STUDIA UNIV. BABEŞ–BOLYAI, INFORMATICA, Volume **XLVIII**, Number 1, 2003

STOCHASTIC OPTIMIZATION OF QUERYING DISTRIBUTED DATABASES I. THEORY OF FOUR RELATIONS JOIN

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Abstract. Stochastic query optimization problem for multiple join is addressed. In Part I two sites model of Drenick and Smith (1993) is extended to four relations stored at four different sites. Our model leads to a special kind of nonlinear optimization problem (P). It is proved (Theorem 5.1) that this problem has at least one solution. In Part II an ad hoc constructive model for solving problem (P) is proposed. In Part III a new evolutionary technique is used for solving problem (P). Results obtained by the two considered optimization approaches are compared.

Keywords: Distributed Databases, Query Optimization Problem, Genetic Algorithms, Evolutionary Optimization, Adaptive Representation.

1. Introduction

The ability of distributed systems for concurrent processing motivates the distribution of a database in a network. The query optimization problem for a single query in a distributed database system was treated in great detail in the literature. Many algorithms were elaborated for minimizing the costs necessary to perform a single, isolated query in a distributed database system. Some methods can be found in Özsu and Valduriez (1999), Date (2000). Most approaches look for a deterministic strategy assigning the component joins of a relational query to the processors of a network that can execute the join efficiently and determine an efficient strategy for the data transferring.

A distributed system can receives different types of queries and processes them at the same time. Query processing strategies may be distributed over the processors of a network as probability distributions. In this case the determination of the optimal query processing strategy is a stochastic optimization problem. There is a different approach to query optimization if the system is viewed as one which receives different types of queries at different times and processes more than one query at the same time.

Received by the editors: May 15, 2003.

2000 *Mathematics Subject Classification.* 68P15, 68T99.

1998 *CR Categories and Descriptors.* C.2.4 [**Computer Systems Organization**]: Computer-Communication Networks – *Distributed Systems*; H.2.4 [**Information Systems**]: Database Management – *Systems*.

The multiple-query problem is not deterministic; the multiple-query-input stream constitutes a stochastic process. The strategy for executing the multiple-query is distributed over the sites of the network as a probability distribution. The "decision variables" of the stochastic query optimization problem are the probabilities that component operators of the query are executed at particular sites of the network.

Drenick and Smith (1993) extend the state-transition model proposed by Lafortune and Wong (1986) and the original multiprocessing model (see Drenick and Drenick, 1987, Drenick, 1986). The main objective of the state-transition model is to give globally optimal query-processing strategies. Drenick and Smith (1993) treat the single-join model, the general model for the join of two relations and a multiple-join with three relations, which are stored at two different sites. The stochastic model for the join of three relations, which are stored at three different sites is presented in Varga (1998) and Varga (1999).

Stochastic query optimization problem leads to a nonlinear programming problem, which is a specific one. General models of sequential and parallel operation for the specified type queries are treated in Varga (1999). Stochastic query optimization model using semijoins is presented in Markus, Morosan and Varga (2001).

The aim of this paper is to extend the stochastic model to the join of four relations. In Section 2 the case when the relations are stored at four sites is considered. The stochastic query optimization problem in case of four relations leads to a constrained nonlinear optimization problem. Considering the complexity of obtained nonlinear problem two complementary methods for solving this problem are proposed. Theorem 5.1 proves, that the nonlinear optimization problem has at least one solution. In Part II of the paper a constructive method for solving the nonlinear programming problem is given.

Due to the successfully application in the recent past of the evolutionary algorithms for solving very difficult optimization problems evolutionary methods seem to be quite appealing for solving our optimization problem. We will consider evolutionary techniques based on a dynamic representation (Dumitrescu, Grosan and Oltean, 2001, Grosan and Dumitrescu, 2002). This technique called *Adaptive Representation Evolutionary Algorithm* (AREA) is described in Part III. The results obtained by applying these different approaches are presented in Part III. Two sets of values for constants are used in these experiments. Solutions are nearly the same. The CPU time required for solving the optimization problem by using evolutionary algorithm is less than the CPU time required by the constructive method.

2. Four relations join

Consider four relations stored in different sites of the distributed database. The join of these four relations will be defined in the context of stochastic model of

Drenick and Smith (1993). Consider relations A, B, C, D stored at the sites 1,2,3 and 4 respectively.

Denote by Q_4 the single-query type consisting of the join of four relations:

$$Q_4 = A \bowtie B \bowtie C \bowtie D.$$

Initial state of relations referenced by the query Q_4 in the four-site network is the column vector defined as:

$$s_0 = \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}$$

where the i-th component of the vector s_0 is the set of relations stored at site i, $i \in \{1, 2, 3, 4\}$ at time t= 0.

Initial state s_0 is given with time-invariant probability

$$p_0 = p(s_0)$$

i.e. p_0 is the probability that relation A is available at site 1, relation B at site 2, relation C at site 3, and relation D at site 4. The four relations are not locked for updating or are unavailable for query processing for any other reason. We assume that the input to the system consists of a single stream of type Q_4.

For the purpose of stochastic query optimization we enumerate all logically valid joins in the order in which they may be executed. Let us suppose that Q_4 has three valid execution sequences:

$$Q_4S_1 = (((A \bowtie B) \bowtie C) \bowtie D),$$

$$Q_4S_2 = ((A \bowtie B) \bowtie (C \bowtie D)),$$

$$Q_4S_3 = (A \bowtie (B \bowtie (C \bowtie D))).$$

Sequence Q_4S_1 can be applied if

$$A \cap B \neq \emptyset.$$

So the join

$$B' = A \bowtie B$$

is executed before the join

$$C' = B' \bowtie C.$$

The last executed join will be

$$D' = C' \bowtie D.$$

The sequence Q_4S_2 is adequate for parallel execution.

3. Stochastic query optimization model

The system that undergoes transition in order to execute the join of four relations is described in this section as in Drenick and Smith (1993). The strategy for executing the multiple join is distributed over the sites of the network. Conditional probabilities are associated with the edges of the state-transition graph. Executing a multiple join is equivalent to solve a optimization problem. This problem is referred as *stochastic query optimization model*. Theorem 3.1 states, that the stochastic query optimization model for the multiple join query defines a nonlinear optimization problem.

We exemplify with the execution of the join Q_4S_1.

The state-transition graph for sequence Q_4S_1 is given in Figure 1. For one state of the state-transition graph the i^{th} line contains the relations stored at site i. We will associate a transition probability to each transition arc of the state-transition model. Let p_{ij} denote the conditional, time-invariant probability that the system undergoes transition from state s_i to state s_j. Given the initial state s_0, we can execute the first step of Q_4S_1transferring relation B from site 2 to site 1, or transferring relation A from site 1 to site 2.

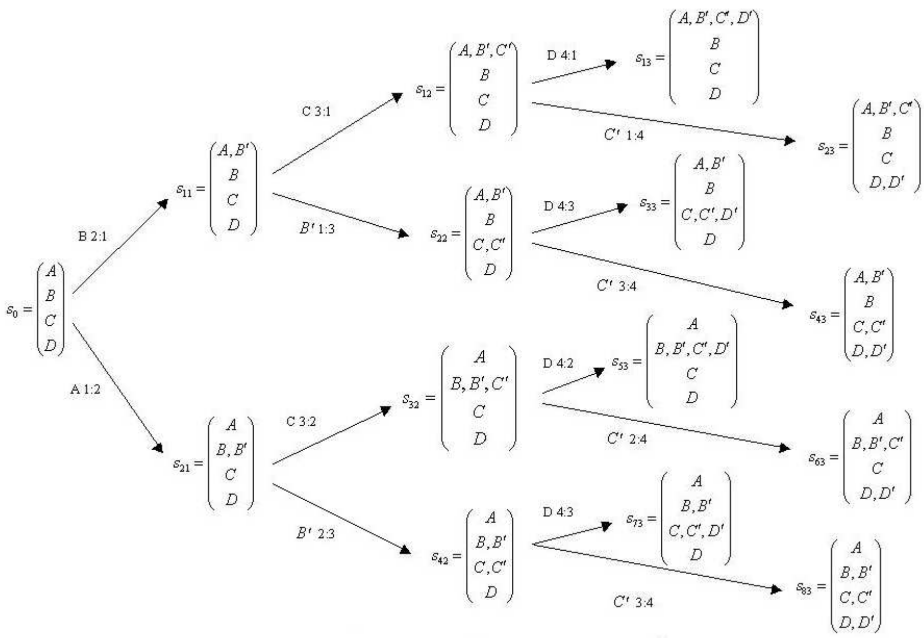

Figure 1. State-transition graph for the join Q_4S_1 of four relations

States of the transition graph are labeled as s_{ij}, where j is the step in computing the multiple join and$i$ denote the number of the selected strategy within the step.

Using the first strategy the system undergoes transition from state s_0 to the state s_{11} with probability $p_{0,11}$. The system may choose the second strategy with probability $p_{0,21}$ when the system undergoes transition to state s_{21}.

In order to compute

$$C' = B' \bowtie C,$$

if the system is in state s_{11} relation B' may be transferred from site 1 to site 3 or relation C from site 3 to site 1 and similar for the other states of the state-transition graph.

With respect to the stochastic query optimization model we can state the following Theorem.

Theorem 3.1 The stochastic query optimization model for the multiple join query of type Q_4 defines a nonlinear optimization problem.

Proof: We will associate the join-processing times with the nodes of the state-transition graph and communication times to the arcs of the graph. Let $T_i(X)$ denote the total processing time required for computing in state i.

So we have:

$$\begin{aligned}
&T_{11}(B') = c_{21}\ (B) + t_1\ (A \bowtie B),\\
&T_{12}(C') = c_{31}\ (C) + t_1\ (B' \bowtie C),\\
&T_{32}(C') = c_{32}\ (C) + t_2\ (B' \bowtie C),\\
&T_{13}(D') = c_{41}\ (D) + t_1\ (C' \bowtie D),\\
&T_{33}(D') = c_{43}\ (D) + t_3\ (C' \bowtie D),\\
&T_{53}(D') = c_{42}\ (D) + t_2\ (C' \bowtie D),\\
&T_{73}(D') = c_{43}\ (D) + t_3\ (C' \bowtie D),\\
&T_{21}(B') = c_{12}\ (A) + t_2\ (A \bowtie B), \qquad (3.1)\\
&T_{22}(C') = c_{13}\ (B') + t_3\ (B' \bowtie C),\\
&T_{42}(C') = c_{23}\ (B') + t_3\ (B' \bowtie C),\\
&T_{23}(D') = c_{14}\ (C') + t_4\ (C' \bowtie D),\\
&T_{43}(D') = c_{34}\ (C') + t_4\ (C' \bowtie D),\\
&T_{63}(D') = c_{24}\ (C') + t_2\ (C' \bowtie D),\\
&T_{83}(D') = c_{34}\ (C') + t_4\ (C' \bowtie D).
\end{aligned}$$

Denote by $c_{ij}(R)$ the time required to transfer the relation R from site i to site j. t_i (E) denotes the necessary time to calculate the expression E in the site i. The expected delay due to computing the join is the product of the delay and the corresponding transition probability. The mean processing time τ_i at site i can be obtained by summing for each state for which there is something to work in the site i, the product of the necessary time for processing multiplied by the probability that the system is in the corresponding state.

Let us suppose that input queries of type Q_4 arrive at the system at average intervals of length δ and successive inputs are statistically independent. It is reasonable to require that none of the processors in the network be allowed to take longer on the average than the period δ to execute its task. If it did, the cumulative delay at each site could increase indefinitely due to queuing, requiring infinite buffer storage at each site. The system may be regarded as overloaded if the mean processing time τ_i is permitted to exceed δ at any site.

Such overload can be avoided if the following inequalities hold:

$$\tau_i \leq \Delta < \delta,$$

where Δ represents a common upper bound on τ_i for each processor i in the network.

In order to maximize the system *query-processing capacity*

$$\lambda = \frac{1}{\delta}$$

the system's mean interarrival time Δ may be minimized, where

$$(\delta - \Delta) > 0,$$

is chosen sufficiently large to provide adequate buffer storage requirements.

The mean processing times $\tau_i, i = 1, 2, 3, 4$ are expressed as:

$$\begin{aligned}
\tau_1 &= T_{11}(B')p_{0,11} + T_{12}(C')p_{0,11}p_{11,12} + T_{13}(D')p_{0,11}p_{11,12}p_{12,13}, \\
\tau_2 &= T_{21}(B')p_{0,21} + T_{32}(C')p_{0,21}p_{21,32} + T_{53}(D')p_{0,21}p_{21,32}p_{32,53} \qquad (3.2) \\
\tau_3 &= T_{22}(C')p_{0,11}p_{11,22} + T_{42}(C')p_{0,21}p_{21,42} + T_{33}(D')p_{0,11}p_{11,22}p_{22,33} \\
&\quad + T_{73}(D')p_{0,21}p_{21,42}p_{42,73}, \\
\tau_4 &= T_{23}(D')p_{0,11}p_{11,12}p_{12,23} + T_{43}(D')p_{0,11}p_{11,22}p_{22,43} \\
&\quad + T_{63}(D')p_{0,21}p_{21,32}p_{32,63} + T_{83}(D')p_{0,21}p_{21,42}p_{42,83}.
\end{aligned}$$

Therefore the stochastic query optimization problem for the query Q_4S_1 is given by:

$$(P_1) \begin{cases}
\text{minimize } \Delta_1 \\
\quad \text{subject to:} \\
\tau_i \leq \Delta_1, i = 1, 2, 3, 4 \\
p_{0,11} + p_{0,21} = 1, \\
p_{11,12} + p_{11,22} = 1, \\
p_{21,32} + p_{21,42} = 1, \\
p_{12,13} + p_{12,23} = 1, \\
p_{22,33} + p_{22,43} = 1, \\
p_{32,53} + p_{32,63} = 1, \\
p_{42,73} + p_{42,83} = 1, \\
p_{0,11}, p_{0,21}, p_{11,12}, p_{11,22}, p_{21,32}, p_{21,42}, p_{12,13} \in [0, 1], \\
p_{12,23}, p_{22,33}, p_{22,43}, p_{32,53}, p_{32,63}, p_{42,73}, p_{42,83} \in [0, 1].
\end{cases}$$

This concludes the proof.

The obtained problem (P_1) is a constrained nonlinear optimization problem. In the next section we propose a constructive approach for solving the optimization problem (P_1).

4. Stochastic query optimization problem

Let us consider the following notations:

$$
\begin{aligned}
h_1(z_1,\, z_2,\, \ldots,\, z_{14}) &= c_1 z_1 + c_2 z_1 z_3 + c_3 z_1 z_3 z_7,\\
h_2(z_1,\, z_2,\, \ldots,\, z_{14}) &= c_4 z_2 + c_5 z_2 z_5 + c_6 z_2 z_5 z_{11}, \qquad (4.1)\\
h_3(z_1,\, z_2,\, \ldots,\, z_{14}) &= c_7 z_1 z_4 + c_8 z_2 z_6 + c_9 z_1 z_4 z_9 + c_{10} z_2 z_6 z_{13},\\
h_4(z_1,\, z_2,\, \ldots,\, z_{14}) &= c_{11} z_1 z_3 z_8 + c_{12} z_1 z_4 z_{10} + c_{13} z_2 z_5 z_{12} + c_{14} z_2 z_6 z_{14},
\end{aligned}
$$

where $z_1 = p_{0,11}$,

$z_2 = p_{0,21}$,
$z_3 = p_{11,12}$,
$z_4 = p_{11,22}$,
$z_5 = p_{21,32}$,
$z_6 = p_{21,42}$,
$z_7 = p_{12,13}$,
$z_8 = p_{12,23}$,
$z_9 = p_{22,33}$,
$z_{10} = p_{22,43}$,
$z_{11} = p_{32,53}$,
$z_{12} = p_{32,63}$,
$z_{13} = p_{42,73}$,
$z_{14} = p_{42,83}$.

Expressing z_{2k-1}, $k = 1, 2, \ldots, 7$, from equality restrictions of problem (P_1) we have:

$$z_{2k-1} = 1 - z_{2k}.$$

By replacing z_{2k-1}, $k = 1, 2, \ldots, 7$, in the inequalities of (P_1)

$$\tau_i \le \Delta_i,$$

the problem (P_1) can be rewritten as the next optimization problem (P_2):

$$
(P_2)\begin{cases}
\text{minimize } \Delta_1\\
\quad \text{subject to:}\\
f_1(x_1, x_2, \ldots, x_7) \le \Delta_1,\\
f_2(x_1, x_2, \ldots, x_7) \le \Delta_1,\\
f_3(x_1, x_2, \ldots, x_7) \le \Delta_1,\\
f_4(x_1, x_2, \ldots, x_7) \le \Delta_1,\\
x_1, x_2, \ldots, x_7 \in [0, 1]\,.
\end{cases}
$$

The number of relations and sites in one distributed database can be different. Resulting nonlinear optimization problem has different number of variables and constraints. Therefore we have to generalize problem (P_2) for an arbitrary number of relations and sites.

Let us consider p continous functions

$$f_1, ..., f_p : [0,1]^n \rightarrow R_+,$$

where p is the number of sites in the distributed database and $f_i, (i = 1, \ldots, p)$ represents the mean processing time at site i.

Our optimization problem (P_2) may be generalized to the following optimization problem (P_p).

$$(P_p) \begin{cases} \text{minimize } \Delta_1 \\ \qquad \text{subject to:} \\ f_1(x_1, x_2, \ldots, x_n) \leq \Delta_1, \\ \vdots \\ f_p(x_1, x_2, \ldots, x_n) \leq \Delta_1, \\ x_1, x_2, \ldots, x_n \in [0,1]. \end{cases}$$

5. General optimization framework

In this section problem (P_p) is considered as an instance of a more general framework. The new framework is necessary for establishing conditions under which problem (P_p) has a solution.

Let (X, d) be a compact metric space and

$$f_1, ..., f_p : X \rightarrow R_+$$

be continuous strictly positive functions.

Consider the next generic optimization problem:

$$(P) \begin{cases} \text{minimize } y, y \in R \\ \qquad \text{subject to:} \\ x \in X, (X \text{ is a compact metric space}), \\ y > 0, \\ f_1(x) \leq y, \\ \vdots \\ f_p(x) \leq y. \end{cases}$$

With respect to problem (P) we can state the following Theorem. For proving it some concepts and results are needed (see for instance Rudin, 1976).

Theorem 5.1: Problem (P) has at least one solution.

Proof. Let X is be compact metric space and $f : X \to R$ be the function defined as

$$f(x) = \max\{f_1(x), ..., f_p(x)\}.$$

Since function f is continuous and X is a compact metric space, according to the Weierstrass theorem, there exists a point $x_0 \in X$ such that x_0 is the global minimum of the function f, i.e.

$$f(x_0) = \min_{x \in X} f(x).$$

We have to prove that

$$f(x_0) = \min y.$$

Let us suppose that it exists $y_0 \in R_+^*$ such that

$$f(x_0) > y_0,$$

and y_0 satisfies the inequalities from the problem (P) for $x_0^* \in X$, i.e.:

$$\begin{aligned} f_1(x_0^*) &\le y_0 \\ &\vdots \\ f_p(x_0^*) &\le y_0. \end{aligned}$$

From these inequalities we obtain

$$\begin{aligned} f(x_0^*) &= \max\{f_1(x_0^*), ..., f_p(x_0^*)\} \\ &\le y_0 \\ &< f(x_0). \end{aligned}$$

But this contradicts the assumption that $x_0 \in X$ is the global minimum of the function f. Therefore the assumption concerning the existence of a value y_0 such that

$$f(x_0) > y_0$$

is false. This completes the proof. □

6. Conclusions

Stochastic optimization model of querying distributed databases, presented by Drenick and Smith (1993), is extended to the join of four relations. These four relations are stored in four different sites. Theorem 3.1 states, that the stochastic query optimization problem in case of four relations leads to a constrained nonlinear programming problem. The problem of querying the distributed database is generalized for p sites. General constrained nonlinear problem (P) is formulated. Theorem 5.1 proofs that problem (P) has at least one solution.

Acknowledgments: We are grateful to professors Cs. Varga and M. Frentiu for their valuable suggestions and to the Journal's anonymous reviewers for useful observations.

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