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ENDGAME STRATEGIES AND SIMULATION RESULTS FOR THE LIAR'S DICE GAME

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Abstract. The Liar's dice is a dice game where deception and the ability to detect the opponents' deception play a crucial role. We analyze this game from different perspectives. First, two-player endgames are analyzed and optimal strategies are calculated. Second, using simulation methods, we examine heuristic playing strategies based on their success against each other. In the simulations, we consider mixed strategies that depend on parameters, populate a parameter space with strategies and perform evolutionary simulation on the strategy population.

1. Introduction

The Liar's Dice [5] game is a dice game where lying and detecting lies is the most important element of the play. It is a thrilling game, interesting from both game theoretical and psychological perspective. There are several known variants, we briefly describe the game that we will consider in the rest of the paper. A continuous variant has been analyzed in [2, 3].

The game can be played by an arbitrary number $P \geq 2$ of players. Initially, all players have $D \geq 1$ dice, and the game proceeds in rounds. In each round, everyone rolls their dice secretly – yielding a hand. Then, the player whose turn it is to start raises a bid. After that, every player raises the bid until someone calls a "challenge" (or doubts the bid). The hands are shown and it is counted if the challenge is successful. If the last player to bid fullfils the challenge then he wins the round, otherwise the player who called the challenge wins the round. Accounting is done after that – meaning that some players lose some or all of their dice (the latter ones are out). The players still in play start a new round with the remaining dice with the starting player position moving clockwise. The rounds continue until only one player remains who is declared the winner.

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The bids are of the form: (count – value), e.g. two threes (two of 3), three fives (three of 5) etc. Two bids are compared as follows: if neither value is 6, then they are compared lexicographically, first by count, then by value. If a value is six, it behaves as if it were twice the count of zeroes. We list the first few valid bids as (count, value) pairs: $(1,1),(1,2),(1,3),(1,4),$ $(1,5),(1,6),(2,1),(2,2),(2,3),$ $(2,4),$ $(2,5),$ $(3,1),\ldots,(3,5),(2,6),(4,1),\ldots,$ $(5,5),(3,6),(6,1),\ldots$. A bid is checked in case of a challenge the following way: if the value field is 6, then the bid is successful if and only if there are at least as many sixes in the hands (altogether) as the bid claimed. If another value is in the bid, then also the 6s are counted as wildcards (jokers), adding to the sum. Let d be the difference of the actual count and the count in the bid. If $d > 0$ then the challenger loses d dice. If $d < 0$, then the challenged player loses $|d|$ dice. If $d = 0$, then all but the challenged player lose 1 die.

Example 1.1. *A two-player example gameplay between Alice and Bob could proceed as follows. Initially they have 2 dice each. In the first round they roll, Alice gets two 5s, Bob gets a 3 and a 6. Alice bids* $(1,5)$*, then Bob bids* $(1,6)$*, then Alice bids* $(2,5)$ *which Bob challenges. The challenge fails because there are really two fives and a joker. The actual count is 3, one more than the challenged bid, so Bob loses a die. In the next round, Alice rolls a 3 and a 4, Bob rolls a 2. It's Bob's turn to bid first: he bids* $(1,2)$*, then Alice bids* $(1,4)$ *then Bob bids* $(2,2)$ *which is challenged and won by Alice. Bob loses his last die, Alice wins the game.*

In this paper we analyze two scenarios of the game. The first analysis is a game-theoretical analysis of the endgame when two players match up with one die each. The earlier parts of the game are extremely complex to analyze. Therefore we chose a largely simplified model. We propose a strategy which encompasses some of the psychological aspects of the game. In a long gameplay, the players may be influenced by the observed behavior of their neighbors. If our right neighbor is known to be a notorious liar (by the game history), it makes challenging slightly more preferable. Also, if our left neighbor is a bad challenger, this makes raising the bid preferable. We introduce parameters which determine how bravely a player is expected to react to such observed behavior of his neighbors.

The paper is built up as follows. In Section 2, the endgame analysis is presented for general dice with n faces without joker, and for dice with 3 faces with joker. In Section 3, the strategy used in the simulation is described. In Section 4, the evolutionary simulation method and the simulation results are shown.

2. Two-player endgame with one die each

Let us consider the case when both players have an n-faced die and there is no joker.

Theorem 2.1. *Player II's probability of win under optimal play by both players is*

$$\frac{\binom{n+1}{2}-1}{n^2}.$$

An optimal strategy for player I is as follows.

- If the value of his die is 1, then he claims j with probability $\frac{1}{n-1}$, $2 \le j \le n$;
- if the value of his die is $j > 1$, then he claims j;
- on the second round (if any), he doubts any claim by player II.

An optimal strategy for player II is as follows.

- If the value of his die j is greater than the bid of player I, then he claims j, except $j = 2$, when he doubts;
- if the value of his die is lower than the bid of player I, then he doubts;
- if the value of his die is equal to the bid of player I, then he claims 2 of that value;
- on the second round (if any), he doubts any claim by player I.

Proof: What is the optimal strategy of player II, if player I uses the aforementioned strategy?

- If the value of player II's die is greater than the bid of player I, then he wins with the claim of that value.
- If the value of player II's die is lower than the bid of player I, then the only possible greater bid is 2 of his die's value, but it can be true only if I's bid was a bluff, so the optimal strategy is to doubt. In this case the probability of player II's victory is
$$\frac{\frac{1}{n-1}}{1+\frac{1}{n-1}} = \frac{1}{n}.$$
- If the value of player II's die is equal to the bid of player I, then with doubting the probability of victory is
$$\frac{\frac{1}{n-1}}{1+\frac{1}{n-1}} = \frac{1}{n},$$
while claiming 2 of that value makes the probability
$$\frac{1}{1+\frac{1}{n-1}} = \frac{n-1}{n},$$

so the latter is optimal.

We can store the probabilities of player II's victory in the matrix W_1 with $n-1$ rows (representing the bid of player I: $n, n-1, \dots, 3, 2$) and n columns (representing the value of player II's die: $n, n-1, \dots, 2, 1$). The elements of W_1 are as follows.

- $w_{1_{ii}} = \frac{n-1}{n}$;
- $w_{1_{ij}} = 1$, if $i > j$;
- $w_{1_{ij}} = \frac{1}{n}$, if $i < j$.

$$W_1 = \begin{pmatrix} \frac{n-1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ 1 & \frac{n-1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ 1 & 1 & \frac{n-1}{n} & \cdots & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & \frac{n-1}{n} & \frac{1}{n} & \frac{1}{n} \\ 1 & 1 & 1 & \cdots & 1 & \frac{n-1}{n} & \frac{1}{n} \end{pmatrix}$$

Each bid of player I and each value of player II distributes independently and uniformly so each case has probability $\frac{1}{n}\frac{1}{n-1}$. The probability of player II's victory is:

$$\frac{1}{n(n-1)}\left(\frac{(n-1)(n-2)}{2}\cdot 1 + (n-1)\frac{(n-1)}{n} + \frac{(n-1)(n-2)}{2}\frac{1}{n} + (n-1)\frac{1}{n}\right) = \frac{\binom{n+1}{2}-1}{n^2}.$$

Let p_{ij} be the probability of player I's bid is j if his die's value is i, so $\sum_{j=1}^{n} p_{ij} = 1$, moreover let P_j =P(player I's bid is j)=$\sum_{i=1}^{n} p_{ij}/n$.

What is the probability of the victory of player II if he uses the aforementioned strategy?

- If the value of player II's die is 2 and the bid of player I is 1, then player II doubts, and he wins if player I bluffed.
- If the value of player II's die $j > 2$ is greater than the bid of player I, then if the value of player I's die is lower than the value of player II wins, else player I can claim a greater bid so he wins.
- If the bid of player I is 2 of something, then player II doubts, because he could claim a valid greater bid only if player I bluffed.
- If the value of player II's die is lower than the bid of player I, then he doubts and wins if player I bluffed.
- If the value of player II's die is equal to the bid of player I, then he claims 2 of that value and wins if player I's bid and his die's value are the same.

Let us consider the matrix W_2 with $2n$ rows (representing the bid of player I: $1, 2, 3, \ldots, n-1, n, 2$ of $1, 2$ of $2, \ldots, 2$ of n) and n columns (representing the value of player II's die: 1, 2,..., n). Let $w_{2_{ij}}$ be the sum of p_{kj} for those k for which player II wins if his die's value is j and player I's bid is i for his value k.

We use the matrix W_2, which – due to its size – appears in the Appendix.

The probability of the victory of player II if the bid of player I is i is:

$$\frac{1}{n}\sum_{j=1}^{n}\frac{w_{2_{ij}}}{n \cdot P_i}.$$

So the probability of the victory of player II is:

$$\sum_i P_i \frac{1}{n}\sum_{j=1}^{n}\frac{w_{2_{ij}}}{n \cdot P_i} = \frac{1}{n^2}\sum_{i,j} w_{2_{ij}}.$$

For $2 < j \leq n$ we get

$$\sum_{i<j} w_{2_{i,j}} + \sum_{i \geq j} w_{2_{i,j-1}} \geq \sum_{k=1}^{j-1}\sum_i p_{k,i} = (j-1)\cdot 1,$$

and

$$\sum_{i=1}^{n} w_{2_{i,i}} + \sum_{i=2}^{n} w_{2_{i,1}} + w_{2_{1,2}} = n\sum_{i=1}^{n} P_i,$$

so the sum of the elements of W_2 is at least:

$$n + \sum_{j=2}^{n-1} j = \binom{n+1}{2} - 1.$$

This means that the probability of player II's victory is at least

$$\frac{\binom{n+1}{2} - 1}{n^2}.$$

Let us consider the case when both players have a 3-faced die where 3 is joker.

Theorem 2.2. *Player II's probability of win under optimal play by both players is*

$$\frac{4}{9}.$$

An optimal strategy for player I is as follows.

- If the value of his die is i, then he claims 2 of i;
- on the second round (if any), he doubts any claim by player II.

An optimal strategy for player II is as follows.

- If the bid of player I is 1, 2 or 3, then he claims 2 of the value of his die;
- if the bid of player I is 2 of 1, 2 of 2 or 2 of 3, then he doubts;
- on the second round (if any), he doubts any claim by player I.

Proof: If player I plays the aforementioned strategy, he wins with probability $\frac{2}{3}$ if his die's value is 1 or 2 (in the cases player II has the same value or 3) and wins with probability $\frac{1}{3}$ if his die's value is 3. So the probability of the victory of player II is

$$1 - \frac{1}{3}\left(\frac{2}{3} + \frac{2}{3} + \frac{1}{3}\right) = 1 - \frac{5}{9} = \frac{4}{9}.$$

Let p_{ij} be the probability of player I's bid is j if his die's value is i, so

$$p_{i1} + p_{i2} + p_{i3} + p_{i,11} + p_{i,22} + p_{i,33} = 1,$$

moreover let P_j =P(player I's bid is j)=$p_{1j}/3 + p_{2j}/3 + p_{3j}/3$.

Playing with the aforementioned strategy when does player II win?

If the value of the die of player II is 1, and the bid of player I is 1, 2 or 3, then player II claims 2 of 1 and wins if the value of player I's die is 1 or 3.

If the value of the die of player II is 1, and the bid of player I is 2 of 1, 2 of 2 or 2 of 3, then player II doubts and loses only if the bid is 2 of 1 and the value of player I's die is 1 or 3.

If the value of the die of player II is 2, and the bid of player I is 1, 2 or 3, then player II claims 2 of 2 and wins if the value of player I's die is 2 or 3.

If the value of the die of player II is 2, and the bid of player I is 2 of 1, 2 of 2 or 2 of 3, then player II doubts and loses only if the bid is 2 of 2 and the value of player I's die is 2 or 3.

If the value of the die of player II is 3, and the bid of player I is 1, 2 or 3, then player II claims 2 of 3 and wins if the value of player I's die is 3.

If the value of the die of player II is 3, and the bid of player I is 2 of 1, then player II can claim 2 of 3 and wins if the value of player I's die is 3, or can doubt and wins if the value of player I's die is 2.

If the value of the die of player II is 3, and the bid of player I is 2 of 2, then player II can claim 2 of 3 and wins if the value of player I's die is 3, or can doubt and wins if the value of player I's die is 1.

If the value of the die of player II is 3, and the bid of player I is 2 of 3, then he doubts and wins if the value of player I's die is 1 or 2.

Assuming that the probability of bidding 2 of 3 when player I's bid is 2 of 1 and the value of player II's die is 3 is q_1, and the probability of bidding 2 of 3 when player I's bid is 2 of 2 and the value of player II's die is 3 is q_2, then we can use the following matrix W_3 of 6 rows (representing the bid of player I: 1, 2, 3, 2 of 1, 2 of 2, 2 of 3) and 3 columns (representing the value of player

II's die: 1, 2, 3) to calculate the probability of the victory of player II, which is similarly to the previous section the sum of the elements of W_3 divided by 9.

$$W_3 = \begin{pmatrix} p_{11}+p_{31} & p_{21}+p_{31} & p_{31} \\ p_{12}+p_{32} & p_{22}+p_{32} & p_{32} \\ p_{13}+p_{33} & p_{23}+p_{33} & p_{33} \\ p_{2,11} & 3P_{11} & q_1 p_{3,11}+(1-q_1)p_{2,11} \\ 3P_{22} & p_{1,22} & q_2 p_{3,22}+(1-q_2)p_{1,22} \\ P_{33} & 3P_{33} & p_{1,33}+p_{2,33} \end{pmatrix}$$

The sum of the elements of W_3 is

$$\begin{aligned} p_{1,1}+p_{3,1}+p_{2,1}+p_{3,1}+p_{3,1}+ \\ +p_{1,2}+p_{3,2}+p_{2,2}+p_{3,2}+p_{3,2}+ \\ +p_{1,3}+p_{3,3}+p_{2,3}+p_{3,3}+p_{3,3}+ \\ +p_{2,11}+3P_{11}+q_1p_{3,11}+(1-q_1)p_{2,11}+ \\ +3P_{22}+p_{1,22}+q_2p_{3,22}+(1-q_2)p_{1,22}+ \\ +3P_{33}+3P_{33}+p_{1,33}+p_{2,33}= \\ p_{1,1}+p_{1,2}+p_{1,3}+p_{1,11}+(2+1-q_2)p_{1,22}+3p_{1,33}+ \\ +p_{2,1}+p_{2,2}+p_{2,3}+(2+1-q_1)p_{2,11}+p_{2,22}+3p_{2,33}+ \\ +3p_{3,1}+3p_{3,2}+3p_{3,3}+(1+q_1)p_{3,11}+(1+q_2)p_{3,22}+2p_{3,33}= \\ 1+(2-q_2)p_{1,22}+2p_{1,33}+ \\ +1+(2-q_1)p_{2,11}+2p_{2,33}+ \\ +1+2p_{3,1}+2p_{3,2}+2p_{3,3}+q_1p_{3,11}+q_2p_{3,2}+p_{3,33}. \end{aligned}$$

If we choose $q_1 = q_2 = 1$, then the sum is

$$4+p_{1,22}+p_{1,33}+p_{2,11}+p_{2,33}+p_{3,1}+p_{3,2}+p_{3,3} \geq 4.$$

So the probability of the victory of player II playing the aforementioned strategy is at least $\frac{4}{9}$.

3. Model of the game for simulation

We present simulation result about strategies in the liar's dice game. In the following, we describe the simulation's details. As the game flow is quite complex, we made several simplifying assumptions about the player's strategies. When a player has to make a decision about his move, he should compare the risks of raising the bid with that of challenging the previous bid. The decision is made more complex by psychological factors, since the bidding strategies of the previous player and the challenge strategies of the following player also influence the decision. In the following we describe a simplified model of the player's strategies. The motivation behind the model is that we want to make challenge slightly more likely against a player who is known to be a frequent liar, and similarly, we would like to be a bit cautious when raising a bid if the next player is known to be a good challenger.

Raise. If we raise the bid, we raise it to the one with highest probability of success. For example, if we have to answer to a bid of "seven threes', we will prefer "eight twos" to "seven fours" if we happen to have a hand with several twos and no fours. The probability of a successful bid is calculated based on our hand. We do not assume anything about the likeliness of a challenge by the next player.

Challenge. We calculate the probability of a successful challenge based on our hand and the bid of the previous player.

Behavior of the previous player. We record the success of our challenges against the previous player. If the previous challenge against this player was successful, we slightly increase the probability of a challenge (details are formalized later).

Behavior of the next player. We record the success of challenges by the next player. If the previous challenge against us was successful, we slightly decrease the probability of a bid raise (details are formalized) later.

The player's strategy is a mixed strategy, parametrized by a pair (s, t). On each turn, the player determines the probability p_1 that the previous bid was a lie (if this is the first move, then we always rise). The player also determines the next preferable bid and its success probability p_2. If the last challenge against the previous player was successful, then let $c_s \in [0, s]$ uniformly randomly chosen, otherwise $c_s = 0$. If the last challenge by the next player failed, then $c_t \in [0, t]$ is uniformly randomly chosen, otherwise $c_t = 0$. We decide to raise if and only if $p_2 - p_1 + c_s - c_t > 0$.

The intuition behind this mixed strategy is as follows: it is natural to assume that we chose out of the two possibilities (raising the bid and challenging) the one that is more likely to succeed. That's why we compare p_1 and p_2. But if the player with the last bid has been caught lying before, then we slightly increase the possibility of a challenge. Likewise, if the player who follows after us has a bad challenge to his record, then we are possibly a bit braver with raising the bid. The parameters correspond to the sensitiveness of the player to such behavior.

We evaluate the strategies using evolutionary simulation methods in the following section.

4. Evolutionary simulation of strategy population

Our simulation methods are inspired by investigations about the iterated prisoner's dilemma, see e.g. [1, 4]. Our evolutionary simulation proceeds as follows. The strategy population S is a mesh of points in a box $[0,a] \times [0,b]$ (E.g. $S = \{(i/10, j/10) \mid 0 \leq i, j \leq 10, i, j \in \mathbb{Z}\}$). Each element $s \in S$ holds a score $f(s)$ which is initially 1. Each simulation is performed in rounds.

We fix the number P of players and D for a simulation run. Each simulation round starts with choosing P strategies from the overall population based on the strategy's score – the probability of choosing a strategy is proportional to its weight. We perform a P-player game with D dice. After the game we add one to the winner's strategy. Then a new simulation round is started. The goal of the simulation is to see which strategies are the most successful ones. In the following section we discuss how the strategy population's score changes.

4.1. **Simulation results.** In the first group of experiments 300 simulation rounds were performed with X a 10×10 mesh on $[0,1]$. We performed experiments with $P = 2, D = 10$, $P = 2, D = 30$, $P = 2, D = 100$, $P = 3, D = 30$, $P = 8, D = 30$. We show some of the results in Figure 1. It is apparent in the figures that the value of $y = c_t$ plays a more decisive role in the success of a strategy than that of $x = c_s$. It is also visible that the exact values of P and D do not influence the results to a large extent.

Based on the results, in the second group of experiments we refined the space to $[0,1] \times [0,0.2]$. The simulation results can be seen in Figure 2. No obvious pattern is visible here. Based on evidence so far, it seems reasonable that a small value of $c_t \sim 0.1$ is beneficial, that is, being a bit braver with raises might pay off. An exact quantification of this statement still needs refined models and further simulation.

5. Summary

In the present paper about the Liar's dice game, we presented an analysis of endgame strategies for two players and simulation results using a simplified model of the game strategies. We plan to extend both the theoretical and simulation results. In the endgame analysis it would be interesting to give optimal strategies for more than two players with one die each, and for two players with several dice. The analysis becomes much more complicated, but we think it can be carried out. For the simulation, a possible direction is to model the information that may be hidden in previous calls by a player in the same round – does someone calling a large number of fives really have a hand full of fives, or is it just a bluff? Another possibility is to apply learning

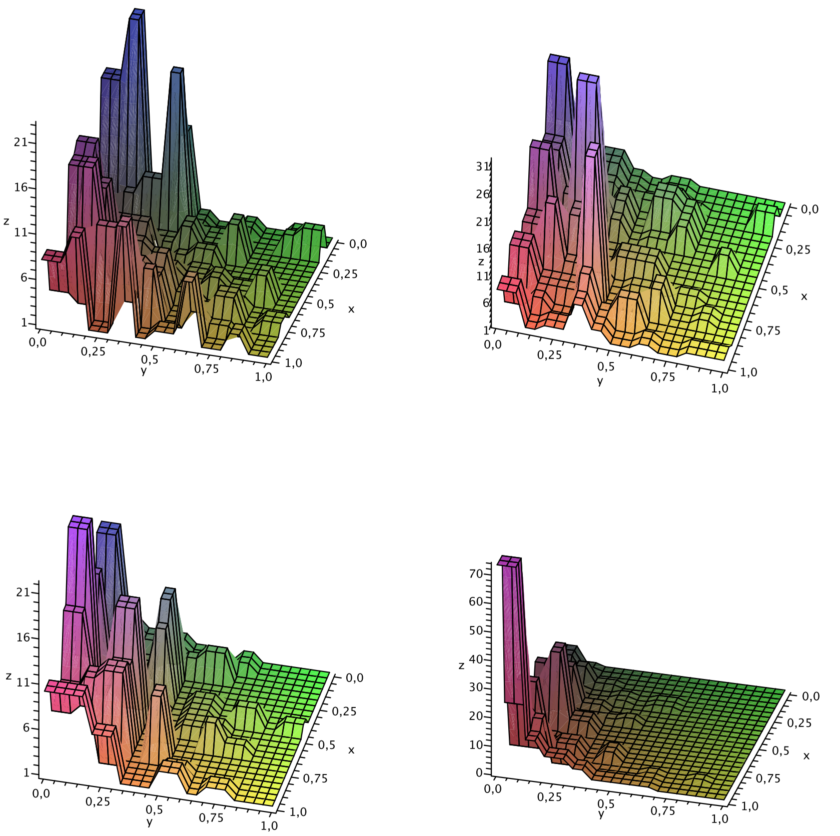

FIGURE 1. Strategy population growth after 300 matches. From left to right: $(P, D) = (2, 30), (2, 100), (3, 30), (8, 30)$.

theory and see how good a player a computer program can become when playing against humans or other programs.

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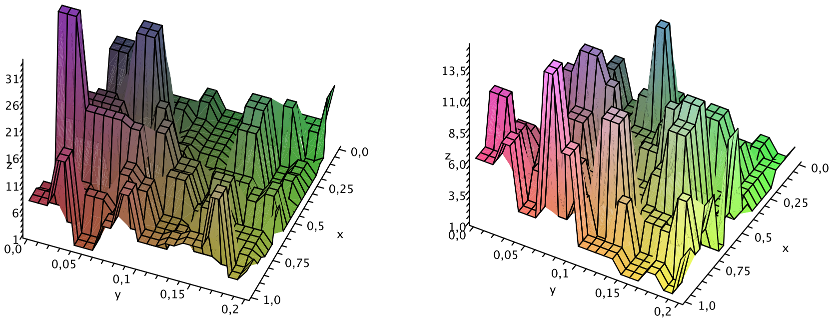

FIGURE 2. Strategy population in finer resolution. Here $c_t \in [0, 1/5]$. The values $(P, D) = (2, 30)$ and $(P, D) = (2, 50)$.

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Appendix

We present matrix W_2 from Section 2.

$$\left(\begin{array}{ccccc}
p_{11} & nP_1-p_{11} & p_{11}+p_{21} & p_{11}+p_{21}+p_{31} & \cdots \\
nP_2-p_{22} & p_{22} & p_{12}+p_{22} & p_{12}+p_{22}+p_{32} & \cdots \\
nP_3-p_{33} & nP_3-p_{33} & p_{33} & p_{13}+p_{23}+p_{33} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
nP_{n-1}-p_{n-1,n-1} & nP_{n-1}-p_{n-1,n-1} & nP_{n-1}-p_{n-1,n-1} & nP_{n-1}-p_{n-1,n-1} & \cdots \\
nP_n-p_{nn} & nP_n-p_{nn} & nP_n-p_{nn} & nP_n-p_{nn} & \cdots \\
nP_{11}-p_{1,11} & nP_{11} & nP_{11} & nP_{11} & \cdots \\
nP_{22} & nP_{22}-p_{2,22} & nP_{22} & nP_{22} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
nP_{n-1,n-1} & nP_{n-1,n-1} & nP_{n-1,n-1} & nP_{n-1,n-1} & \cdots \\
nP_{nn} & nP_{nn} & nP_{nn} & nP_{nn} & \cdots
\end{array}\right.$$

$$\left.\begin{array}{ccc}
\cdots & p_{11}+p_{21}+\cdots+p_{n-2,1} & p_{11}+p_{21}+\cdots+p_{n-1,1} \\
\cdots & p_{12}+p_{22}+\cdots+p_{n-2,2} & p_{12}+p_{22}+\cdots+p_{n-1,2} \\
\cdots & p_{13}+p_{23}+\cdots+p_{n-2,3} & p_{13}+p_{23}+\cdots+p_{n-1,3} \\
\ddots & \vdots & \vdots \\
\cdots & p_{n-1,n-1} & p_{1,n-1}+p_{2,n-1}+\cdots+p_{n-1,n-1} \\
\cdots & nP_n-p_{nn} & p_{nn} \\
\cdots & nP_{11} & nP_{11} \\
\cdots & nP_{22} & nP_{22} \\
\ddots & \vdots & \vdots \\
\cdots & nP_{n-1,n-1}-p_{n-1,(n-1,n-1)} & nP_{n-1,n-1} \\
\cdots & nP_{nn} & nP_{nn}-p_{n,(n,n)}
\end{array}\right)$$

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