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EVOLVING NETWORK TOPOLOGIES FOR CELLULAR AUTOMATA

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Abstract. The problem of evolving network topologies for celular automata has been approached by means of circular evolutionary algorithms. This application is based on Watts proposal to consider small-world topologies for CAs. He has shown that small-world networks could give a better performance for problems like the density task, compared to the performance obtained when considering regular lattices for CAs. The circular evolutionary algorithm proposed in this paper has been successfully applied for evolving network topologies for the density task.

Key words: Cellular automata, Heuristic methods, Evolutionary optimization

1. Introduction

A new class of evolutionary techniques called Circular Evolutionary Algorithms (CEA) is proposed. The main feature of these evolutionary algorithms is a new selection scheme according to which each individual is recombined. The philosophy behind this new model is a gradual propagation of the fittest genetic material into the population. This goal is achieved by considering and interpreting both a time dimension and a space dimension for the algorithm.

CEA selection and recombination take place asynchronously, which allows an improvement of the individuals during the process of selection and recombination in one generation. The circular settlement of all the individuals from the population according to their fitness allows us to define a new notion of neighborhood, recombination taking place only between individuals belonging to the same neighborhood. The problem of evolving networks topologies for cellular automata is addressed by using the proposed model.

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1998 *CR Categories and Descriptors.* I.2.8 [**Artificial Intelligence**]: Problem solving, Control methods, and Search – *Heuristic methods*; F.1.1 [**Computation by Abstract Devices**]: Models of Computation – *Automata*.

Numerical experiments reported in this paper are just preliminary results referring to the performance of obtained networks. Their study and classification is the subject of future work.

The paper is organized as follows. The new circular search model is described in the second section. The problem of evolving network topologies for cellular automata and existing methods are described in the third section. Results obtained after applying a circular evolutionary algorithm for this problem are presented in the fourth section. Conclusions are presented in the last section of the paper.

2. Circular Search Model

A new evolutionary model is proposed in what follows. A new way of understanding the role of the selection process is the foundation of this model. A new population topology and an asynchronous application of the search operators are the main features that arise from this new philosophy of selecting individuals for recombination. The aim of the proposed technique is to ensure a good exploitation of the good genetic material already obtained by the search process, but in the same time to allow the increase of diversity in the population. This aim is achieved by transferring to all individuals from the population genetic material that is believed to be relevant for the search process in a step by step manner that will be exhaustively explained in what follows.

Let us suppose that $P(t)$ is the current population at the time step t. The size of the population is fixed during all stages of the algorithm and is chosen to be a square number, in order to allow a certain topology of the population. Let n^2 be the size of the population (n is an even number). The algorithm ends after a certain number of generations, given as parameter of the algorithm.

2.1. **Space Dimension.** All the individuals from the population are sorted according to their fitness relative to the problem to solve. They will be distributed over $\frac{n}{2}$ concentric circles following the next constraint: the fittest individuals will be placed on the smallest circle, while the less fit individuals will be placed on the larger circle. Moreover, the number of individuals placed on circle $i (i = 0, \frac{n}{2} - 2)$ is $4(n - 2i - 1)$. This means that the individuals belonging to the concentric circles can be easily transposed into a two-dimensional grid. Figure 1 describes proposed topology using both concentric circles and the corresponding two-dimensional grid.

Let us suppose that we obtain the sorted population $P(t) = x_1, x_2, \ldots, x_{n^2}$, where x_1 is the fittest individual and x_{n^2} is the worst individual in the population. On the smallest circle are placed the fittest four individuals (x_1, x_2, x_3, x_4) from the population (their order does not matter). The next circle will hold

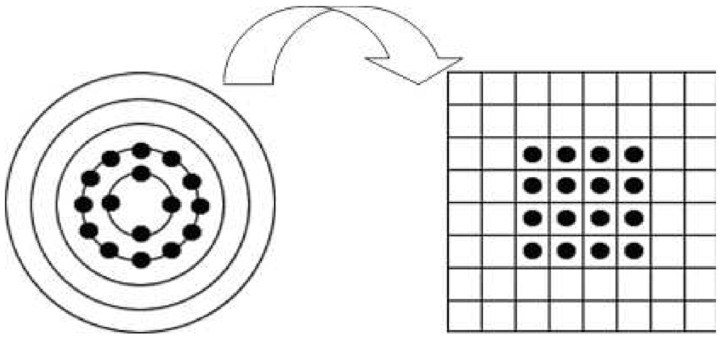

FIGURE 1. Concentric circles topology of the population and the corresponding two-dimensional grid topology

12 individuals $(x_5, \ldots, x_{16})$, the individuals with the next best fitness values. The largest circle will have the less fit individuals from the population, and their number depends on the size of the population.

First of all, the individuals from the smallest circle (the fittest individuals of the population) will always be copied in the next population just as they are. This elitist choice is very suitable especially for algorithms that are using a relative fitness that is slightly different for each generation, because copying the best individuals in the next generation will mean that these individuals will be then tested again but using a different fitness function and they will survive only if they have a very good quality in this generation as well.

Each individual from the population will get the chance of being improved by involving it in a recombination process. The diversity will be thus increased, because considering each individual for recombination means to use genetic material of both very fit individuals and less fit individuals. The selection scheme will therefore decide the second parent involved in each recombination, and this is where the exploitation of the search space is pursued.

Therefore, for each individual except the best four that will be copied in the next generation, the selection scheme will choose its mate in the following way. Let us number the concentric circles on which the individuals are placed, so that the most exterior circle will have the value 0, and the most interior circle will have the biggest value. For a population size of n^2 (n even number), we will obtain $\frac{n}{2}$ circles, therefore the value $\frac{n}{2} - 1$ will be assigned to the most interior circle. For one individual belonging to circle $i, i = 0, \frac{n}{2} - 2$, we will always choose a mate from the circle $i + 1$. Because of the way individuals are placed on the circles, according to their fitness, this means that each individual from the population will be recombined with a better individual, but still close to it regarding the fitness value. This means that individuals from the smallest circle, even if they are not directly involved in recombination, will be chosen as mates for the individuals belonging to circle $\frac{n}{2} - 2$. We therefore have a so-called local selection that refers to the fact that individuals are selected only from a certain circle.

The local selection is done by using one of the existing selection operators like proportional selection, tournament selection and so on. A tournament selection scheme (Dumitrescu, 2000) is considered for all the experiments performed in this paper.

Once we have selected a pair of individuals, they will be recombined by using an existing recombination scheme, depending on their encoding (Back, 1997).

2.2. **Time Dimension.** The entire process described before takes place asynchronously, which is another distinctive and strong feature of the proposed search scheme. Both selection and recombination are done asynchronously. First, individuals from the circle $\frac{n}{2} - 2$ are considered for recombination. For each of them, an individual from the circle $\frac{n}{2} - 1$ is chosen according to a local selection and the two individuals will be recombined. The best offspring obtained after recombination will be mutated and the resulting individual will be accepted only if it has a better quality. The offspring, mutated or not, will then replace the first parent if it has a better quality. The elitist scheme that allows only better individuals to replace the first parents is counteracted by the fact that all individuals from the population are involved in recombination.

From the improved individuals of the circle $\frac{n}{2} - 2$ we will then choose mates for individuals belonging to the circle $\frac{n}{2} - 3$, according to the same local selection scheme. The process that results from the described scheme is a propagation process where the good genetic material of the fittest individuals will be first transferred to the closest fit individuals, and they will transfer it, together with their good genetic material, to the next fit individuals, and so on, from close to close, until the good genetic material collected from the entire population will reach the less fit individuals from the population.

2.3. **Circular Evolutionary Algorithm.** The algorithm that results from the proposed search scheme is called Circular Evolutionary Algorithm and is described in what follows.

Circular Evolutionary Algorithm
begin
    $t := 0$
    Initialize $P(t)$
    while (not stop-criteria) do
    begin

```
        Evaluate P(t)
        CircularSort P(t)
        for each circle c (c := n/2 − 2, 0)
        begin
            for each individual i from c
            begin
                j :=LocalSelection(c-1)
                k :=Recombination (i, j)
                Mutation(k)
            end
        end
        t := t + 1
    end
end
```

3. Evolving Network Topologies for Cellular Automata

The density-classification task is a prototypical distributed computational task for CAs defined as follows. We denote by ρ_0 the fraction (the density) of 1s in the initial configuration. The task requires deciding whether $\rho_0 > \frac{1}{2}$. If so, then the CA must go to a fixed-point configuration of 1s, otherwise it must go to a fixed-point configuration of 0s. The lattice size is chosen to be odd in order to avoid the case $\rho_0 = \frac{1}{2}$. Because finding the density of the initial configuration is a global task, and CA only relies on local interactions, this task is not trivial.

Due to the similarities between the ring lattice where each cell is linked to its r neighbors on each side and a graph where each node is connected to a limited number of nodes, even if not in the topological neighborhood, Watts proposed the use of a small-world graph instead of a ring lattice for CAs (Watts, 1999). He computed the performance of hand-constructed small-world graphs for the density task, and he obtained performance values bigger than 0.8, while the best performances of a cellular automaton based on a ring lattice topology were around 0.76. In order to obtain a different topology, he fixed the rule to a majority rule which states that a node will receive the state of the majority of its neighbors in the graph. Therefore, the problem that arises from Watts proposal is to evolve small-world networks topologies for the density task of CAs.

Besides the hand-constructed small-world networks proposed in (Watts, 1999), an evolutionary technique for evolving small-world networks for the density task has been proposed in (Tomassini, 2005). The authors used a cellular evolutionary algorithm (Alba, 2002) and they obtained topologies with a performance around 0.8, similar to the hand-constructed small-world networks of Watts. Moreover, this performance was obtained in most of the runs of the algorithm, while a good performance of ring lattice topology is difficult to obtain. When evolving small-world networks, they have started both from regular lattices and from random networks, and have studied the results obtained for both cases.

4. Detecting network configuration for density task using CEA

The proposed circular search model is applied for evolving network topologies for cellular automata, for the density task. The resulting algorithm is called Circular Evolution of Network Topologies (CENTA) and is described in what follows.

Encoding and Population Model

A potential solution of the problem represents an undirected graph describing the network topology. A two-dimensional grid is used to encode it. The fixed number of individuals from a population are distributed over the two-dimensional square grid. An array of integers represents all the nodes of the graph, and for each node we have an array of nodes connected to it.

The initial population consists of randomly generated regular lattices of size N =149, with a radius of 3, meaning that each node is connected to 3 nodes on both sides. One node in a graph can have a maximum of max connections, max being a parameter of the algorithm. The set of initial configurations is generated anew for each generation of the algorithm.

Fitness Assignment

The fitness function is a real-valued function $f : X \to [0, 1]$, where X denotes the search space of the problem. $f(x)$ represents the fraction of correct classifications over 100 initial configurations randomly generated but with a uniformly distributed density (Das, 1994).

Selection Operator

For each individual belonging to circle $i(i = 0, \frac{n}{2} - 2)$ a mate will be chosen from the circle $i + 1$. Because of the way individuals are placed on the circles, according to their fitness, this means that each individual from the population will be recombined with a better individual, but still close to it regarding the fitness value. The local selection used for choosing a mate from the circle $i + 1$ is a tournament scheme with a tournament size of $2(n - 2i - 1)$, where $4(n - 2i - 1)$ represents the number of individuals that belong to the circle $i(i = 0, \frac{n}{2} - 2)$. The selection for recombination is performed asynchronously, starting with the individuals belonging to circle $\frac{n}{2} - 2$ and continuing until we select mates for the individuals belonging to circle 0.

Recombination Operator

Once we have selected two individuals for recombination, a two-point crossover is used for our experiments. We start with the recombination of the fittest individuals from the population, thus giving them the opportunity to improve their fitness before they will be recombined with less fit individuals.

Mutation Operator

The individual resulted after each recombination will be mutated similar to the mutation proposed in (Tomassini, 2005), only that they consider a different scheme of choosing the individuals that will be subject of mutation. Each node of a graph that represents a possible solution for the problem will be mutated with a certain probability, parameter of the algorithm. For a node chosen for mutation we will either add or remove a link to another randomly chosen node, with a given probability.

Selection for Replacement and Survival

The replacement of the first parent with the best offspring obtained after recombination and mutation takes place asynchronously, due to the asynchronous selection and recombination scheme. The offspring will replace the first parent only if it has a better fitness.

The circular evolutionary algorithm is applied for evolving network topologies for CAs, for the density task. The parameters of the algorithm are written in Table 1.

TABLE 1. CENTA algorithm parameters

Population size	**100**
Probability of mutation	**0.5**
Max	**30**
Probability of adding a new link to node	**0.5**
Number of generations	**100**

The algorithm successfully evolves, in most of the runs, networks with performances around 0.8 for the density task. These results confirm the hypothesis of Watts regarding the fact that network topologies seem to be a better environment for local interactions that lead to a global behavior for the density task. On the other hand, the results could be interpreted as an indicator for the efficiency of the new proposed evolutionary technique.

Future work will investigate several static structural properties of obtained networks, such as degree distribution, clustering coefficient and average path length. The results will indicate the nature of evolved networks.

5. Conclusions

A new evolutionary search model has been proposed in this paper. The main features of the proposed model are a new population topology, which is distributed over concentric circles, according to the fitness of the individuals and an asynchronous selection and recombination of the individuals, which allows involving in recombination individuals that improve their quality, their adaptation to the environment from close to close.

The algorithm is applied for evolving network topologies for cellular automata, for the density task. The results obtained have been compared with the results reported by the authors of other techniques for these problems, and they can be considered as a proof for the efficiency of the proposed circular evolutionary model. The study of obtained networks will be the subject of a future paper.

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