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AXIOMATIZATION OF CREDULOUS REASONING IN RATIONAL DEFAULT LOGIC

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ABSTRACT. Nonmonotonic reasoning is succesfully formalized by the class of default logics. In this paper we introduce an axiomatic system for credulous reasoning in rational default logic. Based on classical sequent calculus and anti-sequent calculus, an abstract characterization of credulous nonmonotonic default inference in this variant of default logic is presented.

Keywords: default logic, nonmonotonic reasoning, inference relation.

1. INTRODUCTION

Default logics formalize a particular type of nonmonotonic reasoning, called *default reasoning*. These logical systems have special inference rules, called *defaults* which permit drawing conclusions in the absence of complete information, using default assumptions. In default logics a set of facts is extended with new formulas, using the classical inference rules and the defaults obtaining *default extensions*. The elements of extensions are called non-monotonic theorems, or *beliefs*. The beliefs are only consistent formulas, not necessarily true and they can be later invalidated by adding new facts.

The versions (classical([13]), justified ([6]), constrained([14]), rational([11]) of the default logic, use different meanings of the default assumptions in the reasoning process.

The computational problems specific to default logics are:

Search problem: computing the extensions of a default theory, is $\sum_2^P -complete$.

Decision problems:

- Deciding whether a formula belongs to at least one extension of a default theory - *credulous* default reasoning, is $\sum_2^P -complete$.
- Deciding whether a formula belongs to all extensions of a default theory - *skeptical* default reasoning, is $\prod_2^P -complete$.

Due to their very high level of theoretical complexity, caused by the great power of the inferrential process, the above computational problems can be solved in an

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efficient manner only for particular classes of default theories.

Credulous reasoning *versus* skeptical reasoning

Accepting alternative possibilities for extending a default theory characterizes the *credulous reasoning*. The commonsense reasoning is the human model of reasoning, by making default assumptions for overcoming the lack of information. This type of reasoning belongs to the credoulous perspective of the reasoning.

Skeptical reasoning is imposed in prediction problems, because the nonmonotonic consequences cannot be later modified, which means that derived formulas does not depend on the alternative assumptions made during the reasoning process. It is considered irrational to have the possibility to chose one belief or another one if they are contradictory.

The specific of the problem will decide the appropriate perspective for the nonmonotonic reasoning used to solve the problem.

Related Work

The problem of finding efficient algorithms and building automated proof systems for default logics was the most approached in the literature [1, 5, 7, 12, 15].

An important theoretical aspect in the formalization of nonmonotonic reasoning is the study of the inference relations and operations associated to different nonmonotonic formalisms. The specific properties (cumulativity, distribution, cautious monotonicity, proof by cases, absorption, cut) of inference operations for default logics are presented in the papers [2, 8, 16].

The research in the domain of the axiomatization of nonmonotonic reasoning formalized by default logics has begun with the paper [3] and continued with [4, 10]. The proposed axiomatic systems are based on sequent and anti-sequent calculi and characterize the credulous/skeptical default inference in propositional/predicate classical default logic.

In this paper we propose an abstract characterization of credulous default inference associated to rational default logic using the *credulous rational default sequent calculus*. This axiomatic system combines sequent calculus rules and anti-sequent calculus rules with reduction rules specific to the aplication of the defaults.

The paper is structured as follows. Section 2 presents theoretical aspects of *classical* and *rational* default logics. Two complementary systems, sequent calculus and anti-sequent calculus for propositional logic, are discussed in Section 3. In Section 4 we introduce an axiomatic system for credulous reasoning in rational default logic, based on the sequent calculus. Conclusions and further work are outlined in Section 5.

2. Default logics

A *default theory* ([13]) $\Delta = (D, W)$ consists of a set W of consistent formulas of first order logic (the *facts*) and a set D of *default rules*. A *default* has the form $d = \frac{\alpha:\beta_1,\ldots,\beta_m}{\gamma}$, where: α is called *prerequisite*, $\beta_1, \ldots, \beta_m$ are called *justifications* and γ is called *consequent*.

A default $d = \frac{\alpha:\beta_1,\ldots,\beta_m}{\gamma}$ can be applied and thus derive γ if α is believed and it is consistent to assumed $\beta_1, \ldots, \beta_m$(meaning that $\neg\beta_1, \ldots, \neg\beta_m$ are not believed).

The set of defaults used in the construction of an extension is called the *generating default set* for the considered extension.

The results from [9] show that default theories can be represented by unitary theories (all the defaults have only one justification, $d = \frac{\alpha:\beta}{\gamma}$) in such a way that extensions (classical, justified, constrained, rational) are preserved. In this paper we will use only unitary default theories based on propositional logic.

The versions (classical, justified, constrained, rational) of default logic try to provide an appropriate definition of consistency condition for the justifications of the defaults and thus to obtain many interesting and useful properties for these logical systems. These logics must coexist because each of them models a specific type of default reasoning, based on the reasoning context and the semantics of the assumptions.

Classical default logic was proposed by Reiter[13]. Due to the individual consistency checking of justifications the implicit assumptions are lost when the classical extensions are constructed.

Justified default logic was introduced by Lukaszewicz[6]. The applicability condition of default rules is strengthen and thus individual inconsistencies between consequents and justifications are detected, but inconsistencies among justifications are neglected.

Constrained default logic was developed by Schaub[14]. The consistency condition is a global one and it is based on the observation that in commonsense reasoning we assume things, we keep track of our assumptions and we verify that they do not contradict each other.

Rational default logic was introduced in [11] as a version of classical default logic, for solving the problem of handling disjunctive information. The defaults with mutually inconsistent justifications are never used together in constructing a rational default extension.

We denote by $Th(U) = \{X | U \vdash X\}$ the classical deductive closure of the set U of formulas.

The following definitions of classical and rational default extensions show the applicability conditions of defaults in these two variants of default logic.

Definition 2.1. [13] Let $\Delta = (D, W)$ be a default theory. For any set S of formulas, let $\Gamma(S)$ be the smallest set S' of formulas such that:

1. $W \subseteq S'$;
2. $Th(S') = S'$;
3. For any $\frac{\alpha:\beta}{\gamma} \in D$, if $\alpha \in S'$ and $\neg\beta \notin S$ then $\gamma \in S'$.

A set E of formulas is a *classical extension* of (D, W) if and only if $\Gamma(E) = E$.

The set $GD^E_\Delta = \left\{ \frac{\alpha:\beta}{\gamma} | \alpha \in E \textit{ and } \neg\beta \notin E \right\}$ is called the *set of the generating defaults* for the classical extension E.

Definition 2.2. [15] Let $\Delta = (D, W)$ be a default theory. For any set T of formulas, let $\Psi(T)$ be the pair (S', T') of the smallest sets of formulas such that:
1. $W \subseteq S' \subseteq T'$;
2. $S' = Th(S')$ and $T' = Th(T')$;
3. For any $\frac{\alpha:\beta}{\gamma} \in D$, if $\alpha \in S'$ and $\neg\beta \notin T$ then $\gamma \in S'$ and $\beta \wedge \gamma \in T'$.

A pair (E, C) of sets of formulas is a *rational extension* of (D, W) if and only if $\Psi(C) = (E, C)$. The set E is the *actual rational extension* of the default theory and C is the *reasoning context.*

The set $GD_{\Delta}^{(E,C)} = \left\{ \frac{\alpha:\beta}{\gamma} | \alpha \in E \textit{ and } \neg\beta \notin C \right\}$ is called the *set of the generating defaults* for the rational extension (E, C).

From the above definitions we can express the applicability condition of the generating defaults, in terms of derivability and non-derivability in classical logic as follows:

- $\frac{\alpha:\beta}{\gamma}$ is a generating default for a *classical extension:* E if its prerequisite is derivable from the actual extension: $E \vdash \alpha$ and the negation of its justification is not derivable from the *corresponding extension*: $E \nvdash \neg\beta$.
- $\frac{\alpha:\beta}{\gamma}$ is a generating default for a *rational extension:* (E, C) if its prerequisite is derivable from the actual extension: $E \vdash \alpha$ and the negation of its justification is not derivable from the *corresponding reasoning context*: $C \nvdash \neg\beta$.

The applicability condition for classical default logic is an individual one and for rational default logic is a global one, taking in consideration all the justifications (memorized in the context) used in the reasoning process.

Example 2.1. The default theory *(D,W)* with $W = \{F \vee C\}$ and $D = \left\{ d1 = \frac{:A}{B}, d2 = \frac{:\neg A}{C}, d3 = \frac{:\neg B \wedge \neg F}{G}, d4 = \frac{:\neg B \wedge \neg C}{E} \right\}$ has:

- one classical default extension:

$E1 = Th(\{F \vee C, B, C\})$ with $D1 = \{d1, d2\}$ as generating default set.

- two rational default extensions:

1. $(E2, C2)$ =($Th(\{F \vee C, B\})$,$Th(\{F \vee C, B, A\})$) generated by $D2 = \{d1\}$;
2. $(E3, C3)$ = ($Th(\{F \vee C, C, G\})$,$Th(\{F \vee C, G, \neg A, \neg B \wedge \neg F\})$) generated by the set $D3 = \{d2, d3\}$.

We remark that:

- $F \vee C$, B,C,$B \wedge C$,$B \vee C$, $F \vee C \vee B$, $F \wedge C$ are *credulous* and also *skeptical classical* default consequences;
- $F \vee C$, $B \vee C$, $F \vee C \vee G$, $F \vee C \vee B$, $B \vee G$ are *skeptical rational* default consequences belonging to both rational extensions;
- all skeptical rational consequences are also credulous rational consequences;

- B, C, G, $C \vee G$, $C \wedge G$, $F \wedge C$, $(F \vee C) \wedge G$ are *credulous* (but not skeptical) *rational* default consequences belonging to at least one of the rational extensions.

3. Sequent and anti-sequent calculi in propositional logic

The *sequent calculus*, as an improvement of Gentzen natural deduction system, is an axiomatization of classical logic and also provides a direct and syntactic proof method. The *anti-sequent calculus* for propositional logic was introduced in [3] as a complementary system of sequent calculus.

These two axiomatic systems are used to check the derivability and non-derivability in propositional logic.

A *sequent* has the form: $U \Rightarrow V$ and an *anti-sequent* has the form $U \not\Rightarrow V$, where U and V are finite sets of propositional formulas. U is called *antecedent* and V is called *succedent*.

A *basic sequent* contains the same formula, A, in both antecedent and succedent: $U, A \Rightarrow V, A$ or has the form $U \Rightarrow true$.

An anti-sequent $U \not\Rightarrow V$ is called a *basic anti-sequent* if all the formulas of U and V are atomic formulas and $U \cap V = \emptyset$.

Semantics

- The sequent $U \Rightarrow V$ is *true* if each model of U is also a model for at least one of the formulas of V.
- $U \not\Rightarrow V$ is *true* if there is a model M of U in which all the formulas of V are false. M is called an *anti-model* for this anti-sequent.

Axioms

- All basic sequents are true, therefore they are the *axioms* of sequent calculus.
- The basic anti-sequents are true and represent the *axioms* of anti-sequent system.

The rules of sequent calculus and anti-sequent calculus are presented in TABLE 1 and TABLE 2. These rules can be applied as *inference rules*: from the premises (sequents/ anti-sequent above the line) to the conclusion (sequent/anti-sequent below the line) or backwards from the conclusion to the premises as *reduction rules*.

The theorems of soundness and completness for these two systems can be expressed in an uniform manner as follows:

Theorem 3.1.
A sequent/an anti-sequent is true if and only if it can be reduced to basic sequents/anti-sequent using the reduction rules.

From TABLE 1 and TABLE 2 we remark that the rules with two premisses of sequent calculus are splitted into pairs of rules in anti-sequent calculus. Thus the exhaustive search in sequent calculus becomes nondeterminism in anti-sequent calculus and the reduction process is a linear one.

Table 1. Sequent and anti-sequent *left* rules - *Introduction into antecedent*

Connective	Sequent rules	Anti-sequent rules
$\neg$	$(\neg_l)\ \dfrac{U \Rightarrow V, A}{U, \neg A \Rightarrow V}$	$(\neg^c{}_l)\ \dfrac{U \not\Rightarrow V, A}{U, \neg A \not\Rightarrow V}$
$\wedge$	$(\wedge_l)\ \dfrac{U, A, B \Rightarrow V}{U, A \wedge B, \Rightarrow V}$	$(\wedge^c{}_l)\ \dfrac{U, A, B \not\Rightarrow V}{U, A \wedge B, \not\Rightarrow V}$
$\vee$	$(\vee_l)\ \dfrac{U, A \Rightarrow V \quad U, B \Rightarrow V}{U, A \vee B, \Rightarrow V}$	$(\vee^c{}_{l1})\ \dfrac{U, A \not\Rightarrow V}{U, A \vee B \not\Rightarrow V}$ $(\vee^c{}_{l2})\ \dfrac{U, B \not\Rightarrow V}{U, A \vee B \not\Rightarrow V}$
$\rightarrow$	$(\rightarrow_l)\ \dfrac{U \Rightarrow A, V \quad U, B \Rightarrow V}{U, A \rightarrow B \Rightarrow V}$	$(\rightarrow^c{}_{l1})\ \dfrac{U \not\Rightarrow A, V}{U, A \rightarrow B \not\Rightarrow V}$ $(\rightarrow^c{}_{l2})\ \dfrac{U, B \not\Rightarrow V}{U, A \rightarrow B \not\Rightarrow V}$

Table 2. Sequent and anti-sequent *right* rules - *Introduction into succedent*

Connective	Sequent rules	Anti-sequent rules
$\neg$	$(\neg_r)\ \dfrac{U, A \Rightarrow V}{U \Rightarrow V, \neg A}$	$(\neg^c{}_r)\ \dfrac{U, A \not\Rightarrow V}{U \not\Rightarrow V, \neg A}$
$\wedge$	$(\wedge_r)\ \dfrac{U \Rightarrow A, V \quad U \Rightarrow B, V}{U \Rightarrow A \wedge B, V}$	$(\wedge^c{}_{r1})\ \dfrac{U \not\Rightarrow A, V}{U \not\Rightarrow A \wedge B, V}$ $(\wedge^c{}_{r2})\ \dfrac{U \not\Rightarrow B, V}{U \not\Rightarrow A \wedge B, V}$
$\vee$	$(\vee_r)\ \dfrac{U \Rightarrow A, B, V}{U \Rightarrow A \vee B, V}$	$(\vee^c{}_r)\ \dfrac{U \not\Rightarrow A, B, V}{U \not\Rightarrow A \vee B, V}$
$\rightarrow$	$(\rightarrow_r)\ \dfrac{U, A \Rightarrow B, V}{U \Rightarrow A \rightarrow B, V}$	$(\rightarrow^c{}_r)\ \dfrac{U, A \not\Rightarrow B, V}{U \not\Rightarrow A \rightarrow B, V}$

The *derivability* in propositional logic is expressed in sequent calculus as follows:
$U1, U2, \ldots, Un \vdash V1 \vee V2 \vee \ldots \vee Vm$ *if and only if*
the sequent $U1, U2, \ldots, Un \Rightarrow V1, V2, \ldots, Vm$ is **true**,
meaning that from the conjunction of the hypothesis at least one of the formulas from the succedent can be proved.

The *non-derivability* in propositional logic is expressed in anti-sequent calculus as follows:
$U1, U2, \ldots, Un \not\vdash V1 \wedge V2 \wedge \ldots \wedge Vm$ *if and only if*
the anti-sequent $U1, U2, \ldots, Un \not\Rightarrow V1, V2, \ldots, Vm$ is **true**,
meaning that from the conjunction of the hypothesis none of the formulas from the succedent can be proved.

The following theorem shows the complementarity of these two systems:

Theorem 3.2.([4])
The anti-sequent $U \not\Rightarrow V$ *is true* if and only if the sequent $U \Rightarrow V$ *is not true.*

These two axiomatic systems will be used in the following section to check the applicability conditions of the defaults: the derivability of the prerequisites and the non-derivability of the justifications.

4. Axiomatization of credulous reasoning in rational default logic

Based on the credulous sequent calculus for classical default logic proposed in [3], in this section we introduce an abstract characterization of the credulous nonmonotonic inference in rational default logic. An axiomatic system called *credulous rational default sequent calculus* is introduced.

Definition 4.1. Let (D, W) be a propositional default theory.
A *credulous rational default sequent* has the *syntax*:
$(Pre, Just); (W, D, Just_c) \longmapsto U.$
U is a set of propositional formulas and is called *succedent*. The *antecedent* contains two components:

- the first component represented by Pre and $Just$ contains constraints regarding the prerequisites and the justifications of the defaults. The constraints are expressed using the modalities M (possibility) and L (necessity).
- the second component is composed of W, D representing the propositional default theory and $Just_c$ containing the justifications assumed to be true during the reasoning process. In this variant of the default logic we need $Just_c$ in order to check the global applicability condition of the justifications of the defaults.

Remarks:
A constraint of the form $M\alpha$ is satisfied by a set E of sentences if $\neg\alpha$ is not derivable from E, and this can be expressed in anti-sequent calculus as: $E \not\Rightarrow \neg\alpha$.
A set E of sentences satisfies a constraint of the form $L\delta$ if δ is derivable from E, expressed as: $E \Rightarrow \delta$ in sequent calculus.

Definition 4.2. The *semantics* of a credulous rational default sequent:
The *credulous rational sequent* $(Pre, Just); (W, D, Just_c) \longmapsto U$ *is true* if $\vee U$ belongs to at least one rational extension of the theory (W, D), that satisfies the constraints from Pre and $Just$ and is guided by the reasoning context $Th(W \cup Just_c)$.

Definition 4.3. The *axiomatic system* is $Cr = (\Sigma_{Cr}, F_{Cr}, A_{Cr}, R_{Cr})$, where:

- Σ_{Cr} contains all the symbols used to build propositional formulas, modal propositional formulas and defaults defined on the underlying language of the default theory.

- F_{Cr} contains all classical sequents, all classical anti-sequents and all credulous rational default sequents defined on the underlying language of the default theory.
- A_{Cr}, the set of axioms of this formal system, contains all the basic sequents and basic anti-sequents defined on the underlying language of the default theory.
- R_{Cr} = reduction rules = $\{sequent\ rules, anti-sequent\ rules\} \cup$
  $\cup \{R1, R2, R3, R4, R5, R6, R7, R8\}$

We propose the following specific reduction rules based on Definition 2.2.

Sequent rules for rational default logic:

$$\text{(R1)}\ \frac{W \Rightarrow U}{(\emptyset, \emptyset)\ ;\ (W, D, \emptyset)\ \longmapsto U}$$

$$\text{(R2)}\ \frac{W \Rightarrow U}{(\emptyset, \emptyset)\ ;\ (W, \emptyset, Just_c)\ \longmapsto U}$$

$$\text{(R3)}\ \frac{W \Rightarrow \alpha \qquad (Pre, Just \cup \{M\beta\})\ ;\ (W \cup \{\gamma\}, D, Just_c \cup \{\beta\})\ \longmapsto U}{(Pre, Just); (W, D \cup \left\{\frac{\alpha:\beta}{\gamma}\right\}, Just_c)\ \longmapsto U}$$

$$\text{(R4)}\ \frac{(Pre \cup \{M\neg\alpha\}, Just)\ ;\ (W, D, Just_c)\ \longmapsto U}{(Pre, Just)\ ;\ (W, D \cup \left\{\frac{\alpha:\beta}{\gamma}\right\}, Just_c)\ \longmapsto U}$$

$$\text{(R5)}\ \frac{(Pre, Just \cup \{L\neg\beta\})\ ;\ (W, D, Just_c)\ \longmapsto U}{(Pre, Just)\ ;\ (W, D \cup \left\{\frac{\alpha:\beta}{\gamma}\right\}, Just_c)\ \longmapsto U}$$

$$\text{(R6)}\ \frac{W \not\Rightarrow \alpha \qquad (Pre, Just)\ ;\ (W, \emptyset, Just_c)\ \longmapsto U}{(Pre \cup \{M\neg\alpha\}, Just)\ ;\ (W, \emptyset, Just_c)\ \longmapsto U}$$

$$\text{(R7)}\ \frac{W \cup Just_c \not\Rightarrow \neg\beta \qquad (\emptyset, Just)\ ;\ (W, \emptyset, Just_c)\ \longmapsto U}{(\emptyset, Just \cup \{M\beta\})\ ;\ (W, \emptyset, Just_c)\ \longmapsto U}$$

$$\text{(R8)}\ \frac{W \cup Just_c \Rightarrow \neg\beta \qquad (\emptyset, Just)\ ;\ (W, \emptyset, Just_c)\ \longmapsto U}{(\emptyset, Just \cup \{L\neg\beta\})\ ;\ (W, \emptyset, Just_c)\ \longmapsto U}$$

Remarks:

- The rule $R1$ shows that default logic extends the classical logic: if the succedent is derivable from the set of facts (W), it can be deduced from the whole default theory also.
- When all the defaults were introduced (as applicable or non-applicable) and the corresponding constraints were checked ($Pre = \emptyset$, $Just = \emptyset$) then the default rational sequent is reduced to a classical one using $R2$.
- In the reasoning process the introduction of an applicable default, $d = \frac{\alpha:\beta}{\gamma}$, is formalized by the rule $R3$.

-*First premise*: the derivability of the premise α is checked using the classical sequent calculus.

-*Second premise*: the justification β is added to $Just_c$, the consequent γ is added to the set of facts and the corresponding constraint $M\beta$ for justification is introduced.

- The rules $R4$ and $R5$ are used to introduce a default $\frac{\alpha:\beta}{\gamma}$ as non-applicable either by considering its prerequisite as non-derivable (constraint: $M\neg\alpha$) or its justification inconsistent in the context (constraint: $L\neg\beta$).
- $R6$ is used to check the constraints (M) corresponding to the prerequisites of the non-applicable defaults.
- Applying the rules $R7/R8$, the constraints (M/L) corresponding to the justifications are checked in the reasoning context, using anti-sequent/ sequent calculus.
- The order of applying the specific reduction rules is as follows:
  - the rules $R3$, $R4$ and $R5$ are used for introducing all the defaults as applicable or non-applicable until $D = \emptyset$;
  - $R6$ is applied to check the constraints corresponding to the prerequisites until $Pre = \emptyset$;
  - the constraints for justifications are checked using the rules $R7$ and $R8$ until $Just = \emptyset$;
  - when $D = \emptyset, Pre = \emptyset$ and $Just = \emptyset$, the default sequent is reduced to a classical one using $R2$;
  - the classical sequents/anti-sequents are further reduced using the reduction rules from sequent/anti-sequent calculus.
- From the point of view of the classical default logic the above axiomatic system is a reformulation of the one proposed in [3], if we eliminate $Just_c$.

Theorem 4.1. The credulous rational default sequent calculus is *sound and complete*: a credulous sequent is derivable if and only if it is true (can be reduced to classical basic sequents and basic anti-sequents).

Proof: The proof is based on the proof from [3] which can be easily adapted for rational default logic using Definition 2.2.

Consequence:

A formula X is a credulous rational default consequence of the default theory (D, W) if the credulous rational sequent $(\emptyset, \emptyset); (W, D, \emptyset) \longmapsto X$ is true.

Example 4.1. We will show that the formula $C \wedge G$ is a credulous rational belief of the default theory (D, W) from Example 2.1.

$W = \{F \vee C\}$ and $D = \left\{d1 = \frac{:A}{B}, d2 = \frac{:\neg A}{C}, d3 = \frac{:\neg B \wedge \neg F}{G}, d4 = \frac{:\neg B \wedge \neg C}{E}\right\}$.

The reasoning process modeled by the credulous rational default sequent calculus is represented by the following up-side-down binary tree.

$$\dfrac{S6{:}\overline{F \vee C, C, G \Rightarrow G} \qquad S7{:}\overline{F \vee C, C, G \Rightarrow C}}{} \; \wedge_r$$

$$F\vee C,C,G\Rightarrow G\wedge C$$
$$\text{- - - - - - - - - - - - - - - - - -}\ R2$$
$$S5 \qquad (\emptyset,\emptyset);(\{F\vee C,C,G\},\emptyset,\{\neg A,\neg B\wedge\neg F\}) \longmapsto G\wedge C$$
$$\text{- -}\ R7$$
$$S4 \qquad (\emptyset,\{M(\neg B\wedge\neg F)\});(\{F\vee C,C,G\},\emptyset,\{\neg A,\neg B\wedge\neg F\}) \longmapsto G\wedge C$$
$$\text{- -}\ R7$$
$$S3 \qquad (\emptyset,\{M\neg A,M(\neg B\wedge\neg F)\});(\{F\vee C,C,G\},\emptyset,\{\neg A,\neg B\wedge\neg F\}) \longmapsto G\wedge C$$
$$\text{- -}\ R8$$
$$S2 \qquad (\emptyset,\{M\neg A,M(\neg B\wedge\neg F),L\neg(\neg B\wedge\neg C\});(\{F\vee C,C,G\},\emptyset,\{\neg A,\neg B\wedge\neg F\}) \longmapsto G\wedge C$$
$$\text{- -}\ R8$$
$$(\emptyset,\{M\neg A,M(\neg B\wedge\neg F),L\neg A,L\neg(\neg B\wedge\neg C\});(\{F\vee C,C,G\},\emptyset,\{\neg A,\neg B\wedge\neg F\}) \longmapsto G\wedge C$$
$$\text{- -}\ R5$$
$$(\emptyset,\{M\neg A,M(\neg B\wedge\neg F),L\neg A\});(\{F\vee C,C,G\},\left\{d4=\tfrac{:\neg B\wedge\neg C}{E}\right\},\{\neg A,\neg B\wedge\neg F\}) \longmapsto G\wedge C$$
$$\text{- -}\ R5$$
$$S1 \qquad (\emptyset,\{M\neg A,M(\neg B\wedge\neg F)\});(\{F\vee C,C,G\},\left\{\tfrac{:A}{B},\tfrac{:\neg B\wedge\neg C}{E}\right\},\{\neg A,\neg B\wedge\neg F\}) \longmapsto G\wedge C$$
$$\text{- -}\ R3$$
$$S1 \qquad (\emptyset,\{M\neg A\});(\{F\vee C,C\},\left\{d1=\tfrac{:A}{B},d3=\tfrac{:\neg B\wedge\neg F}{G},d4=\tfrac{:\neg B\wedge\neg C}{E}\right\},\{\neg A\}) \longmapsto G\wedge C$$
$$\text{- -}\ R3$$
$$(\emptyset,\emptyset);(\{F\vee C\},\left\{d1=\tfrac{:A}{B},d2=\tfrac{:\neg A}{C},d3=\tfrac{:\neg B\wedge\neg F}{G},d4=\tfrac{:\neg B\wedge\neg C}{E}\right\},\emptyset) \longmapsto G\wedge C$$

- The initial credulous rational default sequent is:
  $(\emptyset,\emptyset);(\{F\vee C\},\left\{\frac{:A}{B},\frac{:\neg A}{C},\frac{:\neg B\wedge\neg F}{G},\frac{:\neg B\wedge\neg C}{E}\right\},\emptyset) \longmapsto G\wedge C$
- The defaults $d2$ and $d3$ are introduced as applicable defaults using $R3$. Their prerequisites are checked in the first premise $S1$: $F\vee C\Rightarrow true$ which is a classical basic sequent. The corresponding M-constraints for their justifications are introduced in the second premise.
- Using twice the rule $R5$, $d1$ and $d4$ are considered non-applicable defaults and thus L-constraints are added to the set of constraints.
- All the defaults were either introduced as applicable or non-applicable and we used the rules $R7$ and $R8$ to check the satisfiability of their corresponding constraints (L and M) for justifications in the reasoning context: $Th(\{F\vee C,G,C\}\cup\{\neg A,\neg B\wedge\neg F\})$.
- The premises $S2$ and $S3$ are classical basic sequents:
  $S2: F\vee C,G,C,\neg A,\neg B\wedge\neg F\Rightarrow\neg A$,
  $S3: F\vee C,G,C,\neg A,\neg B\wedge\neg F\Rightarrow B\vee C$
- The premises $S4$ and $S5$ are classical basic anti-sequents:
  $S4: F\vee C,G,C,\neg A,\neg B\wedge\neg F\not\Rightarrow A$,
  $S5: F\vee C,G,C,\neg A,\neg B\wedge\neg F\not\Rightarrow B\vee F$
- $R2$ reduces the credulous rational sequent (with $D=\emptyset$, $Pre=\emptyset$, $Just=\emptyset$), to a classical sequent which can be further reduced to basic sequents ($S6$ and $S7$) using $\wedge_r$.

The initial credulous rational default sequent was reduced to basic sequents($S1$, $S2$,$S3$,$S6$,$S7$) and basic anti-sequents ($S4$, $S5$), therefore it is a true sequent and according to the soundness of our axiomatic system we have that the formula $C\wedge G$ is a credulous rational default consequence of the default theory (D,W).

$C \wedge G$ belongs only to the actual rational extension $E3 = Th(\{F \vee C, C, G\})$ generated by $D3 = \{d2, d3\}$. The corresponding reasoning context is $C3 = Th(\{F \vee C, G, \neg A, \neg B \wedge \neg F\}$.

5. Conclusions and further work

In this paper we introduced an axiomatic system for credulous reasoning in rational default logic. The proposed system, using specific reduction rules, reduces the nonmonotonic inferential process to a classic inferential process modelled by the sequent calculus and anti-sequent calculus for propositional logic.

As further work we propose an uniform axiomatization of credulous/skeptical reasoning, using sequent calculus, for all the versions (classical, justified, constrained and rational) of default logic. Also of great practical interest is to add to these axiomatic systems proof strategies in order to obtain efficient proof methods.

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