

PLAYERS WITH UNEXPECTED BEHAVIOR: t -IMMUNE STRATEGIES. AN EVOLUTIONARY APPROACH.

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ABSTRACT. An evolutionary detection method based on generative relations for detecting the t -immune strategies of a non-cooperative game is introduced. Numerical experiments on an economic game show the potential of our approach.

1. INTRODUCTION

Game Theory (GT) offers proper models to characterize interactions between agents with conflicting behaviours. The situations where divergent interests interact are modelled as mathematical games. Each player has a set of strategies ("moves") that define his possible actions within the game. Many types of games have been proposed since the concept was first introduced: games with complete information (players have complete information on the entire game), cooperative or non-cooperative games (depending on the players' disposition to build unions or not), one shot games (players play one round only in the same time), etc.

One of GT's main aim was to find patterns and solutions that will allow scholars to accurately anticipate game's outcomes and the behaviour of real players. An equilibrium concept designed for pure rational players, was introduced by Nash [8] and depicts that state where no individual player can gain more by modifying his option within the game (his strategy) while the others keep theirs unchanged. Even if it is one of the central solution concepts in GT, Nash equilibrium was also criticized mostly because of the hard assumptions on players rationality [6]. Experiments conducted with real people lead to the

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conclusion that Nash equilibrium is seldom the output for games with real players.

The search for more realistic models for real players led to entire classes of equilibria. The scholars tried to incorporate the "human" factor in these models but, even if at some point they succeeded at this task, the proper tools to solve them were missing. Evolutionary computation can offer a solution for this. The recent development of fitness solutions for game equilibria detection [5], [4] allows specialized techniques on strategic games to be developed.

In this study we present a tool, based on evolutionary computation, designed to detect a good approximation of a game t -immune equilibria. This equilibria attempts to capture the situations where agents are acting in an unpredictable manner, an irrational behaviour outlined by most of the experiments with real people.

The paper is organized in four sections: an introduction that presents the domain and emphasise the importance of the approached problem; in the second section the proposed technique is presented; the conducted numerical experiments that validate the method are depicted in section three followed by the conclusions and further work section.

2. EVOLUTIONARY EQUILIBRIUM DETECTION FOR t -IMMUNE EQUILIBRIA MODELS

In order to detect the t -immune equilibrium for noncooperative games in strategic form a generative relation is introduced. Using this relation a fitness measure is constructed and two evolutionary methods are modified and adapted to detect t -immune equilibria.

2.1. Prerequisites. A finite strategic game is a system $G = ((N, S_i, u_i), i = 1, \dots, n)$, where:

- N represents a set of players, and n is the number of players;
- for each player $i \in N$, S_i is the set of actions available,

$$S = S_1 \times S_2 \times \dots \times S_n$$

is the set of all possible situations of the game.

Each $s \in S$ is a strategy (or strategy profile) of the game;

- for each player $i \in N$, $u_i : S \rightarrow R$ represents the payoff function of i .

The most used solution concept in Game Theory is the Nash equilibrium [8]. Playing in Nash sense means that no one from the players can change her/his strategy in order to increase her/his payoff while the others keep theirs unchanged.

t -immune equilibrium [1] models situations where players act unpredictable, not in a rational or expected way. A strategy is t -immune when less than t

players change their strategy, but without affecting the payoffs for the other players.

Definition 1. A strategy $s^* \in S$ is t -immune if, for all $T \subseteq N$, with $\text{card}(T) \leq t$, all $s_T \in S_T$, and all $i \notin T$:

$$u_i(s_{-T}^*, s_T) \geq u_i(s^*).$$

t -immune equilibria models the tolerance threshold of players, how many players can behave unpredictable without affecting the other players payoffs.

2.2. Generative relations. In order to compute equilibria we characterize them with adequate relations on the strategy set. Such relations are called *generative relations* of the equilibrium.

We have the quality measure:

$$Q : S \times S \rightarrow \mathbb{N},$$

where S is the set of the strategy profiles.

Let s and s^* be two strategy profiles, $s, s^* \in S$.

In this case $Q(s, s^*)$ measures the quality of strategy s with respect to the strategy s^* .

The quality Q is used to define the relation \prec_Q :

$$s \prec_Q s^*, \text{ if and only if } Q(s, s^*) \leq Q(s^*, s).$$

The first generative relation for the Nash equilibrium has been introduced in [7].

2.2.1. Generative relation for t -immune strategies. Consider a quality measure $t(s^*, s)$, which denotes the number of players who gain by switching from one strategy to the other strategies:

$$t(s^*, s) = \text{card}[i \in N - T, u_i(s_T, s_{-T}^*) \leq u_i(s^*), s_T \neq s_T^*, \text{card}(T) = t, T \subseteq N],$$

where $\text{card}[M]$ represents the cardinality of the set M .

Definition 2. Let $s^*, s \in S$. We say the strategy s^* is better than strategy s with respect to t -immunity, and we write $s^* \prec_T s$, if the following inequality holds:

$$t(s^*, s) < t(s, s^*).$$

Definition 3. The strategy profile $s^* \in S$ is called t -immune non-dominated, if and only if there is no strategy $s \in S, s \neq s^*$ such that

$$s \prec_T s^*.$$

TABLE 1. Parameter settings for t -DE

Parameter	Value
Pop size	100
Max no FFE	5000-2 players; 200000-3 players
Cr	0.8
F	0.3

The relation \prec_T can be considered as the generative relation for t -immune equilibrium, i.e. the set of non-dominated strategies, with respect to \prec_T , induces the t -immune strategies.

2.3. Evolutionary detection method. Two evolutionary algorithms are adapted for detecting t -immune equilibrium: NSGA-II (Non-dominated Sorting Genetic Algorithm II)[9] and DE (Differential Evolution) [10].

In both algorithms the Pareto dominance relation is replaced with the generative relation \prec_T and two new evolutionary methods for t -immune equilibrium detection, called t -NSGA-II, respectively t -DE are obtained.

3. NUMERICAL EXPERIMENTS - THE COURNOT GAME

In the normal Cournot model [3] n companies produce q_i , $i = 1, \dots, n$ quantities of a homogeneous product.

$Q = \sum_{i=1,n} q_i$ is the aggregate quantity on the market, a is a constant, and the market clearing price is $P(Q) = a - Q$ if $Q < a$ and 0 otherwise.

The total cost for company i for producing quantity q_i is $C_i(q_i) = cq_i$. The marginal cost c is constant, and $c < a$.

The payoff for the company i can be described as follows:

$$u_i(q) = q_i P(Q) - C_i q_i,$$

where $q = (q_1, \dots, q_n)$. The final form of the payoff function can be described as:

$$u_i(q) = q_i [a - \sum_{j=1,n} q_j - c], i = 1, \dots, n.$$

In our experiments we consider $a = 50$, $c = 10$, $q_i \in [0, 10]$, $i = 1, \dots, n$ and the two- and three player version of the Cournot game.

Parameter settings for t -DE are presented in Table 1. Parameter settings for t -NSGA-II are presented in Table 2. Ten different runs are considered, and the mean and standard deviation is reported.

Result for the two players variant of the Cournot game are presented in Table 3. Table 4 depicts the three player version of the Cournot game, for $t = \{1, 2\}$.

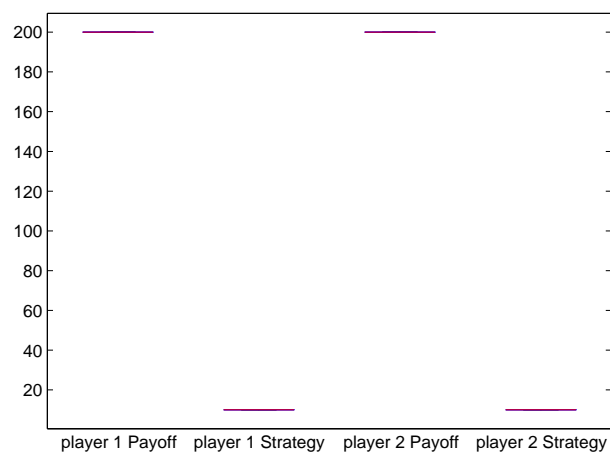


FIGURE 1. Strategies and payoffs for the two player version of the Cournot game

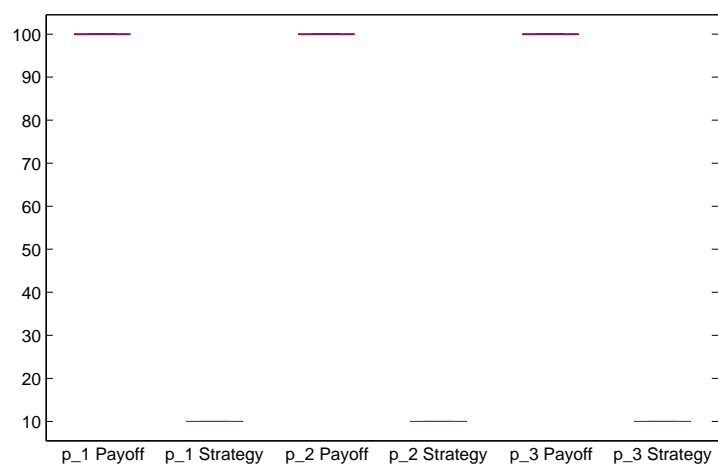


FIGURE 2. Detected 1-immune strategies and payoffs for the three player version of the Cournot game

TABLE 2. Parameter settings for t -NSGA-II

Parameter	Value
Pop size	100
Max no FFE	5000-2 players; 185108-3 players
prob. of crossover	0.2
prob. of mutation	0.2

TABLE 3. Results for 2 player Cournot game for 1-immune equilibrium (mean values for 10 different runs).

Algorithm	t_{immune}	Strategy		Payoff	
		s_1	s_2	u_1	u_2
$t-DE$	1	10	10	200	200
$t-NSGA2$	1	10	10	200	200

TABLE 4. Results for 3 player Cournot game for $t_{immune} \in \{1, 2\}$ (mean values for 10 different runs).

Algorithm	t_{immune}	Strategy			Payoff		
		s_1	s_2	s_3	u_1	u_2	u_3
$t-DE$	1	10	10	10	100	100	100
	2	10	10	10	100	100	100
$t-NSGA2$	1	10	10	10	100	100	100
	2	10	10	10	100	100	100

Figures 1, 2 and 3 depict the t -immune equilibria for two and three players.

Numerical experiments indicate that the players' threshold is minimal in both cases. The firms need to produce maximal quantity of products in order to avoid the unexpected behavior of some firms.

4. CONCLUSIONS AND FURTHER WORK

In this study a generative relation for the t -immune equilibrium is proposed. This generative relation is used in two different evolutionary algorithms to guide the search towards desired equilibria. The Cournot oligopoly model - an economic game, is considered for numerical experiments. Results underline the stability and the potential of the proposed method.

Further experiments will focus on large games.

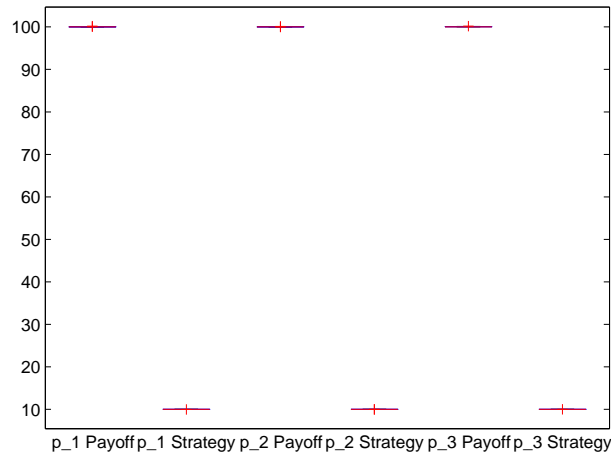


FIGURE 3. Detected 2-immune strategies and payoffs for the three player version of the Cournot game

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