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LEARNING TO PLAY THE GUESSING GAME

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Abstract. We present two models from literature (Nagel's Quantitative model and Stahl's Boundedly Rational Rule Learning model) that describe people's behaviour when playing the guessing game. Although these models were defined based on experimental data, when they are implemented and their result compared to experimental data, the results are not good. We define a new model, called Refined Boundedly Rational Rule Learning, based on an existing one, and show that its results are closer to experimental data than the results of the other two.

1. Introduction

Game theory has defined different equilibrium concepts, probably the most famous of them being the Nash equilibrium. These concepts can be applied to games to show which strategies would be the best for people playing them. Unfortunately, experiments show that people rarely play the strategy given by the equilibrium, but there is no clear explanation about why they do not. Many different experiments were performed to see how people play exactly, trying to define general rules that describe their behaviour. Finding such models of people's behaviour is important, because some games can model different economic phenomena, such as: bargaining, auctions, social networks and so on. This paper presents two algorithms based on existing models that try to simulate people's behaviour when playing the Guessing Game. The first is a simple model, called Quantitative model, while the second is a more complex one, called Boundedly Rational Rule Learning model. Since these models, when using to simulate people's choices do not give good results, we propose a new one, which is a modification of the second.

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2. Related Work

The guessing game (also called beauty contest game) was developed and introduces by John Maynard Keynes in "General Theory of Employment Interest and Money" in 1936, as a way to explain price fluctuations in equity markets [4]. Since its introduction, many different articles have been written about this game, trying to analyse different aspects of it, such as: the importance of complete (or incomplete) information for players ([3]), the importance of the size of groups in games and many different criteria. The Museum of Money & Financial Institutions even has a flash applet of the game on their webpage ([5]), where people can choose a number, and see the average result of choices so far. Currently the average guess is 23.
[1] presents some detailed experiments, and defines a "step-k" model, that is later used by [2] for his own model, which is a complex model, with many different parameters, for which values were estimated based on experimental data.

3. The Guessing Game

The guessing game is usually played by N players ($N \geq 2$), for T ($T \geq 1$) periods. For each period, each player simultaneously chooses a number from the [0, 100] interval. The winner of the game is the person whose chosen number is closer to p (usually p is 2/3, although [1] treats also the case when p =4/3) times the mean of all numbers, the rest of the players win nothing.
When p is less than 1 the only Nash equilibrium of the game is when all players play 0, but many different studies and experiments (for example [1, 2]) show that people rarely play this equilibrium at first. When the game is repeated many times, usually the numbers chosen by people are lower and lower for each round, and there are players who learn to play the equilibrium.
In [1] Nagel presents a model that tries to explain the way people play this game, by introducing the "step-k" model. In her idea, there are players who choose their numbers randomly, without forming any idea of the game, and these are the ones having zero-order belief [1]. Players with first order beliefs think that the rest of them are zero-order belief players, and choose their numbers according to this idea and so on. Although theoretically there could be defined an infinite number of such levels, experiments show that the highest order present when people play is usually 3.

4. Two existing models

4.1. **Quantitative model.** This model was described by Nagel in [1] based on some experiments she performed, during which people played the game for four sessions, and it is used to describe how people change the value they play from one session of the game to another. First of all, an *adjustment factor* a_{it} is defined for player i for period t, after the player has chosen a number (x_{it}). Its value is computed using the formula

$$a_{i,t} = \begin{cases} \frac{x_{i,t}}{50} & \text{for t = 1} \\ \frac{x_{i,t}}{(mean)_{t-1}} & \text{for t = 2,3,4} \end{cases} \tag{1}$$

After all players have chosen their numbers, the *optimal adjustment factor* can be computed which gives the optimal deviation from the previous mean, leading to the current one:

$$a_{opt,t} = \begin{cases} \frac{x_{opt,t}}{50} = \frac{p\times(mean)_t}{50} & \text{for t = 1} \\ \frac{x_{opt,t}}{(mean)_{t-1}} = \frac{p\times(mean)_t}{(mean)_{t-1}} & \text{for t = 2,3,4} \end{cases} \tag{2}$$

After a round a player can compute his own *adjustment factor* and he can also compute the *optimal adjustment factor*. He then compares the two, and if $a_{i,t}$ is less than $a_{opt,t}$ then in the next round he will choose a number that will increase $a_{i,t+1}$, otherwise he will chose a number that will decrease it.

4.2. **The Boundedly Rational Rule Learning Model.** The Boundedly Rational Rule Learning Model is based on Nagel's "step-k" model, but adds learning to it, which means that players of a given step, can learn to play numbers corresponding to another step. The model is presented in [2] and it is a complex model, which depends on 14 parameters, whose value was estimated based on data gathered from experiments. In the following we will shortly present the model as described in [2]. The model defines K *behavioural rules* (corresponding to the *steps* in Nagel's model), numbered from 0 to $K-1$. They consider the case K = 4. Each player will have a type which corresponds to these rules, but during the game they can learn to use a rule that is different from his type. Each player has a *vector of propensities* which has a value for each rule. This vector (denoted by ω) is used for computing the probability of using a behavioural rule. The probability of a player of type k choosing rule j in a period t (denoted by $\varphi(k,j,t)$) is given by the following formula:

$$\varphi(k,j,t) = e^{\omega(k,j,t)} / \sum_l e^{\omega(k,l,t)} \tag{3}$$

The initial propensities are defined so that the rule corresponding to the player's type will have the most chance of being chosen, but other rules will have a positive probability as well. So, they define $\omega(k,k,1) = \mu > 0$ and $\omega(k,j,1) = 0$ for $k \neq j$. They define a function $f_k : A \to \Delta(A)$ that maps the previous mean of numbers into a probability density on the set of current choices. If the mean of choices in the previous round was $\overline{x_t}$, then the probability density of x_{t+1} for rule k is denoted by $f_k(x_{t+1};\overline{x_t})$. Since $f_k(p \times \overline{x_t};\overline{x_{t-1}})$

is the probability density for rule k evaluated after the numbers were chosen, it can be used as a *performance measure*. Both functions will be of normal distribution, but, because making a decision is different from evaluating one, the standard deviation of this *performance measure* (denoted by g_k) is defined using different parameters. In every round, after the player has chosen a number, the vector of propensities is updated in the following way:

$$\omega(k, j, t) = \beta_0 \times \omega(k, j, t-1) + \beta_1 \times g_j(p \times \overline{x_{t-1}}; \overline{x_{t-2}}) \tag{4}$$

The parameter β_0 shows how important the current propensity is, while parameter β_1 shows how important the feedback over the current choice is for the player. The value of g_k does not depend on the current value chosen by the player, it will have the highest value for the k which represents the rule that would have been the best choice taking into consideration the previous and the current means. The model does not specify how a number is chosen, but given a number $x(i, t)$ (the choice of player i in period t) and a player's propensities towards the behavioural rules (which lead to the probabilities of choosing the rules) they define a formula to compute the probability of that number being chosen.

Finally, Stahl defines four parameters (α_0, α_1, α_2 and α_3) to represent the percentage of the players that have type zero, one, two and three respectively. They also add a fifth type, called α_{-1}, which represents the players that does not learn at all, but choose random numbers in every period.

5. Refined Boundedly Rational Rule Learning Model

We have implemented the two models described above to simulate the game, and see if the results are close to actual experimental results. Unfortunately, neither the Quantitative, nor the Boundedly Rational Rule Learning model gives a method of generating the next number of a player. For the Quantitative model, we choose to compute the difference between $a_{i,t}$ and $a_{opt,t}$ and modify $a_{i,t}$ in the given direction with a random value that is at most equal to the difference (chosen from a uniform distribution). When we have the new value of $a_{i,t}$ we can compute x using the formula from 1 (only that this time x is the unknown). The other model contains a formula to compute the probability of a given number being chosen. We used this formula and randomly generated one hundred numbers using a uniform distribution and choose the one for which the probability was the highest.

When testing these models, the only conclusion we could draw was that the numbers chosen by the models are lower and lower, but this was not sufficient. So we decided that instead of randomly generating first session choices, use numbers taken from experimental data and see if numbers chosen for subsequent session will be similar to those from experiments. Unfortunately we did

not have the opportunity to perform experiments and gather data, but Nagel describes some experiments in detail in [1]. She gives the mean of choices for the four sessions, moreover, the first period choices of peoples are also presented on a graph (Figure 1.B) from which we were able to deduce the numbers chosen by people with a good accuracy. Using these values as first period choices, we ran our simulation, but it did not give very good results.
When analysing the models, we observed a problem with the Boundedly Rational Rule Learning model: the way of computing the probabilities based on the propensities will smooth out very big differences in the propensities. To try to solve this problem we propose a modified version of the model, called the Refined Boundedly Rational Rule Learning Model. First we changed Formula 3 to the following one:

$$\varphi(k,j,t) = \omega(k,j,t) / \sum_{l} \omega(k,l,t) \tag{5}$$

The only problem with this Formula is, that all but one probability will be zero initially. This is why we changed the original propensity values from 0 to 0.107 (the number was chosen so that it will give the same probability in the first period as with the old formula). Experimental results were a little better, but learning was still very slow, so we decided to modify the value of β_1 from Formula 4. We performed tests with $\beta_1 = \beta_0$, $\beta_1 = 2 \times \beta_0$ and $\beta_1 = 4 \times \beta_0$.

6. Experimental Results

We performed test with all three modified values for the value of β_1 mentioned above. They will be noted RBRRL 1, RBRRL 2 and RBRRL 3, respectively. Results of the test can be seen on Figure 1, where the column "Nagel" contains the mean choices from Nagel's experimental data, while the following columns contain the results of simulations for the models implemented by us: Quantitative model, Boundedly Rational Rule Learning model, and the three above mentioned models. The values are averages for 100 runs. Values for the first period are similar, because those values were given to the algorithm as input to have initial values similar to the ones in Nagel's experiment.
Comparing the results in the columns, we can see that values closest to the experimental data are in the RBRRL 1 model, where $\beta_1 = \beta_0$, which means that the initial type of a player is equally important as the performance in the last period.

7. Conclusion

We have proposed to implement two models from literature, for simulating the way people play the guessing game: the Quantitative model from Nagel's paper, and the Boundedly Rational Rule Learning model from Stahl's paper,

	All Data					
	Nagel	Quant. Mod	BRRL	RBRRL 1	RBRRL 2	RBRRL 3
Period 1	36.7425	36.55	36.55	36.55	36.55	36.55
Period 2	23.25	22.375	18.075	24.8	21.3	20.075
Period 3	15.7	11.5	10.325	14.625	12.475	11.625
Period 4	9.355	4.95	6.575	9.625	8.125	7.375

Figure 1. Mean choices for four periods in Nagel's experiments, the Quantitative Model, the BRRL model and the RBRRL model with three different values for β_1.

to test how well are they doing at predicting the next number a player will choose. Since they did not give good results, we defined a new model, the Refined Boundedly Rational Rule Learning, based on the second one.
In lack of own experimental results, we compared the performance of our model with experimental results found in Nagel's paper. Results show that all models give values closer and closer to the Nash equilibrium, just like Nagel's experimental results do. Moreover considering the average results for the four sessions, we can conclude that our model gives the values that are closest to the ones in Nagel's paper.
As further work we propose to perform experiments with human players and repeat the simulations using those values as initial numbers. Also, we propose finding other models in the literature, implementing and testing them, to see if they can better model people's behaviour.

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