given by intervals of trapezoidal fuzzy numbers. In Section 5 we give an example inspired from [1] and [5] and modified accordingly to our assumption on input data.

## 2. Preliminaries

We begin by recalling some basic definitions used in this paper.

**Definition 1.** *(see [4]) Let $X$ be a set and $A$ be a subset of $X$ ($A \subseteq X$). A fuzzy set $A$ (fuzzy subset of $X$) is defined as a mapping*

$$A : X \to [0,1],$$

*where $A(x)$ is the membership degree of $x$ to the fuzzy set $A$.*

**Definition 2.** *(see [4]) A fuzzy subset of the real line $A : \mathbb{R} \to [0,1]$ is a fuzzy number if it satisfies the following properties:*
*(i) $A$ is normal, i.e. $\exists x_0 \in \mathbb{R}$ with $A(x_0) = 1$;*
*(ii) $A$ is fuzzy convex, i.e. $A(tx + (1-t)y) \geq min\{A(x), A(y)\}, \forall t \in [0,1]$, $x, y \in \mathbb{R}$;*
*(iii) $A$ is upper semicontinuous on $\mathbb{R}$, i.e. $\forall \epsilon > 0$ $\exists \delta > 0$ such that $A(x) - A(x_0) < \epsilon$, $|x - x_0| < \delta$;*
*(iv) $A$ is compactly supported, i.e. $cl\{x \in \mathbb{R};\ A(x) > 0\}$ is compact, where $cl(M)$ denotes the closure of a set $M$.*

Let us denote by $\mathcal{F}(\mathbb{R})$ the space of fuzzy numbers.

The $r$-level set, $r \in [0,1]$, of a fuzzy number $A$ (see [4]) is defined as

$$A_r = [A_r^-, A_r^+] = \{x \in \mathbb{R}; A(x) \geq r\},$$

where

$$A_r^- = inf\{x \in \mathbb{R} : A(x) \geq r\},$$

$$A_r^+ = sup\{x \in \mathbb{R} : A(x) \geq r\},$$

for $r \in (0,1]$, with the convention $A_0 = [A_0^-, A_0^+] = cl\{x \in \mathbb{R} : A(x) > 0\}$.

For $A, B \in \mathcal{F}(\mathbb{R})$, $A_r = [A_r^-, A_r^+]$, $B_r = [B_r^-, B_r^+]$, $r \in [0,1]$ and $\lambda \in \mathbb{R}$, $\lambda \geq 0$, the sum $A+B$, the scalar multiplication $\lambda \cdot A$ and the product $A \otimes B$ are defined by $r$-level sets as follows:

$$(A+B)_r = A_r + B_r = [A_r^- + B_r^-, A_r^+ + B_r^+],$$
$$(\lambda \cdot A)_r = \lambda A_r = [\lambda A_r^-, \lambda A_r^+],$$

and

$$(A \otimes B)_r = [(A \otimes B)_r^-, (A \otimes B)_r^+],$$

where

$$(A \otimes B)_r^- = min\{A_r^- B_r^-, A_r^- B_r^+, A_r^+ B_r^-, A_r^+ B_r^+\},$$
$$(A \otimes B)_r^+ = max\{A_r^- B_r^-, A_r^- B_r^+, A_r^+ B_r^-, A_r^+ B_r^+\}.$$

Is easily seen that fuzzy arithmetic extends interval arithmetic.

**Definition 3.** *An interval of fuzzy numbers is a pair $\widetilde{A} = [A^L, A^U]$, where $A^L$ and $A^U$ are fuzzy numbers such that $(A^L)_r^- \leq (A^U)_r^-$ and $(A^L)_r^+ \leq (A^U)_r^+$, for every $r \in [0,1]$.*

We denote by $\widetilde{F}(\mathbb{R})$ the set of all intervals of fuzzy numbers.

**Definition 4.** *(see [4]) A trapezoidal fuzzy number $A = (a,b,c,d)$, $a \leq b \leq c \leq d$, is the fuzzy set*

$$A(x) = \begin{cases} 0 & if\ x < a \\ \frac{x-a}{b-a} & if\ a \leq x < b \\ 1 & if\ b \leq x \leq c \\ \frac{d-x}{d-c} & if\ c < x \leq d \\ 0 & if\ d < x \end{cases}.$$

Let $(a^L, b^L, c^L, d^L), (a^U, b^U, c^U, d^U)$ be two trapezoidal fuzzy numbers. It is obvious that $[(a^L, b^L, c^L, d^L), (a^U, b^U, c^U, d^U)]$ is an interval of trapezoidal fuzzy numbers if and only if $a^L \leq a^U, b^L \leq b^U, c^L \leq c^U$ and $d^L \leq d^U$.

We denote by $\widetilde{F}^T(\mathbb{R})$ the set of all intervals of trapezoidal fuzzy numbers.

Let $\widetilde{A} = [A^L, A^U], \widetilde{B} = [B^L, B^U]$ and $\lambda \geq 0$. We introduce the sum of two intervals of fuzzy numbers, such as

$$\widetilde{A} + \widetilde{B} = [A^L + B^L, A^U + B^U],$$

the multiplication between a real pozitive number and an interval of fuzzy numbers, such as

$$\lambda \cdot \widetilde{A} = [\lambda \cdot A^L, \lambda \cdot A^U]$$

and the product of two intervals of fuzzy numbers, such as

$$\widetilde{A} \otimes \widetilde{B} = [A^L \otimes B^L, A^U \otimes B^U].$$

**Remark 5.** *Any fuzzy number $A$ can be considered as an interval of fuzzy numbers $\widetilde{A} = [A, A]$, therefore the above operations extend the operations between fuzzy numbers.*

## 3. Expected value of an interval of fuzzy numbers

In this section we recall the definition of the expected value of a fuzzy number, we give the definition of the expected value of an interval of fuzzy numbers and we prove some properties useful in the paper.

**Definition 6.** *(see* [6]*) The expected value $EV(A)$ of a fuzzy number $A$ is given by*

$$EV(A) = \frac{1}{2}\int_0^1 (A_r^- + A_r^+)dr.$$

The expected interval of a set of fuzzy numbers $\{A_1, \ldots, A_n\}$ was already introduced (see [3]) by

$$EI(A_1, \ldots, A_n) = \frac{1}{n}(EI(A_1) + \ldots + EI(A_n)).$$

As usually, the expected value is considered the middle of the expected interval, therefore it is natural to introduce

$$EV(A_1, \ldots, A_n) = \frac{1}{n}(EV(A_1) + \ldots + EV(A_n))$$

and the expected value of an interval of fuzzy numbers

$$EV(\widetilde{A}) = EV([A^L, A^U]) = \frac{1}{2}(EV(A^L) + EV(A^U)).$$

Taking into account the liniarity of the expected value (see [6]), we obtain:

**Proposition 7.** *(i) $EV(\lambda \cdot \widetilde{A}) = \lambda \cdot EV(\widetilde{A})$, for every $\lambda \in \mathbb{R}_+$, $\widetilde{A} \in \widetilde{F}(\mathbb{R})$;*

*(ii) $EV(\widetilde{A} + \widetilde{B}) = EV(\widetilde{A}) + EV(\widetilde{B})$, for every $\widetilde{A}, \widetilde{B} \in \widetilde{F}(\mathbb{R})$.*

*Proof.* Let $\widetilde{A} = [A^L, A^U]$ and $\widetilde{B} = [B^L, B^U]$

$$\begin{aligned}
(i) EV(\lambda \cdot \widetilde{A}) &= EV(\lambda \cdot A^L, \lambda \cdot A^U) = \\
&= \tfrac{1}{2}(EV(\lambda \cdot A^L) + EV(\lambda \cdot A^U)) = \\
&= \lambda(\tfrac{1}{2}EV(A^L) + \tfrac{1}{2}EV(A^U)) = \\
&= \lambda \cdot EV([A^L, A^U]) = \\
&= \lambda \cdot EV(\widetilde{A});
\end{aligned}$$

$$\begin{aligned}
(ii) EV(\widetilde{A} + \widetilde{B}) &= EV([A^L, A^U] + [B^L, B^U]) = \\
&= EV([A^L + B^L, A^U + B^U]) = \\
&= \tfrac{1}{2}(EV(A^L + B^L) + EV(A^U + B^U)) = \\
&= \tfrac{1}{2}(EV(A^L) + EV(A^U)) + \tfrac{1}{2}(EV(B^L) + EV(B^U)) = \\
&= EV([A^L, A^U]) + EV([B^L, B^U]) = \\
&= EV(\widetilde{A}) + EV(\widetilde{B}).
\end{aligned}$$

□

According to the conclusion of [2] the best choice in the ranking of fuzzy numbers is a simple method with suitable properties. The expected value is such a choice and we use it in the case of the ranking of pairs of fuzzy numbers as follows:

$$\widetilde{A} = [A^L, A^U] \preceq [B^L, B^U] = \widetilde{B} \text{ if and only if } EV(\widetilde{A}) \leq EV(\widetilde{B}).$$

We give the following properties with respect to the expected value of the intervals of trapezoidal fuzzy numbers.

**Proposition 8.**

$$\begin{aligned}
&(i) EV([(a^L, b^L, c^L, d^L), (a^U, b^U, c^U, d^U)]) = \\
&= \frac{a^L + a^U + b^L + b^U + c^L + c^U + d^L + d^U}{8};
\end{aligned}$$

$$(ii) EV([(a_1^L, b_1^L, c_1^L, d_1^L), (a_1^U, b_1^U, c_1^U, d_1^U)] \otimes$$
$$\otimes [(a_2^L, b_2^L, c_2^L, d_2^L), (a_2^U, b_2^U, c_2^U, d_2^U)]) =$$
$$= \frac{1}{24}(2a_1^L a_2^L + a_2^L b_1^L + a_1^L b_2^L + 2b_1^L b_2^L) +$$
$$+ \frac{1}{24}(2c_1^L c_2^L + c_2^L d_1^L + c_1^L d_2^L + 2d_1^L d_2^L) +$$
$$+ \frac{1}{24}(2a_1^U a_2^U + a_2^U b_1^U + a_1^U b_2^U + 2b_1^U b_2^U) +$$
$$+ \frac{1}{24}(2c_1^U c_2^U + c_2^U d_1^U + c_1^U d_2^U + 2d_1^U d_2^U).$$

*Proof.*
$(i)$ Because $EV(a, b, c, d) = \frac{a+b+c+d}{4}$, we have

$EV([(a^L, b^L, c^L, d^L), (a^U, b^U, c^U, d^U)]) =$

$= \frac{1}{2}(EV(a^L, b^L, c^L, d^L) + EV(a^U, b^U, c^U, d^U)) =$

$= \frac{1}{8}(a^L + a^U + b^L + b^U + c^L + c^U + d^L + d^U);$

$(ii) EV([(a_1^L, b_1^L, c_1^L, d_1^L), (a_1^U, b_1^U, c_1^U, d_1^U)] \otimes$

$\otimes [(a_2^L, b_2^L, c_2^L, d_2^L), (a_2^U, b_2^U, c_2^U, d_2^U)]) =$

$= EV([(a_1^L, b_1^L, c_1^L, d_1^L) \otimes (a_2^L, b_2^L, c_2^L, d_2^L),$

$(a_1^U, b_1^U, c_1^U, d_1^U) \otimes (a_2^U, b_2^U, c_2^U, d_2^U)]) =$

$= \frac{1}{2} EV((a_1^L, b_1^L, c_1^L, d_1^L) \otimes (a_2^L, b_2^L, c_2^L, d_2^L)) +$

$+ \frac{1}{2} EV((a_1^U, b_1^U, c_1^U, d_1^U) \otimes (a_2^U, b_2^U, c_2^U, d_2^U)) =$ (see Proposition 1 in [1]) $=$

$= \frac{1}{24}(2a_1^L a_2^L + a_2^L b_1^L + a_1^L b_2^L + 2b_1^L b_2^L) +$

$+ \frac{1}{24}(2c_1^L c_2^L + c_2^L d_1^L + c_1^L d_2^L + 2d_1^L d_2^L) +$

$+ \frac{1}{24}(2a_1^U a_2^U + a_2^U b_1^U + a_1^U b_2^U + 2b_1^U b_2^U) +$

$+ \frac{1}{24}(2c_1^U c_2^U + c_2^U d_1^U + c_1^U d_2^U + 2d_1^U d_2^U).$

$\square$

## 4. The method

A standard multicriteria decision making problem assumes a committee of $k$ decision-makers $D_1,\dots,D_k$ which is responsible for evaluating $m$ alternatives $A_1,\dots,A_m$, under $n$ criteria $C_1,\dots,C_n$. We consider that $C_1,\dots,C_h$ are subjective criteria, $C_{h+1},\dots,C_p$ are objective criteria of benefit kind and $C_{p+1},\dots,C_n$ are objective criteria of cost kind. In addition, as a generalization of the fuzzy multicriteria decision making method proposed in [1] we consider that the evaluations are given by intervals of trapezoidal fuzzy numbers. If $\widetilde{r_{ijt}} = [(e^L_{ijt}, f^L_{ijt}, g^L_{ijt}, h^L_{ijt}), (e^U_{ijt}, f^U_{ijt}, g^U_{ijt}, h^U_{ijt})]$, $i\in\{1,\dots,m\}$, $j\in\{1,\dots,h\}$, $t\in\{1,\dots,k\}$ is the performance of alternative $A_i$ versus subjective criterion $C_j$ in the opinion of the decision-maker $D_t$ then

(1)

$$\widetilde{r_{ij}} = [(e^L_{ij}, f^L_{ij}, g^L_{ij}, h^L_{ij}), (e^U_{ij}, f^U_{ij}, g^U_{ij}, h^U_{ij})] =$$
$$= \left[\left(\sum_{t=1}^{k}\frac{e^L_{ijt}}{k}, \sum_{t=1}^{k}\frac{f^L_{ijt}}{k}, \sum_{t=1}^{k}\frac{g^L_{ijt}}{k}, \sum_{t=1}^{k}\frac{h^L_{ijt}}{k}\right), \left(\sum_{t=1}^{k}\frac{e^U_{ijt}}{k}, \sum_{t=1}^{k}\frac{f^U_{ijt}}{k}, \sum_{t=1}^{k}\frac{g^U_{ijt}}{k}, \sum_{t=1}^{k}\frac{h^U_{ijt}}{k}\right)\right],$$
$i\in\{1,\dots,m\}$, $j\in\{1,\dots,h\}$

is the averaged rating of $A_i$ under $C_j$. If $\widetilde{x_{ij}} = [(a^L_{ij}, b^L_{ij}, c^L_{ij}, d^L_{ij}), (a^U_{ij}, b^U_{ij}, c^U_{ij}, d^U_{ij})]$, $i\in\{1,\dots,m\}$, $j\in\{h+1,\dots,n\}$ is the performance of alternative $A_i$ versus objective criterion $C_j$, then

$$\widetilde{r_{ij}} = [(e^L_{ij}, f^L_{ij}, g^L_{ij}, h^L_{ij}), (e^U_{ij}, f^U_{ij}, g^U_{ij}, h^U_{ij})],\ i\in\{1,\dots,m\},\ j\in\{h+1,\dots,p\}$$

is given by

$$(2)\qquad e^L_{ij} = \frac{a^L_{ij} - a^{L^*}_j}{m^{L^*}_j}, f^L_{ij} = \frac{b^L_{ij} - a^{L^*}_j}{m^{L^*}_j}, g^L_{ij} = \frac{c^L_{ij} - a^{L^*}_j}{m^{L^*}_j}, h^L_{ij} = \frac{d^L_{ij} - a^{L^*}_j}{m^{L^*}_j},$$

$$(3)\qquad e^U_{ij} = \frac{a^U_{ij} - a^{U^*}_j}{m^{U^*}_j}, f^U_{ij} = \frac{b^U_{ij} - a^{U^*}_j}{m^{U^*}_j}, g^U_{ij} = \frac{c^U_{ij} - a^{U^*}_j}{m^{U^*}_j}, h^U_{ij} = \frac{d^U_{ij} - a^{U^*}_j}{m^{U^*}_j}$$

and

$$\widetilde{r_{ij}} = [(e^L_{ij}, f^L_{ij}, g^L_{ij}, h^L_{ij}), (e^U_{ij}, f^U_{ij}, g^U_{ij}, h^U_{ij})],\ i\in\{1,\dots,m\},\ j\in\{p+1,\dots,n\}$$

is given by

$$(4)\qquad e_{ij}^L = \frac{d_j^{L^*} - d_{ij}^L}{m_j^{L^*}}, f_{ij}^L = \frac{d_j^{L^*} - c_{ij}^L}{m_j^{L^*}}, g_{ij}^L = \frac{d_j^{L^*} - b_{ij}^L}{m_j^{L^*}}, h_{ij}^L = \frac{d_j^{L^*} - a_{ij}^L}{m_j^{L^*}},$$

$$(5)\qquad e_{ij}^U = \frac{d_j^{U^*} - d_{ij}^U}{m_j^{U^*}}, f_{ij}^U = \frac{d_j^{U^*} - c_{ij}^U}{m_j^{U^*}}, g_{ij}^U = \frac{d_j^{U^*} - b_{ij}^U}{m_j^{U^*}}, h_{ij}^U = \frac{d_j^{U^*} - a_{ij}^U}{m_j^{U^*}},$$

where

$$a_j^{L^*} = min_{i\in\{1,\ldots,m\}} a_{ij}^L,\ a_j^{U^*} = min_{i\in\{1,\ldots,m\}} a_{ij}^U,$$

$$d_j^{L^*} = max_{i\in\{1,\ldots,m\}} d_{ij}^L,\ d_j^{U^*} = max_{i\in\{1,\ldots,m\}} d_{ij}^U,$$

$$m_j^{L^*} = d_j^{L^*} - a_j^{L^*},\ m_j^{U^*} = d_j^{U^*} - a_j^{U^*},\ \text{for } j \in \{h+1, \ldots, n\}.$$

are the normalized values of performances with respect to benefit and cost criteria, respectively. Let $\widetilde{w_{jt}} = [(o_{jt}^L, p_{jt}^L, q_{jt}^L, s_{jt}^L), (o_{jt}^U, p_{jt}^U, q_{jt}^U, s_{jt}^U)]$, $j \in \{1, \ldots, n\}$, $t \in \{1, \ldots, k\}$ the weight of the criterion $C_j$ in opinion of the decision-maker $D_t$. The averaged weight of the criterion $C_j$ assessed by decision-makers $D_1, \ldots, D_k$ is

(6)

$$\widetilde{w_j} = [(o_j^L, p_j^L, q_j^L, s_j^L), (o_j^U, p_j^U, q_j^U, s_j^U)] =$$
$$= \left[\left(\sum_{t=1}^k \frac{o_{jt}^L}{k}, \sum_{t=1}^k \frac{p_{jt}^L}{k}, \sum_{t=1}^k \frac{q_{jt}^L}{k}, \sum_{t=1}^k \frac{s_{jt}^L}{k}\right), \left(\sum_{t=1}^k \frac{o_{jt}^U}{k}, \sum_{t=1}^k \frac{p_{jt}^U}{k}, \sum_{t=1}^k \frac{q_{jt}^U}{k}, \sum_{t=1}^k \frac{s_{jt}^U}{k}\right)\right],$$

for $j \in \{1, \ldots, n\}$.

The final evaluation value of alternatives $A_i$ is the aggregation of the weighted ratings by interval of fuzzy numbers $\widetilde{G_i}$, developed as

$$\widetilde{G_i} = \frac{1}{n}((\widetilde{r_{i1}} \otimes \widetilde{w_1}) + \ldots + (\widetilde{r_{in}} \otimes \widetilde{w_n})), i \in \{1, \ldots, m\}.$$

Taking into account Proposition 7 we easily obtain

$$EV(\widetilde{G_i}) = \frac{1}{n}(EV(\widetilde{r_{i1}} \otimes \widetilde{w_1}) + \ldots + EV(\widetilde{r_{in}} \otimes \widetilde{w_n})),$$

where the calculus of each $EV(\widetilde{r_{ij}} \otimes \widetilde{w_j})$, $j \in \{1, \ldots, n\}$ can be performed according to Proposition 8.

The following procedure can be elaborated to ranking $m$ alternatives $A_1, \ldots, A_m$ under $n$ criteria $C_1, \ldots, C_n$ by a committee of $k$ decision-makers $D_1, \ldots, D_k$.

**Algorithm**

---

Step 1. Compute $\widetilde{r_{ij}}$ for $i \in \{1, \ldots, m\}$, $j \in \{1, \ldots, h\}$ following (1).

Step 2. Compute $\widetilde{r_{ij}}$ for $i \in \{1, \ldots, m\}$, $j \in \{h+1, \ldots, p\}$ given by (2)-(3) and $\widetilde{r_{ij}}$ for $i \in \{1, \ldots, m\}$, $j \in \{p+1, \ldots, n\}$ given by (4)-(5).

Step 3. Compute $\widetilde{w_j}$ for $j \in \{1, \ldots, n\}$ following (6).

Step 4. Compute $EV(\widetilde{G_i}) = \frac{1}{n}(EV(\widetilde{r_{i1}} \otimes \widetilde{w_1}) + \ldots + EV(\widetilde{r_{in}} \otimes \widetilde{w_n}))$ for $i \in \{1, \ldots, m\}$, where

$$EV(\widetilde{r_{ij}} \otimes \widetilde{w_j}) =$$

$$= \frac{1}{24}(2e_{ij}^L o_j^L + f_{ij}^L o_j^L + e_{ij}^L p_j^L + 2f_{ij}^L p_j^L) + \frac{1}{24}(2g_{ij}^L q_j^L + h_{ij}^L q_j^L + g_{ij}^L s_j^L + 2h_{ij}^L s_j^L) +$$

$$+ \frac{1}{24}(2e_{ij}^U o_j^U + f_{ij}^U o_j^U + e_{ij}^U p_j^U + 2f_{ij}^U p_j^U) + \frac{1}{24}(2g_{ij}^U q_j^U + h_{ij}^U q_j^U + g_{ij}^U s_j^U + 2h_{ij}^U s_j^U),$$

for every $j \in \{1, \ldots, n\}$.

Step 5. If $EV(\widetilde{G_{i_1}}) \geq EV(\widetilde{G_{i_2}}) \geq \ldots \geq EV(\widetilde{G_{i_n}})$ then the descending order of alternatives is $A_{i_1}, A_{i_2}, \ldots, A_{i_m}$, that is $A_{i_1}$ is better than $A_{i_2}$ and so on, $A_{i_m}$ is the worst alternative.

---

## 5. Numerical example

We illustrate the theoretical part by giving the following example inspired from [1] and [5].

**Example 9.** A company must select from three distribution centers $A_1, A_2, A_3$ to better serve its customers. Four decision-makers $D_1, D_2, D_3, D_4$, four subjective criteria (transportation availability-$C_1$, human resource-$C_2$, market potential-$C_3$ and climate condition-$C_4$) and one objective criterion (cost in million US$-$C_5$) are considered. The decision-makers use the linguistic rating set $S = \{VP, P, F, G, VG\}$, where

**Table 1**. Ratings of alternatives versus criteria.

| Criteria/ alternatives | Decision-makers $D_1$ | $D_2$ | $D_3$ | $D_4$ | $\widetilde{r_{ij}}$ |
|---|---|---|---|---|---|
| $C_1/A_1$ | $[G, G]$ | $[G, VG]$ | $[VG, VG]$ | $[G, G]$ | $[(0.550, 0.650, 0.750, 0.775), (0.600, 0.700, 0.800, 0.850)]$ |
| $C_1/A_2$ | $[G, G]$ | $[VG, VG]$ | $[F, G]$ | $[F, F]$ | $[(0.500, 0.600, 0.650, 0.725), (0.525, 0.625, 0.700, 0.750)]$ |
| $C_1/A_3$ | $[VG, VG]$ | $[G, G]$ | $[F, G]$ | $[G, G]$ | $[(0.525, 0.625, 0.700, 0.750), (0.550, 0.650, 0.750, 0.775)]$ |
| $C_2/A_1$ | $[G, G]$ | $[F, F]$ | $[VG, VG]$ | $[G, G]$ | $[(0.525, 0.625, 0.700, 0.750), (0.525, 0.625, 0.700, 0.750)]$ |
| $C_2/A_2$ | $[F, F]$ | $[G, G]$ | $[VG, VG]$ | $[VG, VG]$ | $[(0.575, 0.675, 0.750, 0.825), (0.575, 0.675, 0.750, 0.825)]$ |
| $C_2/A_3$ | $[F, F]$ | $[F, G]$ | $[G, G]$ | $[F, F]$ | $[(0.425, 0.525, 0.550, 0.625), (0.450, 0.550, 0.600, 0.650)]$ |
| $C_3/A_1$ | $[VG, VG]$ | $[G, G]$ | $[G, G]$ | $[G, G]$ | $[(0.550, 0.650, 0.750, 0.775), (0.550, 0.650, 0.750, 0.775)]$ |
| $C_3/A_2$ | $[G, G]$ | $[F, F]$ | $[VG, VG]$ | $[G, G]$ | $[(0.525, 0.625, 0.700, 0.750), (0.525, 0.625, 0.700, 0.750)]$ |
| $C_3/A_3$ | $[F, G]$ | $[F, F]$ | $[G, G]$ | $[G, G]$ | $[(0.450, 0.550, 0.600, 0.650), (0.475, 0.575, 0.650, 0.675)]$ |
| $C_4/A_1$ | $[F, F]$ | $[P, P]$ | $[F, F]$ | $[F, F]$ | $[(0.325, 0.425, 0.450, 0.550), (0.325, 0.425, 0.450, 0.550)]$ |
| $C_4/A_2$ | $[F, F]$ | $[F, F]$ | $[G, VG]$ | $[G, G]$ | $[(0.450, 0.550, 0.600, 0.650), (0.500, 0.600, 0.650, 0.725)]$ |
| $C_4/A_3$ | $[G, G]$ | $[F, F]$ | $[G, G]$ | $[F, F]$ | $[(0.450, 0.550, 0.600, 0.650), (0.450, 0.550, 0.600, 0.650)]$ |
| $C_5/A_1$ | $[(3.5, 4.7, 4.9, 6.1), (4.1, 4.3, 6.1, 6.3)]$ | | | | $[(0.395, 0.674, 0.721, 1.000), (0.405, 0.459, 0.946, 1.000)]$ |
| $C_5/A_2$ | $[(4.7, 4.8, 4.9, 5.2), (4.7, 4.8, 4.9, 5.2)]$ | | | | $[(0.605, 0.674, 0.698, 0.721), (0.703, 0.784, 0.811, 0.838)]$ |
| $C_5/A_3$ | $[(6.2, 7.0, 7.4, 7.8), (6.7, 6.9, 7.3, 7.8)]$ | | | | $[(0.000, 0.093, 0.186, 0.372), (0.000, 0.135, 0.243, 0.297)]$ |

**Table 2**. The importance weights of the criteria and the aggregated weights.

| Criteria | Decision-makers $D_1$ | $D_2$ | $D_3$ | $D_4$ | $\widetilde{w_j}$ |
|---|---|---|---|---|---|
| $C_1$ | $[VH, VH]$ | $[VH, VH]$ | $[H, H]$ | $[VH, VH]$ | $[(0.650, 0.825, 0.925, 1.000), (0.650, 0.825, 0.925, 1.000)]$ |
| $C_2$ | $[L, M]$ | $[M, M]$ | $[M, M]$ | $[M, M]$ | $[(0.275, 0.450, 0.450, 0.725), (0.300, 0.500, 0.500, 0.800)]$ |
| $C_3$ | $[L, L]$ | $[L, M]$ | $[M, M]$ | $[M, H]$ | $[(0.250, 0.400, 0.400, 0.650), (0.325, 0.475, 0.500, 0.775)]$ |
| $C_4$ | $[M, M]$ | $[H, H]$ | $[VH, VH]$ | $[VH, VH]$ | $[(0.550, 0.725, 0.800, 0.950), (0.550, 0.725, 0.800, 0.950)]$ |
| $C_5$ | $[H, H]$ | $[VH, VH]$ | $[VH, VH]$ | $[H, VH]$ | $[(0.600, 0.750, 0.850, 1.000), (0.650, 0.825, 0.925, 1.000)]$ |

$VP$ = Very Poor = $(0, 0, 0.1, 0.2)$,
$P$ = Poor = $(0.1, 0.2, 0.3, 0.4)$,
$F$ = Fair = $(0.4, 0.5, 0.5, 0.6)$,
$G$ = Good = $(0.5, 0.6, 0.7, 0.7)$ and
$VG$ = Very Good = $(0.7, 0.8, 0.9, 1.0)$,
to evaluate the subjective criteria $C_1, C_2, C_3, C_4$ and a linguistic weighting set $W = \{VL, L, M, H, VH\}$, where
$VL$ = Very Low = $(0, 0.1, 0.2, 0.3)$,
$L$ = Low = $(0.2, 0.3, 0.3, 0.5)$,
$M$ = Medium = $(0.3, 0.5, 0.5, 0.8)$,
$H$ = High = $(0.5, 0.6, 0.7, 1.0)$ and
$VH$ = Very High = $(0.7, 0.9, 1.0, 1.0)$,
to assess the importance of criteria $C_1, C_2, C_3, C_4, C_5$.

The ratings of alternatives versus criteria under the opinion of decision-makers (Step 1 and Step 2 of our algorithm) are presented in Table 1 and the importance weights of the five criteria from the four decision-makers (Step 3 of our algorithm) are displayed in Table 2.

Obviously values $\widetilde{r_{ij}}$ from Table 1 and respectively $\widetilde{w_j}$ from Table 2 are obtained after running the C# program that implements the algorithm described in Section 4.

Finally, as a result of this program too, we get that

$$EV(\widetilde{G_3}) = 0.345 < EV(\widetilde{G_1}) = 0.445 < EV(\widetilde{G_2}) = 0.458$$

which means that the best selection is $A_2$ and the worst selection is $A_3$.

## 6. Conclusions

Intervals of trapezoidal fuzzy numbers are used in the present paper to model a real situation in problems of multicriteria decision making. The expected value of an interval of fuzzy numbers is introduced, its properties and calculus on intervals of trapezoidal fuzzy numbers are given (Propositions 7 and 8) to elaborate the method and the corresponding algorithm. Our method is suitable to be applied when an intermediate answer or two answers are chosen in a survey and it is illustrated by a concrete example in Section 5. Taking into account Remark 5 and the fact that $EV([A, A]) = EV(A)$ for every $A \in \mathcal{F}(\mathbb{R})$, we obtain that the proposed method extends the method from [1].

## References

[1] A. Ban, O. Ban, *Optimization and extensions of a fuzzy multicriteria decision making method and applications to selection of touristic destinations*, Expert Systems with Applications, 39 (2012), pp. 7216-7225.
[2] A. Ban, L. Coroianu, *Simplifying the Search for Effective Ranking of Fuzzy Numbers*, to apper in IEEE Transactions on Fuzzy Systems, DOI 10.1109/TFUZZ.2014.2312204.
[3] A. Ban, L. Coroianu, P. Grzegorzewski, *Trapezoidal approximation and aggregation*, Fuzzy Sets and Systems, 177 (2011), pp. 45-59.
[4] B. Bede, *Mathematics of fuzzy sets and fuzzy logic*, Springer, Studies in fuzziness and soft computing, Heidelberg, New York, 2013.
[5] T.-C. Chu, Y. Lin, *An extension to fuzzy MCDM*, Computers and Mathematics with Applications, 57 (2009), pp. 445-454.
[6] S. Heilpern, *The expected value of a fuzzy number*, Fuzzy Sets and Systems, 47 (1992), pp. 81-86.

Department of Mathematics and Informatics, University of Oradea, 410087 Oradea, Romania
*E-mail address*: delia.tuse@yahoo.com