STUDIA UNIV. BABEŞ–BOLYAI, INFORMATICA, Volume **LII**, Number 2, 2007

GENERALIZED CYLINDERS SURFACES

L. ŢÂMBULEA AND I. GÂNSCA

Abstract. A generalized cylinder surface is generated by moving a 2D continuous curve along a 3D regular spine curve; the generating curve could be scaled and rotated around the spine curve. The shape of generalized cylinder surface induced by the scale functions and the angular velocity of rotation as well as some integral properties are discussed.

Keywords: Generalized cylinders; Generalized cylinder surfaces; Shape; Cusp; Rotation; Angular velocity; Integral properties

1. Motivation

Many industrial and artistic objects can be modelled with the aid of generalized cylinder surfaces. Theoretical and practical investigations in this area have been done by Lee and Requicha [7], Shani and Ballard [9], Bronsvoort and Warts [1], van der Helm, Ebell and Bronsvoort [6], Maekawa, Patrikalakis, Sakkalis and Yu [8], Gansca, Bronsvoort, Coman and Ţâmbulea [5] and others. The paper has the following structure. In Section 2 we recall the vector equation of a generalized cylinder surface. The shape of a generalized cylinder surface induced by the shapes of the scale functions is revealed in Section 3. Section 4 contains generalized cylinder surfaces generated by a scaled curve which makes rotations around the spine curve. Some integral properties of these twisted generalized cylinder surfaces and twisted generalized cylinders (objects) are given in Section 5.

2. Vector equation of a generalized cylinder surface

A generalized cylinder surface is generated by a continuous 2D curve which moves along a 3D regular spine (guide) curve, the plane of curve being perpendicular to the spine curve. The generating curve is referred to the local coordinate system X, Y, situated on the unit principal normal and binomial, respectively, vectors of the spine curve, see Fig.1.

Let us consider that the vector position of an arbitrary point of the spine curve is $\mathbf{C}(u), u \in [a, b]$ and the vector position of an arbitrary point of

Received by the editors: October 1, 2007.

2000 *Mathematics Subject Classification.* 65D18, 68U05, 68U07.

1998 *CR Categories and Descriptors.* J.6 [**Computer Applications**]: Computer-Aided Engineering – *Computer-aided design.*

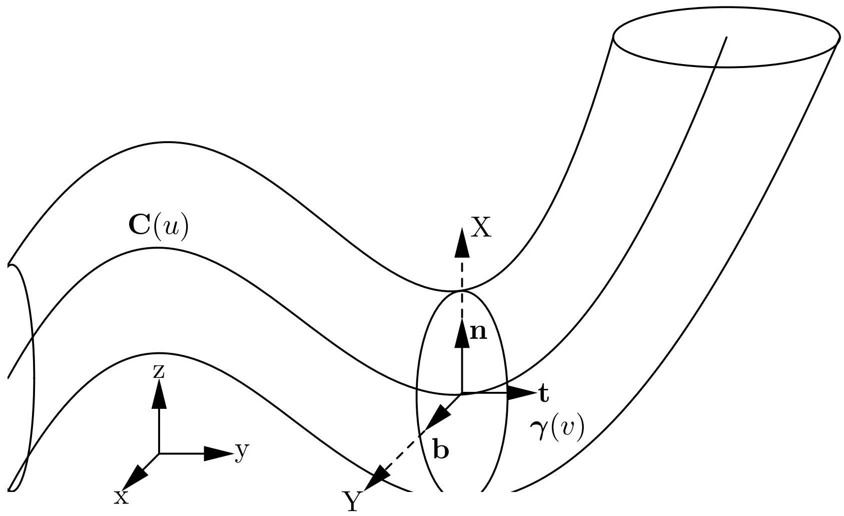


FIGURE 1. A generalized cylinder surface with an intermediary position of the generating curve $\boldsymbol{\gamma}(v)$.

the generating curve, with respect to the X, Y coordinate system, is $\boldsymbol{\gamma}(v) = (\varphi(v), \psi(v))^T$, $v \in [c, d]$. If the generating curve $\boldsymbol{\gamma}(v)$ is scaled into the directions of $\mathbf{n}(u)$ and $\mathbf{b}(u)$ with the aid of the positive and continuous scalar functions $s_1(u)$ and $s_2(u)$, respectively, $u \in [a, b]$, then, from Fig.1, one deduces the following vector equation of the generalized cylinder surface

$$\mathbf{\Gamma}(u, v) = \mathbf{C}(u) + s_1(u)\varphi(v)\mathbf{n}(u) + s_2(u)\psi(v)\mathbf{b}(u), \quad (u, v) \in D, \tag{1}$$

where $D = [a, b] \times [c, d]$.

The unit vectors $\mathbf{t}(u), \mathbf{n}(u)$ and $\mathbf{b}(u)$ form the Frenet trihedrom and are given by the formulas

$$\mathbf{t}(u) = \frac{\mathbf{C}'(u)}{|\mathbf{C}'(u)|}, \quad \mathbf{b}(u) = \frac{\mathbf{C}'(u) \times \mathbf{C}''(u)}{|\mathbf{C}'(u) \times \mathbf{C}''(u)|}, \quad \text{and} \quad \mathbf{n}(u) = \mathbf{b}(u) \times \mathbf{t}(u). \tag{2}$$

Remark 1. From (2) results that the vector function $\mathbf{C}(u)$ must be of the second order continuity class.

In what follows we will firstly focus on the $\mathbf{\Gamma}(u, v)$ surface shape control with the aid of scale functions $s_1(u)$ and $s_2(u)$. Next we will deduce the vector equation of the generalized cylinder surface resulted by rotation of the scaled generating curve $\boldsymbol{\gamma}_s(v; u) = (s_1(u)\varphi(v), s_2(u)\psi(v))^T$ around the spine curve $\mathbf{C}(u)$, with a variable angular velocity.

3. Shape of $\mathbf{\Gamma}(u,v)$ induced by the shapes of $s_1(u)$ and $s_2(u)$

Information about the shape of $\mathbf{\Gamma}(u,v)$ one obtains analysing its coordinate lines $\mathbf{\Gamma}(u=\text{const},v), v\in[c,d]$ and $\mathbf{\Gamma}(u,v=\text{const}), u\in[a,b]$, respectively. From the vector equation (1) we observe that the coordinate line $\mathbf{\Gamma}(u,v=\text{const}), u\in[a,b]$ is, in fact, the scalled generating curve $\boldsymbol{\gamma}_s(v;u), u=\text{const}$ relative to the $xOyz$ coordinate system.

Important information about the shape of coordinate line $\mathbf{\Gamma}(u,v=$const), $u\in[a,b]$ results from its tangent vector $\mathbf{\Gamma}_u(u,v)$. From (1), taking into account the Frenet-Serret formulas,

$$\begin{aligned}\mathbf{t}'(u) &= \mathcal{K}(u)|\mathbf{C}'(u)|\mathbf{n}(u),\\ \mathbf{n}'(u) &= |\mathbf{C}'(u)|[-\mathcal{K}(u)\mathbf{t}(u)+\mathcal{T}(u)\mathbf{b}(u)],\\ \mathbf{b}'(u) &= -\mathcal{T}(u)|\mathbf{C}'(u)|\mathbf{n}(u),\end{aligned}$$

one obtains

$$\begin{aligned}\mathbf{\Gamma}_u(u,v) = & |\mathbf{C}'(u)|\left[1-\mathcal{K}(u)s_1(u)\varphi(v)\right]\mathbf{t}(u)+\\ &+\left[s_1'(u)\varphi(v)-\mathcal{T}(u)s_2(u)|\mathbf{C}'(u)|\psi(v)\right]\mathbf{n}(u)+\\ &+\left[s_2'(u)\psi(v)+\mathcal{T}(u)s_1(u)|\mathbf{C}'(u)|\varphi(v)\right]\mathbf{b}(u),\end{aligned} \tag{3}$$

where $\mathcal{K}(u)>0$ and $\mathcal{T}(u)$ are the curvature and torsion, respectively, of the spine curve $\mathbf{C}(u)$, and are given by the formulas

$$\mathcal{K}(u)=\frac{|\mathbf{C}'(u)\times\mathbf{C}''(u)|}{|\mathbf{C}'(u)|^3} \quad\text{and}\quad \mathcal{T}(u)=\frac{(\mathbf{C}'(u)\times\mathbf{C}''(u))\,\mathbf{C}'''(u)}{|\mathbf{C}'(u)\times\mathbf{C}''(u)|^2}.$$

Next we recall

Definition. An interior point of a curve $\mathbf{g}(t), t\in I, I\subset\Re$, say $\mathbf{g}(t_0)$, is called a cusp of $\mathbf{g}(t)$ if

$$\lim_{t\to t_0^-}\mathbf{g}'(t)=-\lim_{t\to t_0^+}\mathbf{g}'(t). \tag{4}$$

Remark 2. In the special case when $\mathbf{g}(t)=(t,f(t))^T, t\in I$, the interior point $\mathbf{g}(t_0)$ is a cusp of $\mathbf{g}(t)$ if and only if

$$\lim_{t\to t_0^-}f(t)=-\lim_{t\to t_0^+}f(t)=\infty,(or-\infty). \tag{5}$$

With other words, the interior point $t_0\in I$ is a cusp of the scalar function $f(t)$, if and only if (5) holds.

Regarding to an arbitrary coordinate line $\mathbf{\Gamma}(u,v_0), v_0\in[c,d]$ we will prove the following

Proposition 1. *If $s_1(u)$ and $s_2(u)$ have cusps for $u = u_0$ and*

$$\lim_{u\to u_0} = \frac{s_1^{'}(u)}{s_2^{'}(u)} = m, \tag{6}$$

then the coordinate line $\mathbf{\Gamma}(u, v_0)$ does have cusp for $u = u_0$ in the direction of the vector

$$\varphi(v_0)\mathbf{n}(u_0) + m\psi(v_0)\mathbf{b}(u_0), \tag{7}$$

provided that $|\boldsymbol{\gamma}(v_0)| \neq 0$.

Proof. Let us consider a vicinity of u_0, say V_0, such that $s_1^{'}(u) \neq 0$, if $u \in V_0$. Similar reasoning one does if $s_2^{'}(u) \neq 0, u \in V_0$. From (3), if $u \in V_0$, we can write

$$\begin{aligned}\mathbf{\Gamma}_u(u, v_0) = \;& s_1^{'}(u)\left\{\varphi(v_0)\mathbf{n}(u) + \tfrac{s_2^{'}(u)}{s_1^{'}(u)}\psi(v_0)\mathbf{b}(u) + \right.\\ & + \tfrac{|\mathbf{C}^{'}(u)|}{s_1^{'}(u)}\left[(1 - \mathcal{K}(u)s_1(u)\psi(v_0))\,\mathbf{t}(u) - \mathcal{T}(u)s_2(u)\psi(v_0)\mathbf{n}(u) + \right.\\ & \left.\left. + \;\mathcal{T}(u)s_1(u)\varphi(v_0)\mathbf{b}(u)\right]\right\}.\end{aligned}$$

If the scalar functions s_1 and s_2 do have cusps in u_0, then, taking into account Remark 2, (5) and (6) results

$$\lim_{u\to u_0^-} \mathbf{\Gamma}_u(u, v_0) = -\lim_{u\to u_0^+} \mathbf{\Gamma}_u(u, v_0),$$

and the direction of $\mathbf{\Gamma}_u(u, v_0)$, when $u \to u_0$, approaches to the direction of vector given at (7).

Remark 3. If $\varphi(v_0) = 0$ and $\psi(v_0) \neq 0$, then from (7) results that the cusp of coordinate line $\mathbf{\Gamma}(u, v_0)$, in $u = u_0$, is in the direction of $\mathbf{b}(u_0)$. Analogously, if $\varphi(v_0) \neq 0$ and $\psi(v_0) = 0$, then the cusp of $\mathbf{\Gamma}(u, v_0)$, in $u = u_0$, is in the direction of $\mathbf{n}(u_0)$.

Figures 2 (a), (b) and Figure 3 illustrate this theoretical part. The generalized cylinder surface from Figure 3 has the spine curve

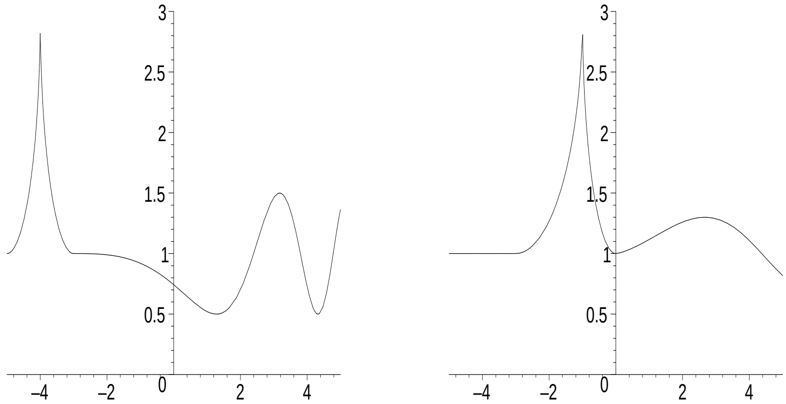
$$\mathbf{C}(u, a^*) = (u, a^* u^2, 0)^T, u \in [-5, 5], \quad a^* = 0.05,$$

and the generating curve with

$$\varphi(v) = cos(v),\ \psi(v) = sin(v),$$

$$s_1(u) = s((u + 5)/10, 0.1, 1, 0.1, 2, 0.5, 20, 3, \frac{\pi}{2}),$$

$$s_2(u) = s((u + 5)/10, 0.4, 1, 0.2, 0.1, 2, -0.3, 10, 1.4, \frac{\pi}{2}), \quad u \in [-5, 5],$$



(a) (b)

FIGURE 2. Cusps of $s_1(u)$ and $s_2(u)$.

where $s(t, u_0, a, b, c, d, e, p, q, r) =$

$$\begin{cases} a, & t \in [0, u_0 - b], \\ a + d - \frac{d}{b}\sqrt{b^2 - (t - u_0 + b)^2}, & t \in (u_0 - b, u_0], \\ a + d - \frac{d}{c}\sqrt{c^2 - (t - u_0 - c)^2}, & t \in (u_0, u_0 + c], \\ a + e \cdot cos(p(t - u_0 - c)^q + r), & t \in (u_0 + c, 1]. \end{cases}$$

Figures 2 (a) and (b) present the cusps of $s_1(u)$ and $s_2(u)$ respectively, which determine the cusps to the coordinate lines $\mathbf{\Gamma}(u, v = \text{const})$, shown in Figure 3.

4. Rotation of the scaled generating curve

In what follows we consider that the scaled generating curve

$$\boldsymbol{\gamma}_s(v; u) = (s_1(u)\varphi(v), s_2(u)\psi(v))^T, \ v \in [c, d],$$

makes rotations around the spine curve $\mathbf{C}(u)$, with angular velocity $\omega = \omega(u), u \in [a, b]$, while it moves along $\mathbf{C}(u)$.

The angle of rotation between the initial and an intermediary position of $\boldsymbol{\gamma}_s(v; u)$, if $\omega(u) \geq 0$ (or $\omega(u) \leq 0$) is

$$\alpha(u) = \int_a^u \omega(t)dt. \tag{8}$$

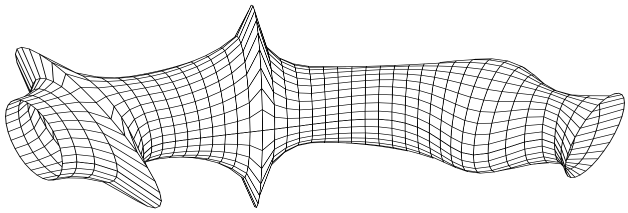


FIGURE 3. Cusps of coordinate lines $\mathbf{\Gamma}(u, v = \text{const})$.

The twisted generalized cylinder surface, in this case, is represented by the following vector equation

$$\begin{aligned} &\mathbf{\Gamma}_1(u,v) = \mathbf{C}(u)+ \\ &+\left[s_1(u)\varphi(v)cos(\alpha(u)+\alpha_0)+s_2(u)\psi(v)sin(\alpha(u)+\alpha_0)\right]\mathbf{n}(u)+ \\ &+\left[-s_1(u)\varphi(v)sin(\alpha(u)+\alpha_0)+s_2(u)\psi(v)cos(\alpha(u)+\alpha_0)\right]\mathbf{b}(u), \end{aligned} \tag{9}$$

$(u, v) \in D$; α_0 is the angle of $\boldsymbol{\gamma}_s(v, u)$-rotation around $\mathbf{C}(u)$, before the starting generation of $\mathbf{\Gamma}_1(u, v)$.

The number of rotations, if $\omega(u) \geq 0$ (or $\omega \leq 0$), when $u \in [0, u^*]$ is

$$n^* = \frac{|\alpha(u^*)|}{2\pi}, \tag{10}$$

where $\alpha(u^*)$ is given by the formula (8).

For example, if the angular velocity $\omega = k|u - u_0|^\beta, u \in [0, 1]$, where the parameter $u_0 \in [0, 1]$ and α, β are real and positive numbers, using formula (10), one obtains

$$\alpha(u) = \begin{cases} \frac{k}{\beta+1}\left[u_0^{\beta+1} - (u_0 - u)^{\beta+1}\right], & 0 \leq u \leq u_0, \\ \\ \frac{k}{\beta+1}\left[u_0^{\beta+1} + (u - u_0)^{\beta+1}\right], & u_0 \leq u \leq 1. \end{cases} \tag{11}$$

Denoting by ν the rotations number of curve $\boldsymbol{\gamma}_s(v; u)$ around the spine curve $\mathbf{C}(u)$, when $u \in [0, 1]$ then, using formulas (10) and (11), results

$$\nu = \frac{k}{2(\beta+1)\pi}\left[u_0^{\beta+1} + (1 - u_0)^{\beta+1}\right].$$

Therefore, if one wants ν rotations then, the angular velocity must be

$$\omega(u)=\frac{2\pi\nu(\beta+1)}{u_0^{\beta+1}+(1-u_0)^{\beta+1}}|u-u_0|^{\beta},\ \ u\in[0,1]. \tag{12}$$

Corresponding to this angular velocity, the angle of rotation is

$$\alpha(u)=\begin{cases}\frac{2\pi\nu}{u_0^{\beta+1}+(1-u_0)^{\beta+1}}\left[u_0^{\beta+1}-(u_0-u)^{\beta+1}\right], & 0\le u\le u_0,\\ \frac{2\pi\nu}{u_0^{\beta+1}+(1-u_0)^{\beta+1}}\left[u_0^{\beta+1}+(u-u_0)^{\beta+1}\right], & u_0\le u\le 1.\end{cases} \tag{13}$$

In Figs. 4 and 5 are presented two particular twisted cylinder surfaces $\mathbf{\Gamma}_1(u,v)$ of equation (9), for which

$$\mathbf{C}(u)=\sum_{i=0}^{6}\mathbf{b}_iB_i^6(u),\ u\in[0,1], \tag{14}$$

where $B_i^6(u)=\binom{6}{i}(1-u)^{6-i}u^i$, $\mathbf{b}_0=(3,0,9)$, $\mathbf{b}_1=(8,1,5)$, $\mathbf{b}_2=(11,9,2)$, $\mathbf{b}_3=(13,25,0)$, $\mathbf{b}_4=(7,29,2)$, $\mathbf{b}_5=(3,26,6)$, $\mathbf{b}_6=(0,23,11)$ and the angle of rotation is of the form (13). Fig.4 corresponds to

$$\begin{array}{l}\varphi(v)=cos(v),\ \psi(v)=sin(v),\ v\in[0,2\pi],\\ s_1(u)=1+0.5sin(12u),\ s_2(u)=1/s_1(u),\ u\in[0,1],\\ \nu=1.5,\ \beta=0.01,\ u_0=0.1\end{array} \tag{15}$$

and the defining elements of Fig.5 are

$$\begin{array}{l}\varphi(v)=4cos^3(v),\ \psi(v)=4sin^3(v),\ v\in[0,2\pi],\\ s_1(u)=s_2(u)=1,\ u\in[0,1],\\ \nu=0.5,\ \beta=0.2,\ u_0=0.3.\end{array} \tag{16}$$

5. Some integral properties

Firstly we recall that a generalized cylinder (solid) is the body bounded by a generalized cylinder surface and two planes perpendicular to the spline curve in its initial and final points. Next we will give some formulas regarding twisted generalized cylinder surfaces and twisted generalized cylinder, without self-intersections.

Throughout this section we will make use of the

Remark 4. Rotations and other maps, characterized by orthonormal matrices, leave lengths, areas and angles unchanged (Farin, 1990).

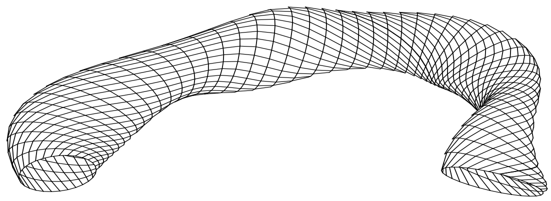


FIGURE 4. Twisted generalized cylinder surface $\mathbf{\Gamma}_1(u, v)$ corresponding to (14) and (15).

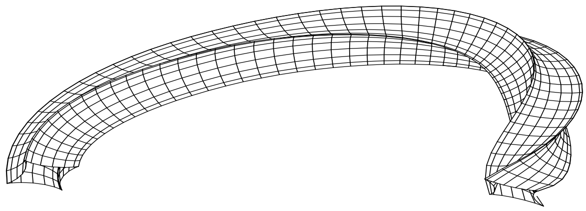


FIGURE 5. Twisted generalized cylinder surface $\mathbf{\Gamma}_1(u, v)$ corresponding to (14) and (16).

5.1. **Gravity center line and area of $\mathbf{\Gamma}_1(u, v)$.** Gravity center line of $\mathbf{\Gamma}_1(u, v)$ is evidently the locus of the gravity centers of the generating curve

$$\boldsymbol{\gamma}_r(v; u) = (x(v; u), y(v; u))^T ,$$

where

$x(v;u) = s_1(u)\varphi(v)cos(\alpha(u)+\alpha_0) + s_2(u)\psi(v)sin(\alpha(u)+\alpha_0),$
$y(v;u) = -s_1(u)\varphi(v)sin(\alpha(u)+\alpha_0) + s_2(u)\psi(v)cos(\alpha(u)+\alpha_0),\ v \in [c,d].$

Let $G\left(X_g(u), Y_g(u)\right)$ be the gravity center of the curve

$$\boldsymbol{\gamma}_s(v;u) = \left(s_1(u)\varphi(v), s_2(u)\psi(v)\right)^T.$$

The coordinates $X_g(u)$ and $Y_g(u)$ are given in our paper [5], by formulas (13) and (17). Denoting by $G_r\left(X_g^r(u), Y_g^r(u)\right)$ the gravity centre of the curve $\boldsymbol{\gamma}_r(v;u)$, in virtue of the Remark 4 we have,

$$(17)\qquad \begin{array}{rl} X_g^r(u) = & X_g(u)cos\left(\alpha(u)+\alpha_0\right) + Y_g sin\left(\alpha(u)+\alpha_0\right), \\ Y_g^r(u) = & -X_g(u)sin\left(\alpha(u)+\alpha_0\right) + Y_g cos\left(\alpha(u)+\alpha_0\right), \end{array}$$

Locating G_r with respect to the $xOyz$ coordinate system we have

$$(18)\qquad \mathbf{G}_r(u) = \mathbf{C}(u) + X_g^r(u)\mathbf{n}(u) + Y_g^r(u)\mathbf{b}(u);\ u \in [a,b].$$

If $\mathbf{C}(u) = \left(x(u), y(u), z(u)\right)^T$, $\mathbf{n}(u) = (a_1(u), a_2(u), a_3(u))$ and $\mathbf{b}(u) = (b_1(u), b_2(u), b_3(u))$, then, from (18) results

$$(19)\ \mathbf{G}_r(u) = \begin{pmatrix} x_g^r(u) \\ y_g^r(u) \\ z_g^r(u) \end{pmatrix} = \begin{pmatrix} x(u) + X_g^r(u)a_1(u) + Y_g^r(u)a_2(u) \\ y(u) + X_g^r(u)b_1(u) + Y_g^r(u)b_2(u) \\ z(u) + X_g^r(u)c_1(u) + Y_g^r(u)c_2(u) \end{pmatrix};$$

where $u \in [a,b]$, $X_g^r(u)$ and $Y_g^r(u)$ being given by (17).

Denoting by CG_r the locus of $G_r(u)$, when $u \in [a,b]$ results

Proposition 2. *The gravity center line CG_r of the surface $\boldsymbol{\Gamma}_1(u,v)$ has the parametric equations (19).*

If S_1 is the area of $\boldsymbol{\Gamma}_1(u,v)$, then, in virtue of the Remark 4 and formula (23) from our paper (2002) results

$$(20)\ \ S_1 = \int_{CG_r} \mathcal{L}(u)ds = \int_a^b \sqrt{\left(x_g^r(u)\right)'^2 + \left(y_g^r(u)\right)'^2 + \left(z_g^r(u)\right)'^2}du.$$

Next we denote by $\mathcal{V}_1$ the twisted generalized cylinder bounded by $\boldsymbol{\Gamma}_1(u,v)$ and the perpendicular planes to the spine curve in its initial and final points.

5.2. **Gravity center line and volume of $\mathcal{V}_1$.** In our paper [5] we have established (formulas (23) and (24)) that if $G^0\left(X_G^0, Y_G^0\right)$ is the gravity centre of the domain bounded by the closed curve $\boldsymbol{\gamma}(v) = \left(\varphi(v), \psi(v)\right)^T, v \in [c,d], \boldsymbol{\gamma}(c) = \boldsymbol{\gamma}(d)$, then the gravity centre of the domain bounded by the curve $\boldsymbol{\gamma}_s(v;u) = \left(s_1(u)\varphi(v), s_2(u)\psi(v)\right)^T, v \in [c,d]$ is $G^*\left(X_G^*, Y_G^*\right)$, where

$$(21)\qquad \begin{array}{l} X_g^*(u) = s_1(u)X_G^0, \\ Y_g^*(u) = s_2(u)Y_G^0. \end{array}$$

Now, if $\boldsymbol{\gamma}_s(v;u)$ makes rotations around the spine curve, with the angular velocity $\omega(u)$, then its gravity centre becomes $G_r^*\left(X_r^*, Y_r^*\right)$, where

$$(22)\quad \begin{array}{l} X_r^*(u) = s_1(u)X_G^0 cos\left(\alpha(u)+\alpha_0\right) + s_2(u)Y_G^0 sin\left(\alpha(u)+\alpha_0\right), \\ Y_r^*(u) = -s_1(u)X_G^0 sin\left(\alpha(u)+\alpha_0\right) + s_2(u)Y_G^0 cos\left(\alpha(u)+\alpha_0\right), \end{array}$$

where $\alpha(u)$ is given by the formula (8).

Denoting by CG_r^* the locus of $G_r^*\left(X_r^*(u), Y_r^*(u)\right)$ when $u \in [a,b]$ and proceedings as before, we can state

Proposition 3. *The gravity center line CG_r of the generalized cylinder $\mathcal{V}_1$ has the following parametric equations*

$$(23)\quad \mathbf{G}_r^*(u) = \begin{pmatrix} x_r^*(u) \\ y_r^*(u) \\ z_r^*(u) \end{pmatrix} = \begin{pmatrix} x(u) + X_r^*(u)a_1(u) + Y_r^*(u)a_2(u) \\ y(u) + X_r^*(u)b_1(u) + Y_r^*(u)b_2(u) \\ z(u) + X_r(^*u)c_1(u) + Y_r^*(u)c_2(u) \end{pmatrix};$$

where $u \in [a,b]$.

With regard to the volume of $\mathcal{V}_1$, similary to the formula (26) from our paper [5], we have

$$(24)\quad \begin{array}{rl} \mathcal{V}_1 & = \int_{CG_r^*} \mathcal{A}(u)du = \\ & = \mathcal{A}_0 \int_a^b s_1(u)s_2(u)\sqrt{\left(x_r^*(u)\right)'^2 + \left(y_r^*(u)\right)'^2 + \left(z_r^*(u)\right)'^2}du. \end{array}$$

References

[1] Bronsvoort W. F. and Waarts J.J. (1992), A method for converting the surface of a generalized cylinder into a B-spline surface. Computers & Graphics, 16(2), 175–178.

[2] Bronsvoort W. F. (1992), A surface-scanning algorithm for displaying generalized cylinders. Visual Computer, 8(3), 162–170.

[3] de Voogt E., van der Helm A., Bronsvoort W. F. (2000), Ray tracing deformed generalized cylinders. Visual Computers, 16, 197–207.

[4] Farin G. (1990), *Curves and Surfaces for Computer Aided Geometric Design.* Academic Press, New York, Second Edition.

[5] Gansca I., Bronsvoort W. F., Coman Gh., Tambulea L. (2002), Self-intersection avoidance and integral properties of generalized cylinders. Computer Aided Geometric Design, 19, 695–707.

[6] van der Helm A., Ebell P. and Bronsvoort W. F. (1998), Modelling mollusc shells with generalized cylinders. Computers & Graphics 22(4), 505–513.

[7] Lee Y.T., Requicha A.A.G. (1982), Algorithms for computing the volume and other integral properties of solids. I. Known methods and open issues. Communications of the ACM 25(9), 635–641.

[8] Maekawa T., Patrikalakis N. M., Sakkalis T. and Yu G. (1998), Analysis and applications of pipe surfaces. Computer Aided Geometric Design, 15(5), 437–458.

[9] Shani U., Ballard D.H. (1984), Splines as embeddings for generalized cylinders. Computers Vision, Graphics and Image Processing, 27, 129–156.

Babes-Bolyai University, Faculty of Mathematics and Computer Science, 1 M. Kogălniceanu St., 400084 Cluj-Napoca, Romania

E-mail address: leon@cs.ubbcluj.ro

16/4 Muscel St., Cluj-Napoca, Romania