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COMPLEX SYSTEMS AND CELLULAR AUTOMATA MODELS IN THE STUDY OF COMPLEXITY

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Abstract. Complex systems consist of a large number of interconnected and mutually interacting components. The real world is full of examples of complex adaptive systems from ancient and modern cultures, biological and social systems to economies and ecosystems. The study of complex systems is crucial for a constructive assessment and understanding of essential aspects of everyday life. Computational approaches to the analysis of complexity are among the most important tools used for this purpose. The current paper aims to offer an overview of complex systems and their main properties of emergence, adaptability and self-organization. Cellular automata are reviewed as major computational techniques engaged for complex systems modelling. In a cellular automaton, simple rules give rise to complex emergent behaviours worth investigated in the context of many real-world complex models. Perspectives of promising research directions to be tackled are discussed.

1. Introduction

Complexity, emergence and self-organization represent essential aspects of today's world real systems. The study and in-depth analysis of these elements needs computational perspectives able to significantly impact the study of complexity and the solving process of dynamic complex problems. Complex systems research has developed and grown tremendously during the past two decades, being subject of studies in a great variety of fields including physics, biology, computer science, sociology and economics. There are three interrelated approaches to the modern study of complex systems: (i) study of the interactions that give rise to complex systems; (ii) models of complex systems; (iii) the process of formation of complex systems.

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A complex system usually involves a large number of components. These components may be simple both in terms of their internal characteristics and in the way they interact. However, when the system is observed over a long time period and length scales, there may be phenomena that are not easily understood in terms of its simple components and their interactions. Complex systems are mainly characterized by emergence, complexity, self-organization, non-linearity, order/chaos dynamic and generic evolution. The current paper offers a survey on the main aspects related to the properties of complex systems and the major computational instruments that can be engaged in the study of these complex systems. Cellular Automata (CA) are reviewed in detail as important tools in the analysis of complex interactions and emergent systems. Furthermore, we discuss potential promising perspectives worth exploring in the analysis and understanding of complex behaviour in real systems.

The structure of the paper is as follows: complex systems are defined and their main properties of emergence and self-organization are detailed; CA are reviewed from definition and rule types to CA applications; some perspectives on future work and concluding remarks are drawn at the end of the paper.

2. Complex Systems

A system is a delineated part of the universe which is distinguished from the rest by an imaginary boundary [21]. A complex system is any system containing a large number of interacting entities (agents, processes, etc.) which are interdependent. The system behaviour cannot be identified by just considering each entity and combining them, but considering how the relationships between the entities affect the behaviour of the whole. This definition applies to systems from a wide range of scientific disciplines including thermodynamics, neural networks, evolution of cooperation, economic systems of interacting trading agents, urban growth, traffic systems and many others.

A system can be defined between two extremes of connectivity among its components: order and chaos. A complex system lies within this state-space of connectivity. At the ordered extreme, elements have no connections between them. Order is the result of no interactions (and therefore no dynamics). At the chaotic extreme, every element is directly connected to every other. This means that all information percolates and is modelled conventionally as a field. A complex system is an intermediate state between an ordered system and a chaotic system.

The main features of complex systems include emergence, self-organization, evolution and adaptability. Emergence occurs when the behaviour of a system cannot be reduced to the sum of the behaviour of the parts. Self-organization is the process by which elements interact to create spatio-temporal patterns of behaviour that are not directly imposed by external forces.

The formation of complex systems, and the structural/functional change of such systems, is a process of adaptation. Evolution is the adaptation of populations through inter-generation changes in the composition of the population. Learning is a similar process of adaptation of a system through changes in its internal patterns. The conventional notion of evolution of a population based upon replication with variation and selection with competition continues to be central. However, additional concepts such as co-evolution, ecosystems, multiple niches and hierarchical or multilevel selection have become important.

3. Emergence and Complexity

Emergence is a concept widely used in sciences, arts and engineering. A short description would state that "emergence" is the notion that the whole is not the sum of its parts [38]. For example, an individual ant or an individual neuron do not exhibit special intelligence, but gathered together and properly connected, 'spontaneous intelligence' emerges. Actually, in nature, some of the most engaging and perplexing phenomena are those in which highly structured collective behaviour emerges over time from the interaction of simple subsystems. Flocks of birds flying in lockstep formation and schools of fish swimming in coherent array abruptly turn together with no leader guiding the group [16]. The emergence of order and organization in systems composed of many autonomous entities or agents is a fundamental process.

Emerging properties are fundamental and yet familiar. According to Holland [38], emerging phenomena in generated systems are typically persistent patterns with changing components, i.e. they are changeless and changing, constant and fluctuating, persistent and shifting, inevitable and unpredictable. True emergent properties are irreducible, they cannot be destroyed or decomposed - but they appear or disappear. Unforeseeable failures and unexpected faults in software or hardware systems are special, undesired forms of emergence. It is necessary to understand the process of emergence in complex systems in order to enhance their robustness. Thus the knowledge of different types of emergence is essential for understanding and mastering complex systems.

Several attempts to classify and formalize the concept of emergence have been made [26]. Formalization attempts using grammars, formal languages and mathematics are proposed in [44, 5, 19]. The ubiquitousness of emergence in various, very different fields prohibits - for the moment - a unified formalization that would fit every form of emergence encountered. However, common characteristics across these fields [88] are as follows: the micro-macro effect is the most important characteristic referred in the literature (which refers to the properties, behaviours, structures or patterns appearing at a higher macro level that cannot be explicitly found at the lower, micro level); radical novelty - individuals at the micro-level have no explicit representation of the global behaviour; coherence or organizational closure; interacting parts - emergents arise from interaction between parts; dynamical - emergents arise as the system evolves in time; decentralized control; two-way link: micro-level parts give rise to an emergent structure which, from a macro-level, influences the parts; robustness and flexibility.

In [26] Fromm delineates four types of emergence as follows:

- *Type I* Simple/nominal emergence without top-down feedback, only "feed forward" relationships;
- *Type II* Weak emergence including simple (positive or negative) top-down feedback (Type II weak emergence may be stable or unstable);
- *Type III* Multiple emergence with many feedbacks appears in very complex systems with many feedback loops or complex adaptive systems with intelligent agents having a large amount of external influence during the process of emergence;
- *Type IV* Strong emergence is the form of emergence which is responsible for structures on a higher level of complexity which cannot be reduced, even in principle, to the direct effect of the properties and laws of the the elementary components.

Currently, emergence as phenomenon is studied in various fields. For example its philosophy is analyzed and defended in [40]. A general framework that allows the treatment of emergence without explicit reference to the specific underlying mechanism is presented in [34]. Current practical computational issues regarding implementing emergent behaviour are discussed in [15].

Application fields for studying the emergence phenomenon have become more complex: emergence of personality from the perspective of dynamical systems is formalized in [62]; emergence of chaos in complex dynamical networks is studied in [95]; emergence issues in managing complex adaptive systems used in natural resource management in [68] and many others.

In [93] the use of evolutionary computation is advocated for the automated, simulation-based design of organic computing systems [94] with emerging behaviour.

4. Self-organization and Adaptability

From cells, organisms and ecosystems to planets, stars and galaxies almost all systems found in nature show organization. Traditional scientific fields attempt to explain these features by referencing the particular micro properties or laws characterizing the system components. However, following "the principle of self-organization" proposed by Ashby [3], the problem can also be approached from a general perspective using properties applicable to every collections of parts of a system, regardless of size or nature. In [3] Ashby notes that a dynamical system, independently of its type or composition, always tends to evolve towards a state of equilibrium, towards an attractor.

The mathematical modelling of systems with many degrees of freedom is very challenging. With the advent of (inexpensive) computer modelling, the scientific study of self-organizing systems and adaptivity has recently grown out from a variety of disciplines including thermodynamics, cybernetics, evolutionary biology. The computer simulation assisted approach is at the base of the new domain of "complex adaptive systems", which was pioneered in the 1980's by a number of researchers from the Santa Fe Institute in New Mexico. Through computer simulations, complexity theorists can study systems consisting of many interacting components, which undergo constant change, both autonomously and in interaction with their environment. The behaviour of such complex systems is typically unpredictable, yet exhibits various forms of adaptation and self-organization.

A defining characteristic of complex systems is their tendency to self-organize globally as a result of many local interactions without explicit pressure or involvement from outside the system. The field of self-organization seeks general rules about the dynamics and evolution of global patterns that might be used to predict future organization of the system, as a result to the changes made to the underlying components [41].

A complex system moves between order and disorder without becoming fixed in either state. Such a system is considered adapting when it responds to information by changing. Complex adaptive systems persists in spite of changes in the underlying components due to how the interactions between those components are realized and changed. From changes in local interactions the system itself engages in adaptation or learning [37].

Random or locally directed changes can instigate self-organization by promoting the exploration of new state space positions. The instability that may arise from a local change triggers some sort of stress upon the whole system, causing it to move along a trajectory to a new attractor which forms the self-organized state. Changes and fluctuations allow the system to escape one basin of attraction and to enter another one.

One could expect that a system perturbed from an equilibrium state should settle to a "minimally" stable state generated by some type of optimization process. In real life, these optimized states often are highly unstable, exhibiting catastrophic breakdown events or avalanches. For example, in traffic flow the idealized state corresponds to a uniform flow of cars with all cars moving at maximum velocity possible. This idealized system is very fragile and unsustainable as traffic jams of all sizes might occur [65].

Complex systems are often situated at the delicate balanced edge between order and disorder in a self-organized critical state. Changes or mutations of a system may take it either towards a more static configuration or towards a more changeable one. If a particular dynamic structure is optimum for the system and the current configuration is too static, then the most changeable configuration will be most successful. If the system is currently too changeable then the most static mutation will be favoured. In this way the system can adapt in both directions to converge on the optimum dynamic characteristics.

Self-organized criticality provides a general mechanism for the emergence of complex behaviour in nature with granular piles [20], traffic [65], river networks [72] and braided rivers [82], the crust of the earth and many other systems operating in this state.

To summarize, the complex adaptive systems have the following properties:

- Local, non-linear interactions of the parts result in self-organization of the system as a whole. Kauffman [42] outlines the importance of the co-evolution of agents and their environments. As an agent changes, so does the environment (including other agents, and vice versa).
- The mixed condition between order and chaos of these systems gives them stability and flexibility simultaneously.
- Rather than striving for ideal but unstable states, complex systems are organized in a way that assures good functioning but also the ability to change.

5. Cellular Automata

Cellular Automata (CA) represent useful and important tools in the study of complex systems and interactions. Introduced by von Neumann more than fifty years ago [61] as formal models for organisms capable of self-reproducing, CA are simple models of computing with an extraordinary complex behaviour. A cellular automaton is a system evolving in discrete time steps with a discrete spatial geometry - usually a regular lattice. Other CA topologies such as networks or irregular graphs have been studied [83, 86, 87, 18]. The cellular automaton is specified in terms of rules that define how it changes and evolves in time. The emergent behaviour and computational complexity of a system can be analyzed and better understood based on CA dynamics.

The main features of CA are the following:

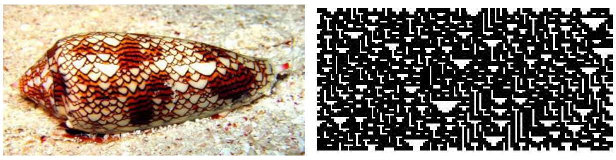

FIGURE 1. Similarities between the pattern growth of a specific shell and the dynamics of Rule 30 in 1D CA (from [71])

- *The state*: the value of each cell in CA (taken from a finite set of states). For example, cell state can have one of two values: 0 or 1 (extensively studied CA).
- *The neighbourhood*: the set of cells which interact locally. In 1D CA, a common neighbourhood of a cell refers to the so-called *first neighbours* $\{-1, 0, 1\}$ composed of the two neighbouring cells (one from the right and one from the left). In 2D CA, most used neighbourhoods include the von Neumann neighbourhood $\{(-1, 0), (1, 0), (0, -1), (0, 1)\}$ (the four neighbouring cells on top, right, left and down side) and Moore neighbourhood $\{-1, 0, 1\}^2$ (all eight neighbouring cells).
- *The transition function or the set of rules governing the evolution of CA*: how the state of a cell changes depending on its current value and that of the neighbourhood.

The global rule of a cellular automaton maps a configuration to the next time step configuration by applying the transition function uniformly in each cell. A space-time diagram is obtained by mapping the cellular automaton from one configuration to the next at all time steps [63]. It is important to emphasize that even simple rules (underlying local interactions only) give rise to complex emergent behaviour. Indeed, an important research direction refers to the analysis of CA space-time diagrams. The study of the underlying CA dynamics can potentially enable the understanding of emergent behaviour in complex systems and their computational capacity [30, 63].

CA are attractive models for many real-world complex systems from fields such as physics, biology and social sciences. For example, the natural pattern growth of a shell can be matched to the dynamics of 1D CA Rule 30 (see next subsection for a description of CA rule types) as illustrated in Figure 1 while the growth of crystals - like snowflakes patterns (see Figure 2) - can be modelled using 2D CA.

During the past few decades, CA have continuously attracted a great deal of interest from researchers and practitioners from various disciplines due to their simplicity and ability to generate highly complex behaviour. One significant property of CA is their capability to perform complex computation based

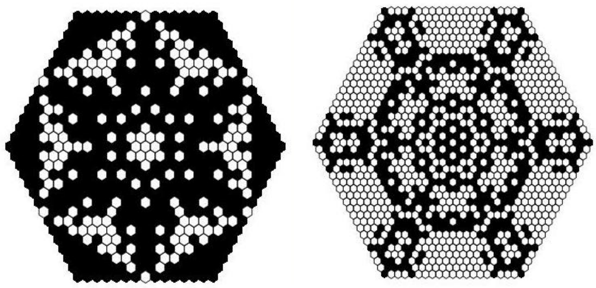

FIGURE 2. The pattern growth in snowflakes modeled with 2D CA (from [96])

on local information. Universality of computing and other theoretical aspects concerning the computational power of cellular automata are consequently considered of great importance [91].

The problem of finding CA rules able to generate a desired global behaviour is considered of great importance and highly challenging. The field of evolutionary computing offers the most promising models for addressing this inverse problem of global to local mapping [30]. The most studied problems include the density classification task [64, 50, 66, 33], the synchronization task [17, 47] and the discovery of structures such as gliders and glider guns, essential in the study of computationally universal cellular automata [75, 79].

5.1. **CA Rule Types.** Conway identified the first binary cellular automaton that supports universal computation, the well known Game of Life (or Life) [31]. This automaton evolves in an infinite two-dimensional grid of cells where each cell can be dead or alive. At each time step, the state of each cell is modified depending on the value of its eight neighbours (cells that are horizontally, vertically, or diagonally adjacent). The following simple rules are applied: any live cell with fewer than two live neighbours dies; any live cell with two or three live neighbours lives on; any live cell with more than three live neighbours dies; any dead cell with exactly three live neighbours becomes a live cell.

In [8] the authors have proved that this CA is able to simulate a Turing machine, meaning that it can compute everything that can be algorithmically computed. Life was one of the first major findings in CA field, being a great example of emergence and self-organization. Indeed, despite its very simple local rules, complex global behaviour and interesting patterns like still lives, oscillators and spaceships are evolved within Life. More interesting patterns have been further discovered: guns, which are able to create gilders and other spaceships, puffers which leave a trail when moving or rakes, which create spaceships while moving through space.

After discovering the Game of Life, identifying other CA capable of universal computation has become a challenge for researchers. In [75, 76] evolutionary techniques are used in order to discover rules that give rise to CA able to simulate logic gates, as this could help identifying automata capable of universal computation. A similar evolutionary approach manages to identify a new cellular automaton that can implement any logic circuit and is a simulation of Life in [77, 78]. Gliders and glider guns have been further investigated in [79, 80, 81] by means of evolutionary techniques.

Wolfram has studied all 256 possible rules for 1D CA with two neighbours. The decimal interpretation of the binary number representing the rule gave the name of the resulting cellular automata. He identified several CA which exhibit an interesting complex behaviour. These CA have been classified into four categories [89]: Class 1 - CA which evolve to a homogeneous state; Class 2 - CA displaying simple separated periodic structures; Class 3 - CA which exhibit chaotic or pseudo-random behavior, and Class 4 - CA which yield complex patterns of localized structures and are capable of universal computation.

CA that exhibit Class 4 features include The Game of Life, HighLife [7], Life-3d [6], Rule 54, Rule 110 and Beehive Rule [92]. Wolfram conjectured that all CA belonging to Class 4 might be able to perform universal computation.

The 1D CA Rule 110 (see Figure 3) is one of the most intensively studied CA, due to its complex behaviour [90]. Its interesting behaviour is given by the evolution of gliders, periodic structures moving through time. Another important finding was the proof that Rule 110 automaton is capable of universal computation, emulating a universal Turing machine [91]. In 2003, Martinez shows that each glider without extensions proposed by Cook [14] can be obtained by collisions [48]. The computational complexity of Rule 110 is investigated in [60], where the prediction of Rule 110 automaton is proved to be a P-complete problem.

Rule 54 is investigated in [11, 35, 91, 1]. In [49] the author shows that all gilders produced in this automaton can be evolved by collisions, similar to Rule 110. The results obtained by this analysis are further used to evolve dynamical logic gates.

The most common approach used for updating the state of a cell is synchronous, where all cells are updated simultaneously. Taking into account the fact that the updating is not synchronized in most of the real complex systems that can be simulated using CA, the asynchronous application of rules has been recognized as an important issue and experimentally investigated in [13, 73, 84, 9]. In [22] the authors show that the behaviour induced by some rules is significantly modified by the asynchronous update of the cells, while

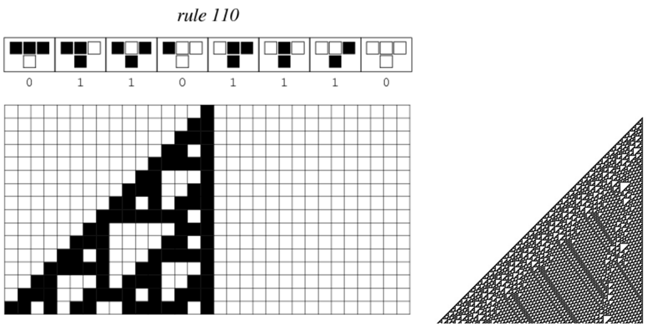

FIGURE 3. Space-time diagram of Rule 110 [91]

for some of them the CA dynamics do not change significantly. Asynchrony was induced by updating each cell with a given probability, called synchrony rate. This study is continued in [25] where directed percolation induced by asynchrony is identified. Such a phenomenon was also observed in the Game of Life [10].

The mathematical approaches to studying the behaviour induced by asynchrony in 1D CA can be found in [23, 24, 27, 28, 69]. 2D CA have been studied in [70, 71], where the authors show that the minority rule exhibits a complex behaviour when applied asynchronously. The importance of this rule and related findings might help improving understanding complex biological systems.

5.2. **Reversible CA.** In reversible CA, for each configuration there is exactly one past configuration. The importance of reversible CA is due to the fact that in such automata the information is conserved through time, making them suitable for the simulation of physical phenomena. This special class of CA has been studied starting with [59, 51, 2]. In [36] the authors investigate the local properties of cellular automata in order for them to be reversible. This study has been further developed in [52] by finding a way to detect the reversible behaviour by means of matrix representations. For this purpose, they are using the results obtained in [12] which show that any 1D CA can be transformed into an automaton of neighbourhood 2.

A method for calculating the ancestors of a configuration of states has been provided in [53, 56]. The ancestors are chosen to be non Garden-of-Eden sequences, which are sequences of states that can not be produced by any CA [51]. Two methods for calculating and classifying all possible reversible 1D CA of 3, 4, 5 and 6 states are proposed in [55]. Another type of invertible behaviour is investigated in [54, 57] and takes into account the spatial reversibility and not the temporal evolution.

5.3. **CA Applications.** The application areas of CA are many and diverse ranging from the simulation of complex systems in nature and society to modelling games and understanding social dynamics.

CA have been used to model various systems in biology (intracellular activity, cell-cell interactions, population of organisms, DNA sequences), chemistry (molecular systems, crystal growth, lattice gas automata), physics (dynamical systems, spin systems, reaction-diffusion systems), computer science (parallel processing architectures, von Neumann machines) and many others [30].

Due to the stability of CA dynamics [30], the most widely spread CA application refers to modelling dynamical physical systems. CA have been used to model the laws of physics as an alternative to differential equations [85]. Moreover, CA have been successfully engaged to model geography and urban growth [43], tumor-immune system interactions and tumor growth [45, 58, 32], controlling highly non-linear dynamical systems [4], image processing [74], traffic control [46], pattern recognition [39, 67, 29] etc.

6. Conclusions and Perspectives

The study of complex systems clearly needs to address essential aspects such as the emergence of complexity, the role of self-organization, cooperation and specialization and the relation between different complex adaptive systems. Basic properties of complex systems may include communication, cooperation, complexity, adaptation, feedback, growth, reproduction, spatial and temporal organization.

Computational approaches to the modelling and analysis of complex adaptive systems can potentially offer a better understanding of the underlying working mechanisms of many real-world systems. In this sense, the current paper offers an extensive review of cellular automata as supporting computational tools in the study of complexity. Although the volume of research material published in this area is very large and continuously growing, there are many issues to be tackled that can offer further insights into the understanding of complexity. Some of the perspectives worth exploring in the author's view include computer simulations using multi-agent modelling and evolutionary computing techniques and the investigation of complex network and cellular automata models for the analysis of complex systems.

Different types of emergence should be modelled by analyzing different interaction models within a population. A micro-level can be considered either at the individual/agent level or even within single entity individuals while a macro-level can be defined at group/population level. Different interaction models at the micro/macro level that induce emergent behavior can be studied using evolutionary computation.

Specific CA perspectives refer to the study of various CA topologies, the design of new CA types based on fuzzy neighbourhoods, the investigation of rule propagation models during CA evolution, the asynchronous application of (mixed) rules, the development of dynamic rules and neighbourhoods in CA, the investigation of various types of hybridizations within CA and novel natural computing models for CA rule detection.

Future scientific and technological developments in many fields will depend on the advances made to understand and harness complex systems behaviour. The study of complexity might facilitate the comprehension of the complex features involved in - for instance - the essential emotion-cognition-action interaction. Furthermore, this study may be really useful in dealing with the management of systems having a great number of components and dimensions. The development of universal methods for estimating and characterizing the behaviour of complex systems would be highly beneficial. A formalization would shed a light on how to (automatically) implement self-organization processes linked to applicative problems and might give us new insights about our surroundings.

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References

[1] A. Adamatzky, *Computing in nonlinear media and automata collectives*, Bristol, Philadelphia: Institute of Physics Publishing, 2001.

[2] S. Amoroso, Y. Patt, *Decision procedures for surjectivity and injectivity of parallel maps for tessellation structures*, Journal of Computer ans System Sciences, 6 (1972), pp. 448-464.

[3] W.R. Ashby, *Principles of the self-organizing system*, Principles of Self-organization (1962), pp. 255-278.

[4] F. Bagnoli, S. El Yacoubi, R. Rechtman, *Synchronization and Control of Cellular Automata*, Cellular Automata - 9th International Conference on Cellular Automata for Research and Industry - ACRI (2010), pp. 188-197.

[5] Y. Bar-Yam, *A mathematical theory of strong emergence using multiscale variety*, Complex., 9 (2004), pp. 15-24.

[6] C. Bays, *Candidates for the game of life in three dimensions*, Complex Syst 1 (1987), pp. 373-400.

[7] D.I. Bell, *HighLife - an interesting variant of life*, 1994. Unpublished article, available on the internet: http://www.tip.net.au/d̃bell/.

[8] E. Berlekamp, J.H. Conway, R. Guy, *Winning Ways for your mathematical plays*, Academic Press, New York, 1982.

[9] H. Bersini, V. Detours, *Asynchrony induces stability in cellular automata based models*, Proceedings of the 4th International Workshop on the Synthesis and Simulation of Living Systems ArtificialLifeIV (Brooks, R. A, Maes, and Pattie, eds.), MIT Press (1994), pp. 382-387.

[10] H.J. Blok, B. Bergersen, *Synchronous versus asynchronous updating in the "game of life"*, Physical Review E 59 (1999), pp. 3876-9.

[11] N. Boccara, J. Nasser, M. Roger, *Particle like structures and their interactions in spatio-temporal patterns generated by one dimensional deterministic cellular automaton rules*, Phys Rev A; 44, 2 (1991), pp. 866-875.

[12] T. Boykett, *Comparism of Radius 1/2 and Radius 1 Paradigms in One Dimensional Reversible Cellular Automata*, http://www.algebra.uni-linz.ac.at/ tim/radiuscomp.ps.gz, Universidad Johannes Kepler, Linz, Austria, 1997.

[13] R.L. Buvel, T.E. Ingerson, *Structure in asynchronous cellular automata*, Physica D 1 (1984), pp. 59-68.

[14] M. Cook, *Introduction to the activity of rule 110* (copyright 1994-1998 Matthew Cook)

[15] S.B. Cooper, *Emergence as a computability-theoretic phenomenon*, Applied Mathematics and Computation, 215, 4 (2009), pp. 1351-1360.

[16] J.P. Crutchfield, *The calculi of emergence: Computation, dynamics, and induction*, Physica D, 75 (1994), pp. 11-54.

[17] R. Das, J. Crutchfield, M. Mitchell, J. Hanson, *Evolving globally synchronized cellular automata*, Proc. of Inter. Conf. on Genetic Algorithms (1995).

[18] C. Darabos, M. Giacobini, M. Tomassini, *Scale-Free Automata Networks are not Robust in a Collective Computational Task*, In El Yacoubi S., Chopard B. & Bandini S. (Eds.), Lecture Notes in Computer Science, Proceedings of Sixth International Conference on Cellular Automata in Research and Industry (ACRI), 4173 (2006), pp. 512 - 521.

[19] J. Deguet, L. Magnin, Y. Demazeau, *Elements about the emergence issue: A survey of emergence definitions*, ComPlexUs, 3 (2006), pp. 24-31.

[20] R.O. Dendy, P. Helander, *Sandpiles, silos and tokamak phenomenology: a brief review*, Plasma Physics and Controlled Fusion, 39:1947 (1997).

[21] P. Erdi ,T. Kiss, *The complexity of the brain: Structural, functional, and dynamic module*, In S.Wermter, J. Austin, D. J. Willshaw (Eds), Emergent Neural Computational Architectures Based on Neuroscience - Towards Neuroscience-Inspired Computing, volume 2036 of Lecture Notes in Computer Science, Springer (2001), pp. 203-211.

[22] N. Fates, M. Morvan, *An experimental study of robustness to asynchronism for elementary cellular automata*, Complex Systems 16 (2005), pp. 1-27.

[23] N. Fates, M. Morvan, N. Schabanel, E. Thierry, *Fully asynchronous behaviour of doublequiescent elementary cellular automata*, Theoretical Computer Science, 362 (2006), pp. 1-16.

[24] N. Fates, D. Regnault, N. Schabanel, E. Thierry, *Asynchronous behaviour of doublequiescent elementary cellular automata*, In Proceedings of LATIN'2006, LNCS 3887 (2006).

[25] N. Fates, *Directed percolation phenomena in asynchronous elementary cellular automata*, 7th International Conference on Cellular Automata for Research and Industry, Perpignan, France, LNCS 4173 (2006), pp. 667-675.

[26] J. Fromm, *Types and forms of emergence*, 2005, Cornell University Library, available from http://arxiv.org/pdf/nlin.AO/0506028.

[27] H. Fuks, *Non-deterministic density classiffcation with diffusive probabilistic cellular automata*, Phys. Rev. E, 66, 2 (2002)

[28] H. Fuks, *Probabilistic cellular automata with conserved quantities*, Nonlinearity, 17:1 (2004), pp. 159-173.
[29] N. Ganguly, P. Maji, S. Dhar, B. K. Sikdar, P. P. Chaudhuri, *Evolving Cellular Automata as Pattern Classifier*, ACRI 2002, LNCS 2493, Springer-Verlag Berlin Heidelberg (2002) pp. 56-68.
[30] N. Ganguly, B. K. Sikdar, A. Deutsch, G. Canright, P. P. Chaudhuri, *A Survey on Cellular Automata*, Technical Report, Centre for High Performance Computing, Dresden University of Technology (2003).
[31] M. Gardner, *Scientific American*, ISBN 0894540017, October 1970.
[32] P. Gerlee, A.R.A. Anderson, *An Evolutionary Hybrid Cellular Automaton Model of Solid Tumour Growth*, J Theor Biol., 246:4 (2007), pp. 583-603.
[33] A. Gog, C. Chira, *Cellular Automata Rule Detection Using Circular Asynchronous Evolutionary Search*, HAIS 2009, LNCS 5572 (2009), pp. 261-268.
[34] J. de Haan, *How emergence arises*, Ecological Complexity, 3, 4 (2006), pp. 293-301.
[35] J.E. Hanson, J.P. Crutchfield, *Computational mechanics of cellular automata: an example*, Physics D 103:1-4 (1997), pp. 169-89.
[36] G.A. Hedlund, *Endomorphisms and automorphisms of the shift dynamical system*, Mathematical Systems Theory, 3 (1969), pp. 320-375.
[37] J. Holland, *Hidden Order: How Adaptation Builds Complexity*, Helix Books. Addison-Wesley, Reading, Massachusetts, 1995.
[38] J. Holland, Emergence: From Chaos to Order, Oxford University Press, 1998.
[39] E. Jen, *Invariant Strings and Pattern Recognizing properties of 1D CA*. Journal of Statistical Physics, 43 (1986), pp. 243-265.
[40] J. Jost, N. Bertschinger, E. Olbrich, *Emergence*, New Ideas in Psychology, 28, 3 (2010), pp. 265-273.
[41] S. Kauffman, *The Origins of Order: Self-Organization and Selection in Evolution*, Oxford University Press, USA, 1 edition, 1993.
[42] S. Kauffman, *At home in the universe: The search for laws of self-organization and complexity*, Oxford University Press, Oxford, 1995.
[43] V. Kocabas, S. Dragicevic, *Assessing cellular automata model behaviour using a sensitivity analysis approach*, Computers, Environment and Urban Systems, 30 (2006), pp. 921-953.
[44] A. Kubik, *Toward a formalization of emergence*, Artif. Life, 9 (2002), pp. 41-65.
[45] D. G. Mallet, L. G. de Pillis, *A cellular automata model of tumor-immune system interactions*, Journal of Theoretical Biology, 239:3 (2006), pp. 334-350.
[46] S. Maerivoet, B.De Moor, *Cellular Automata Models of Road Traffic*, in Physics Reports, 419:1 (2005), pp. 1-64.
[47] A. S. Mariano, G. M. B. de Oliveira, *Evolving one-dimensional radius-2 cellular automata rules for the synchronization task*, AUTOMATA-2008 Theory and Applications of Cellular Automata, Luniver Press (2008), pp. 514-526.
[48] G.J. Martinez, H.V. McIntosh, J.C.S.T Mora, *Production of gliders by collisions in Rule 110*, Lecture Notes in Computer Science, 2801 (2003), pp. 175-182.
[49] G.J. Martinez, A. Adamatzky, H.V. McIntosh, *Phenomenology of glider collisions in cellular automaton Rule 54 and associated logical gates*, Chaos, Solitons & Fractals, 28:1 (2006), pp. 100-111.
[50] M. Mitchell, M. D. Thomure, N. L. Williams, *The role of space in the Success of Coevolutionary Learning*, Proceedings of ALIFE X - The Tenth International Conference on the Simulation and Synthesis of Living Systems (2006).

[51] E.F. Moore, *Machine models of self-reproduction*, In Arthur W. Burks, editor, Essays on Cellular Automata, University of Illinois Press (1970), pp. 204-205.
[52] J.C.S.T. Mora, S.V.C Vergara, G.J. Martinez, H.V. McIntosh, *Spectral properties of reversible one-dimensional cellular automata*, International Journal of Modern Physics C, 14:3 (2003), pp. 379-395.
[53] J.C.S.T. Mora, G. Juarez, H.V. McIntosh, *Calculating ancestors in one-dimensional cellular automata*, International Journal of Modern Physics C, Vol 15, No. 8 (2004), pp 1151-1169.
[54] J.C.S.T. Mora, G.J. Martinez, H.V. McIntosh, *Correspondence between the temporal and spacial behavior in reversible one-dimensional cellular automata equivalent with the full shift*, TUCS Gen. Publ., 32 (2004), pp. 54-63.
[55] J.C.S.T. Mora, S.V.C. Vergara, G.J. Martinez, H.V. McIntosh, *Procedures for calculating reversible one-dimensional cellular automata*, Physica D. v202 (2005), pp. 134-141.
[56] J.C.S.T. Mora, G.J. Martinez, H.V. McIntosh, *The inverse behavior of a reversible one-dimensional cellular automaton obtained by a single welch diagram*, Journal of Cellular Automata, 1:1 (2006), pp. 25-39.
[57] J.C.S.T. Mora, M.G. Hernandez, G.J. Martnez, S.V.C. Vergara, H.V. McIntosh, *Unconventional invertible behaviors in reversible one-dimensional cellular automata*, Int. J. Bifurcation Chaos Appl. Sci. Eng. 18, No. 12 (2008), pp. 3625-3632.
[58] J. Moreira, A. Deutsch, *Cellular Automaton Models of Tumor Development: A Critical Review*, Advances in Complex Systems, 5(2002), pp. 247-269.
[59] J. Myhill, *The converse of Moore's Garden-of-Eden theorem*, Proceedings of the American Mathematical Society, 14 (1963), pp. 685-686.
[60] T. Neary, D. Woods, *P-completeness of cellular automaton Rule 110*, Lecture Notes in Computer Science 4051 (2006), pp. 132-143.
[61] J. von Neumann, *The Theory of Self-Reproducing Automata*, Univ. of Illinois Press, 1966.
[62] A. Nowak, R.R. Vallacher, M. Zochowski, *The emergence of personality: Dynamic foundations of individual variation*, Developmental Review, 25(3-4) (2005), pp. 351-385.
[63] N. Ollinger, *Universalities in Cellular Automata. A (Short) Survey*, Journees Automates Cellulaires (2008), pp. 102-118.
[64] N. H. Packard, *Adaptation toward the edge of chaos. Dynamic Patterns in Complex Systems*, World Scientific (1988), pp. 293-301.
[65] M. Paczuski, K. Nagel, *Self-organized criticality and 1/f noise in traffic*, In D. E.Wolf, M. Schreckenberg, and A. Bachem, editors, Workshop — 1995 Oct: Julich; Germany, pub-WORLD-SCI:adr, World Scientific Publishing Co (1996), pp. 73-86.
[66] L. Pagie, M. Mitchell, *A comparison of evolutionary and coevolutionary search*, Int. J. Comput. Intell. Appl., 2:1 (2002), pp. 53-69.
[67] R. Raghavan, *Cellular Automata in Pattern Recognition*, Information Science, 70 (1993) pp. 145-177.
[68] C. Rammel, S. Stagl, H. Wilfing, *Managing complex adaptive systems - a co-evolutionary perspective on natural resource management*, Ecological Economics, 63, 1 (2007), pp. 9-21.
[69] D. Regnault, *Abrupt behaviour changes in cellular automata under asynchronous dynamics*, In Proceedings of 2nd European Conference on Complex Systems (ECCS), Oxford, UK, 2006.

[70] D. Regnault, N. Schabanel, E. Thierry, *Progresses in the analysis of stochastic 2D cellular automata: a study of asynchronous 2D Minority*, LNCS, vol. 4708 (2007), pp. 320-332.

[71] D. Regnault, N. Schabanel, E. Thierry, *On the analysis of "simple" 2D stochastic cellular automata*, Discrete Mathematics & Theoretical Computer Science; vol. 12:2 (2010), pp. 263-294.

[72] I. Rodriguez-Iturbe, A. Rinaldo, *Fractal river basins: chance and self-organization*, Cambridge Univ Pr, 2001.

[73] A. Roli, F. Zambonelli, *Emergence of macro spatial structures in dissipative cellular automata*, Proc. of ACRI2002: Fifth International Conference on Cellular Automata for Research and Industry, Lecture Notes in Computer Science, vol. 2493 (2002), pp. 144-155.

[74] P. L. Rosin, *Training Cellular Automata for Image Processing*, SCIA 2005, LNCS 3540, Springer-Verlag Berlin Heidelberg (2005) pp. 195-204.

[75] E. Sapin, O. Bailleux, J.J. Chabrier, *Research of a cellular automaton simulating logic gates by evolutionary algorithms*, In EuroGP03. Lecture Notes in Computer Science, 2610 (2003), pp. 414-423.

[76] E. Sapin, O. Bailleux, J.J. Chabrier, *Research of complex forms in the cellular automata by evolutionary algorithms*, In EA03. Lecture Notes in Computer Science, 2936 (2003), pp. 357-367.

[77] E. Sapin, O. Bailleux, J.J. Chabrier, P. Collet, *A New Universal Cellular Automaton Discovered by Evolutionary Algorithms*, In GECCO04. Lecture Notes in Computer Science, 3102 (2004), pp. 175-187.

[78] E. Sapin, O. Bailleux, J. Chabrier, P. Collet, *Demonstration of the Universality of a New Cellular Automaton*, International Journal of Unconventional Computing 3 (2007), pp. 79-103.

[79] E. Sapin, O. Bailleux, J. Chabrier, *Research of complexity in cellular automata through evolutionary algorithms*, Complex Syst 17:3 (2007), pp. 231-241.

[80] E. Sapin, L. Bull, A. Adamatzky, *A genetic approach to search for glider guns in cellular automata*, Proceedings of the IEEE Congress on Evolutionary Computation. IEEE Press (2007), pp. 2456-2462.

[81] E. Sapin, L. Bull, *Searching for Glider Guns in Cellular Automata: Exploring Evolutionary and Other Techniques*, In N. Monmarche et al. (eds) Artificial Evolution: Proceedings of the 8th International Conference on Evolution Artificielle. Springer (2007), pp. 255-265.

[82] V.B. Sapozhnikov, E. Foufoula-Georgiou, *Experimental evidence of dynamic scaling and indications of self-organized criticality in braided rivers*, Water Resources Research, 33, 8 (1997), pp. 1983-1991.

[83] R. Serra, M. Villani, *Perturbing the regular topology of cellular automata: Implications for the dynamics*, Proceedings of the 5th International Conference on Cellular Automata for Research and Industry (2002), pp. 168-177.

[84] B. Schonfisch, A. de Roos, *Synchronous and asynchronous updating in cellular automata*, BioSystems 51 (1999), pp. 123-143.

[85] T. Tofoli, *Cellular Automata as an Alternative to (rather than an approximation of) Diferential Equations in Modeling Physics*, Physica D, 10 (1984), pp. 117-127.

[86] M. Tomassini, M. Giacobini, C. Darabos, *Evolution of small-world networks of automata for computation*, Lecture Notes in Computer Science, Proceedings of Parallel Problem Solving from Nature, PPSN 04, 3102 (2004), pp. 672 - 681.

[87] M. Tomassini, M. Giacobini, C. Darabos, *Evolution and dynamics of small-world cellular automata*, Advances in Complex Systems, 15, 4 (2005), pp. 261 - 284.
[88] T. de Wolf, T. Holvoet, *Emergence versus self-organisation: Different concepts but promising when combined* Springer-Verlag (2005), pp. 1-15.
[89] S. Wolfram, *Undecidability and Intractability in Theoretical Physics*, Phys. Rev. Lett., 54 (1985), pp. 735-738.
[90] S. Wolfram, *Theory and Applications of Cellular Automata*, World Scientific Press, Singapore 1986.
[91] S. Wolfram, *A new kind of science*, Champaign, IL: Wolfram Media, Inc., 2002.
[92] A. Wuensche, *Self-reproduction by glider collisions; the beehive rule*, In: Pollack et al., editors. Int J. Cambridge: MIT Press (2004), pp. 286-291.
[93] R. Wuertz, J. Branke, H. Schmeck, *Evolutionary design of emergent behavior*, In Organic Computing, volume 21 of Understanding Complex Systems, Springer Berlin / Heidelberg (2008), pp. 123-140.
[94] R.P. Wuertz, *Organic Computing*, Springer (2008).
[95] H.F. Zhang, R.X. Wu, X.C. Fu, *The emergence of chaos in complex dynamical networks*, Chaos, Solitons and Fractals, 28, 2, (2006), pp. 472-479.
[96] http://demonstrations.wolfram.com/SnowflakeLikePatterns/

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