STUDIA UNIV. BABEŞ–BOLYAI, INFORMATICA, Volume **XLVIII**, Number 1, 2003

THE LOAN-BANK CONTRACT: A SWAP OPTION

D. AKUME

ABSTRACT. This review paper discusses financial options of the European type, particularly swap options and their pricing using the modified Black Model(1976). It also discusses the theory and modelling of *Contractual Saving for Housing*[1] as practiced by *German "Bauparkassen"*. After a discussion of both *Option Pricing* and *Loan Banking*, the last part of this paper reveals that a *loan-bank contract* is in actual fact a *financial option* on an *amortization swap*. The outlook aims at the valuation of such a *swaption*.

Keywords: Loan-bank, contract, option, swap, swaption.

1. INTRODUCTION

In this paper, we present a scientific perspective of *loan banking* by linking it to one of the cornerstones of modern Mathematical Finance, the theory of option pricing. The more seasoned users are fairly au fait with swaps and swaptions as forms of financial options[1].

The present survey paper was stimulated by the analysis of *loan banking*[25], done in collaboration between German "Bausparkassen" and the Center for Applied Computer Science, Cologne (ZAIK).

Firstly, we illustrate the mathematical valuation of swaps and swaptions following the modified Black Model[2] of 1976 for European options. Secondly, we review

Received by the editors: May 5, 2003.

2000 *Mathematics Subject Classification.* 91B28, 00A69, 46N10.

[1]A (Put)Call option (See Peter Reissner (1991), [24], page 23) gives its holder the right without obligation to (sell)purchase an underlying asset (to)from the writer at a predetermined price (strike price, K), on or before an agreed future date (expiry date T).

An American option is that which permits its holder to exercise the aforementioned right on or before the expiry date. Otherwise it is known as European option.

Boundary conditions for a European call (C_E) and a European put (P_E) option at expiry are as follows:

$$C_E = \max\{V_T - K; 0\} = (V_T - K, 0)_+$$
$$P_E = \max\{K - V_T; 0\} = (K - V_T, 0)_+,$$

whereby V_T is the spot price of the underlying asset at expiry date T.

[2]Here follows the Black formula (see also Biermann, [4], page 159):

$$\begin{aligned} \text{Call} &= B(0,T)[F \cdot N(d_1) - K \cdot N(d_2)] \\ \text{Put} &= B(0,T)[K \cdot N(-d_2) - F \cdot N(-d_1)] \end{aligned}$$

the theory and practice of Contractual Saving for Housing as practiced by German "Bausparkassen"[25], and present the model of a typical, discrete transaction. Finally, we analyse the *loan-bank contract*, compare it to a swaption and infer that it is a swaption on an amortization swap[3]

2. Interest Rate Swaps (IRS)

Definition 1. *An* interest rate swap (IRS[4]) *is a contractual agreement entered into between two counterparties under which they agree to exchange fixed for variable interest rates (mostly LIBOR[5]) periodically, for an agreed period of time based upon a notional amount of principal. The principal amount is notional because there is no need to exchange actual amounts of principal. Equally, however, a notional amount of principal is required in order to compute the actual cash amounts that will be periodically exchanged.*

2.1. **Concepts.** An IRS is an agreement of specified duration between two parties (Swap partners or Counterparties) for the exchange of interest rate payments relative to a nominal value (Notional Principal Amount) at predetermined periods of time. That is, counterparty A makes fixed interest payments to Counterparty B at specific time intervals. On the other hand, A receives from B variable payments relative to an agreed reference interest rate [6].

Counterparty B receives fixed interest payments (receiver position), That is to say, B pays variable. On the other hand, A makes fixed interest payments (payer position), That is, A receives variable payments in the swap. Variable and fixed coupon payments occur in predetermined time intervals. It is however stressed here that in practical terms, the dates of variable and fixed payments may not always coincide. If one denotes the variable cashflow interval[7] with Δt , the interest rate of reference would be agreed upon at time t, with the cashflow occurring at time $t + \Delta t$. Cashflows on both sides coinciding would mean the net value being paid to the beneficiary.

---

with

$$d_1 = \frac{\ln\frac{F}{K} + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \qquad d_2 = \frac{\ln\frac{F}{K} - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \tag{1}$$

whereby $N(.)$ is the gaussian distribution, $B(.,.)$ is the discount function and F is the future price (see also [19]) for details also of the Black/Scholes model.

[3]In actual fact a swaption, since it is an option on a future swap.

[4]In this paper the terms Interest Rate Swap and Swap are used interchangeably.

[5]London Interbank Offered Rate

[6]Rauleder (1993), [23], page 11.

[7]Rauleder (1993), [23], page 13: in the case of the variable cashflow interval being smaller than the fixed, fixed and variable cashflows will fall apart at certain dates.

2.2. **Application of Interest Rate Swaps.** Interest rate swaps have been widely used by the larger corporate institutions for some time as an efficient off-balance sheet method to manage interest rate exposure arising from their assets and liabilities. For example, a floating-rate borrower who expects a rise in interest rates can swap his floating rate obligation to a fixed rate obligation, thus locking in his future cost. Should he subsequently decide that rates have peaked, and that the trend is reversing, the interest obligation could be swapped back to a floating rate basis, thereby gaining advantage from the anticipated fall in rates.

Swaps are particularly useful in the restructuring of risk in an investment. Eventual interest rate risks can be hedged away with swaps. It is for this reason that swaps have become so important in financial management.

2.3. **Swap Pricing Model.** Pricing[8] a swap means determining the fixed interest rate R_{fix} of the swap (swap rate) such that, the value of the swap is zero at time $t = 0$.

It is clear from the definition that a swap is equivalent to a portfolio of two bonds, one short and the other long, one a fixed-rate bond and the other a floating rate bond[9].

Let $0 < t_1 < \cdots < t_n$ represent the reset dates of the swap.

The price of a Floating Rate Note (FRN) is always equal to the nominal value L[10] at time $t = 0$ irrespective of reset interval. Therefore the floating payment X_v at time $t = 0$ is the difference between L and the present value of the nominal value[11].

$$X_v = L - B(0, t_n)L = L\left(1 - B(0, t_n)\right),$$

whereby B(.,.) is the discount factor for the interval (t, T) as introduced in [19], page 2. Total fixed payment X_f at time $t = 0$ is a series of fixed payments at fixed interest rate R_{fix}

$$X_f = \sum_{i=1}^{n} B(0, t_i) R_{fix}(t_i - t_{i-1})L$$

The pricing of a swap is reduced to the problem of determining R_{fix}, such that the following equation holds:

$$\sum_{i=1}^{n} B(0, t_i) R_{fix}(t_i - t_{i-1})L = L\left(1 - B(0, t_n)\right)$$

[8]See B. Luderer; O. Zuschanke, [20], for a rigorous treatment of Swap-Pricing.

[9]Rauleder, [23], page 97; R. Kohn, [15].

[10]Rauleder (1993), [23], page 97.

[11]Unlike for a normal bond, the nominal value L is notional Capital, thus never actually changes hands.

3. Swaptions

A *swaption* is a combination of the following two financial instruments: *Interest Rate Swap (IRS)* and *Option*.

Swaptions first came into vogue in the mid-1980s in the US on the backof structured bonds tagged with a callable option issued by borrowers. With a callable bond, a borrower issues a fixed-rate bond which he may call at par from the investor at a specific date(s) in the future. In return for the issuer having the right to call the bond issue at par, investors are offered an enhanced yield. Bond issuers often issue an IRS in conjunction with the bond issue in order to change their interest profile from fixed to floating. Swaptions are then required by the issuer as protection to terminate all or part of the original IRS in the event of the bonds being put or called.

Definition 2. *A* Swaption[12] (Swap Option) *reserves the right for its holder to purchase a swap at a prescribed time and interest rate in the future (European Option).*

The holder of such a call option has the right, but not the obligation to pay fixed in exchange for variable interest rate. Therefore, this option is also known as "Payer Swaption". The holder of the equivalent put option has the right, but not the obligation to receive interest at a fixed rate (Receiver Swaption) and pay variable.

3.1. Applications of Swaptions.

Actually, a swaption is an option on a forward interest rate. Like interest rate swaps, swaptions are used to mitigate the effects of unfavorable interest rate fluctuations at a future date. The premium paid by the holder of a swaption can more or less be considered as insurance against interest rate movements. In this way, businesses are able to guarantee risk limits in interest rates.

For instance, a five year swaption expiring in six months is the same as an option to contract a swap in six months time, and the swap will be valid for five years. To further buttress the point, an example is in order:

3.1.1. *Example:* Consider the case of a firm that will start servicing its debt six months from now. The debt is serviceable within five years, at a floating interest rate payable every six months.

This firm can protect itself against rising interest rates by purchasing a payer swaption. By paying a premium, the firm obtains the right to recive variable payments (mostly LIBOR) to pay a predetermined fixed interest rate eg. 12% p.a. for a five year period. The swap begins six months from now (expiry date of the Swaption).

There will be two possible outcomes at the expiry date:

[12]Peter Reissner (1991), [24], page 23.

(1) The market swap rates are higher than 12%: The option is exercised and its holder is able to satisfy his variable interest rate commitment at a rate below the market interest rate. Our firm thus gains.
(2) The market swap rate is below the strike rate: The swaption is not exercised and the firm turns to lower interest rates in the market.

3.2. **Pricing Swaptions With The Black-Model. Notation**

T	–	expiry date of Option
F	–	forward swap rate
$B(0,t_i)$	–	discount factor from date t_i, down to 0
X_v	–	variable interest rate
X_f	–	fixed interest rate
σ	–	volatility of swap
K	–	Strike rate
R_{fix}	–	Market swap rate at time T

The input parameter σ is obtained from market data[13].

Let $t_1 < \cdots < t_n$ represent the coupon dates for the swap and $t_0 = T$.

In deriving a pricing formula, we look at the swap underlying the swaption. The swap begins on the expiration date (T) of the swaption - this coincides with the first cashflows - and ends at time t_n. The swap comprises payments at floating interest rate X_v and payments at fixed interest rate X_f , relative to the n cashflows[14].

The variable interest rate payments are based on a benchmark (mostly LIBOR) at time t_k, a notional principal amount L and n interest periods $(t_i - t_{i-1})$

The fixed payments are based on the strike rate K, as well as same notional principal amount and periods.

At each date t_i, the interest rate R_{fix} shall be compared with the Strike rate K.

3.2.1. *Pricing Model for European Swaptions.* In case $R_{fix} > K$, the fixed interest payer gets paid a balance of $L \cdot (R_{fix} - K) \cdot (t_i - t_{i-1})$. Otherwise this balance goes to the variable interest payer.

The swap rate R_{fix} satisfies the following equation(see Section 2.3)

$$\sum_{i=1}^{n} B(T,t_i) R_{fix} (t_i - t_{i-1}) L = (1 - B(T,t_n)) L \tag{2}$$

[13][24], page 48 : historic or implicit Volatility.

[14]Robert V. Kohn, [15].

The left hand side of the above equation represents the value of the fixed payments at the rate of R_{fix} as of time T. The right hand side, on the other hand, represents the value of variable payments. Take note that the present value at variable interest rate will be equal to the notional principal amount.

The holder of an European swaption has the right to pay the fixed rate K and to receive a floating rate (payer swaption). In the case that $R_{fix} > K$, it means for the holder a value of

$$\begin{aligned} X_v - X_f &= (1 - B(T, t_n))L - \sum_{i=1}^{n} B(T, t_i) K (t_i - t_{i-1}) L \\ &= \sum_{i=1}^{n} B(T, t_i) R_{fix} (t_i - t_{i-1}) L - \sum_{i=1}^{n} B(T, t_i) K (t_i - t_{i-1}) L \\ &= (R_{fix} - K) \sum_{i=1}^{n} B(T, t_i)(t_i - t_{i-1}) L \end{aligned}$$

The *ith* term corresponds to the value of an European call option with expiry date T and coupon date t_i

$$L(t_i - t_{i-1})(R_{fix} - K)_+,$$

whereby R_{fix} is as introduced in section 2.3.

According to the Black Model (1976), the option value at time 0 as shown in footnote 1 is as follows:

$$B(0, t_i)(t_i - t_{i-1}) L [F \cdot N(d_1) - K \cdot N(d_2)],$$

whereby F is the forward swap rate and

$$d_1 = \frac{\ln \frac{F}{K} + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \qquad d_2 = \frac{\ln \frac{F}{K} - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

d_1 and d_2 remain as in (1). The forward swap rate F is obtained from (2), by replacing $B(T, t_i)$ with $F(T, t_i) = \frac{B(0,t_i)}{B(0,T)}$.

The value of the swap C_E itself is the summation over all individual call options, i.e. over all i. And we get:

$$C_E = LA[FN(d_1) - KN(d_2)],$$

whereby

$$A = \sum_{i=1}^{n} B(0, t_i)(t_i - t_{i-1})$$

For an European receiver swaption in like manner, if $K > R_{fix}$ the the following holds:

$$\begin{aligned} X_f - X_v &= \sum_{i=1}^{n} B(T,t_i)K(t_i - t_{i-1})L - (1 - B(T,t_n))L \\ &= \sum_{i=1}^{n} B(T,t_i)K(t_i - t_{i-1})L - \sum_{i=1}^{n} B(T,t_i)R_{fix}(t_i - t_{i-1})L \\ &= (K - R_{fix}) \sum_{i=1}^{n} B(T,t_i)(t_i - t_{i-1})L \end{aligned}$$

The *ith* term corresponds to the value of an European put option with expiry date T and coupon date t_i.

$$L(t_i - t_{i-1})(K - R_{fix})_+$$

According to the Black Model (1976), the option value at time 0 as shown in footnote 1 is as follows:

$$B(0,t_i)(t_i - t_{i-1})L[K \cdot N(-d_2) - F \cdot N(-d_1)],$$

whereby F, d_1, d_2 are as defined above, d_1 and d_2 are as introduced in (1). The forward swap rate F is obtained from (2) by substituting[15] $B(T,t_i)$ with $F(T,t_i) = \frac{B(0,t_i)}{B(0,T)}$.

The swap value P_E is the summation over all individual put options, i.e. over all i. And we get:

$$P_E = L \cdot A[K \cdot N(-d_2) - F \cdot N(-d_1)],$$

whereby

$$A = \sum_{i=1}^{n} B(0,t_i)(t_i - t_{i-1})$$

4. Loan Banking

4.1. Concepts.

The idea behind loan banking can be illustrated with the following example:

Assuming that there are ten individuals each of who wants to build a house of the same size with none of them having sufficient capital to do so. If each of them saves ten percent of the required amount per year, each would be capable of building after ten years. However, if these individuals get together, the first person will already build after one year. In the subsequent nine years he will be busy amortizing his loan, so that the second person builds after two years etc. By so doing, the average waiting time to build is reduced from 10 to 5.5 years.

[15] Robert V. Kohn, [15].

The idea of getting together to form a *building cooperative*[16] is the basis upon which *loan banking* is founded.

Loan banking started in the United Kingdom in the late 18th. century in the form of closed cooperatives. These *Building Societies* as they were called had a limited number of members though.

4.2. **Contractual Saving for Housing.** Under this scheme, this loan bank offers loans to individuals and corporate bodies for the following purposes:

(1) construction and acquisition of a home
(2) renovation and completion of building projects
(3) purchase of building plots

However, to be eligible for such a lone, the aspirant has to open a savings account at *Loan Bank*. The Contractual Saving for Housing thus offers the owner of the contract a whole range of opportunities related to ownership of a real estate. The loan bank offers different types of contracts which generally evolve in the same trend.

The evolution of the contract can be classified into four phases - the contracting phase, the saving phase, the disbursement phase and the amortizing phase:

- **Contracting phase:**The owner signs the contract with the loan bank. The contract is a savings agreement for building purposes and locks conditions such as savings sum[17] and tariff[18]. The saving agreement also spells out the interest rate on saving and interest rate on loan. An inial deposit is paid in during this phase.
- **Saving phase:** The owner pays deposits in their saving account during this period. The goal of this phase is to fulfil the minimum requirements[19] that would make the contract eligible for the next phase (disbursement).
- **Disbursement phase:** Disbursement[20] takes place upon fulfillment of minimum requirements and on condition that the loan bank has the means[21] available.

---

[16] The terms building society, loan bank, building cooperative are used in this paper interchangeably.

[17] amount to be saved by customer plus loan to be obtained from building society as specified in the contract upon opening of account.

[18] Interest rates may differ depending on tariffs. Moreover, the fee charged upon conclusion of the contract, waiting period for contract maturity etc. also vary.

[19] As a rule, a minimum period of saving and a minimum amount to be saved are set in the loan agreement.

[20] The owner of the contract gets paid the contract sum (saving in account plus loan).

[21] Contracts are ranked and disbursed according to evaluation number. The evaluation number of a contract is an assessment of the intensity of saving with time $= \int_0^T \text{savings}(t)dt$, t being time.

The owner of a contract that is eligible for disbursement has the option of either accepting disbursement, deferring disbursement or terminating the contract (i.e. desists from taking loan)[22]. One of the reasons for terminating the contract at this stage could be the availability of a cheaper loan in the capital market.

- **Loan amortizing phase:** Systematic reimbursement of loan plus interest[23] levied on loan.

In an attempt here to model the aforementioned loan banking operations mathematically, a discrete model is chosen, given that in reality, financial transactions take place in discrete time steps [25].

Parameters for Building Society are an aggregate of those for individual contracts that make up the cooperative.

4.3. **Individual Contract Model.** Transactions involving a contract can be divided into four main phases as follows: Opening of account, saving, assignment, loan.

4.3.1. *Contracting and Saving.* Upon opening of an account, the contract is signed between the bank and the customer. This contract spells out contract conditions such as *contract volume tariff* etc.

The account once opened, the *saving phase* begins and money is subsequently saved in the account.

The following relationship exists for the savings S of customer i at time t :

$$S_i(t) = S_i(t-1) + P_i(t) + I_i(t),$$

whereby $S_i(t)$, for instance, denotes the cumulative Savings of customer or account i in year t. In the same logic, I is cumulative interest rate and P, cumulative deposit made in the account in the course of the year. The following relationship holds for the interest:

$$I_i(t) = \sum_{j=1}^{n} I_i(t_j)$$

$$I_i(t_j) = \frac{p}{n}\left(S_i(t-1) + \sum_{k=1}^{j} P_i(t_k)\right) \qquad j = 1, \ldots, n.$$

p is interest rate per annum and $t_1 < \cdots < t_n$ are discrete depositing periods within a year.

The saving phase of account i begins soon after opening of the account at time $t = t_{b,i}$ and ends upon assignment $t = t_{a,i}$. At time $t = t_{a,i}$, the owner of account i gets his entire saving plus loan paid out to him/her, except the owner decides to wait.

[22]The contract gets paid just its savings.

[23]The loan agreement contract already guarantees a fixed interest rate for the loan.

Generally, the following system of equations describes the saving phase (t stands for yearly periods, t_j stands for periods under one year, $t_{b,i}-1$ stands for the period prior to signing the contract):

$$S_i(t_{b,i}-1)=0$$

$$S_i(t)=S_i(t-1)(1+p)+\sum_{j=1}^{n}\left(P_i(t_j)+\sum_{k=1}^{j-1}\frac{p}{n}P_i(t_k)\right)$$

$$t=t_{b,i},\ldots,t_{a,i}$$

If at any point in time t_{close} a customer decides to terminate the contract, their saving is paid out to him/her.

4.3.2. *Disbursement.* an account that fulfils certain minimum requirements (is is going to be spelt out below) and whose eligibility coefficient lies above the target eligibility coefficient becomes eligible for disbursement. That is, such an account is paid back its savings plus loan amount. The payment of the loan for an account automatically causes the transition into the next phase i.e., the loan phase. The disbursement date, depending on the tariff in question, is also determined by the following three factors: The *minimum waiting period,* the *minimum saving coefficient* and the *minimum eligibility coefficient.*

- **Eligibility:** an assessment of the intensity of saving with time $=\int_0^T \text{savings}(t)dt$, t being time.
- **Savings coefficient:** savings in an account as a fraction of total nominal amount of accounts not yet approved for loan.
- **Waiting period:** After signing up with the loan bank, you are not eligible for a building loan until after saving a certain stipulated period of time .

4.3.3. *The Loan Phase.* This is the phase during which the loan is amortized. The net loan comprises the nominal amount minus saving. The net loan is paid out to the customer. This net loan is the amount to be amortized.

The customer pays a fixed amount, the amortizing amount (AA) at regular intervals (as agreed) to the cooperative. This so called amortization amount comprises the loan reimbursement plus interest on loan [24] (LI). This amounts to the following model for periods under one year t_j:

$$LI_i(t_j)=\frac{q}{n}\cdot Loan_i(t_{j-1}),$$

$$Loan_i(t_j)=Loan_i(t_{j-1})+LI_i(t_j)-AA_i(t_j),$$

[24]The method interest payment varies from tariff to tariff.

q being the *fixed* interest on loan. The remaining loan for each new period is obtained by subtracting the mount so far amortized from the foregoing loan plus the interest so far paid:

$$Loan_i(t) = (1 + \frac{q}{n})^n \cdot Loan_i(t-1) - \sum_{j=1}^{n} (1 + \frac{q}{n})^{n-j} AA_i(t_j).$$

The contract is automatically terminated when the entire loan is amortized.

4.4. **Aggregate Model.** The parameters required in the mathematical modelling of the building cooperative are outlined as follows:

- saving;
- loan;
- payment into account (during saving phase);
- interest and amortization payments (during loan phase);
- saving payout;
- loan payout;
- interest on saving;
- interest on loan.

With this in mind, a simple generalized model for a loan bank operating on a single tariff is set up. Let the number of customers be N, the number of periods in a year be n. The following relationships involving savings and loans is set up:

$$S(t) = \sum_{i=1}^{N} \left(S_i(t-1)(1+p) + \sum_{j=1}^{n} (P_i(t_j) + \sum_{k=1}^{j-1} \frac{p}{n} (P_i(t_k)) \right)$$

$$Loan(t) = \sum_{i=1}^{N} \left((1 + \frac{q}{n})^n \cdot Loan_i(t-1) - \sum_{i=1}^{n} \left((1 + \frac{q}{n})^{n-j} AA_i(t_j) \right) \right)$$

The savings and interest during saving phase is also modelled as follows:

$$S(t) = \sum_{i=1}^{N} S_i(t)$$

and

$$I(t) = \sum_{i=1}^{N} I_i(t)$$

The rest of the cooperative-wide aggregate parameters such as interest on savings and interest on loan are similarly computed from sum of individual accounts.

Here are a few more parameters that are required in modelling such a cooperative:

- level of fresh business;
- unassigned lot(liquidity);

- assigned lot;
- liquidity coefficient;
- available lot.

The unassigned lot ($UL(t)$) at time t is the overall saving Sin all accounts that are in the saving phase.

$$UL(t) = \sum_{i=1}^{N} S_i(t),$$

$$t_{b,i} \leq t \leq t_{a,i}.$$

The assigned lot ($AL(t)$) at time t is the overall sum of amounts in all accounts that have been assigned.

$$AL(t) = \sum_{i=1}^{N} NA_i(t),$$

$$t_{a,i} \leq t \leq t_{e,i}.$$

whereby $t_{e,i}$ is the final date.

Computing the target eligibility coefficient: A target eligibility coefficient is usually set for the period under consideration.

One however has to know first of all, how much resource is available for assignment. This so called *available lot* (AvL)is computed as a function of the savings and loan of that period. To the saving is added the deposit and in-coming interest payments that is expected upon assignment time. From the saving is also subtracted the saving payout as well as interest on saving. The loan is subtracted the amortization amount and to it is added loan payout.

$$AvL(t) = S(t) + P(t) - I(t) - Loan(t) + AA(t) - SP(t) - LoanP(t),$$

whereby $SP(t)$ stands for saving payout and $LoanP(t)$ for loan payout.

The target eligibility coefficient is determined, based on the available lot. All customers fulfilling the minimum requirements are sorted in descending order of eligibility coefficient. This list is assigned until the available lot gets finished. The eligibility coefficient of the last assigned account is the target eligibility coefficient.

In order to be able to follow-up cooperative development, the *liquidity ratio* ([16] and [17]) could be used to assess the development of the cooperative. certain parameters have been suggested in the past. The liquidity ratio represents the ratio of total loans to total saving.

$$\text{Liquidity ratio}(t) = \frac{Loan(t)}{S(t)}$$

It should be in the best interest of the cooperative to keep this ratio < 1, otherwise she would have to go borrowing. And as you know, borrowing is expensive.

5. The Loan Contract As Swaption

A contract that effectively gets disbursed at the disbursement phase does enter the loan phase and pays interest on the entire building loan at a fixed predetermined interest rate. During the loan phase, such a contract cannot take advantage of lower interest rates in the financial market without having to terminate the contract.

The fact that the customer may terminate the contract at the disbursement phase and desist from taking the loan, and instead obtain the loan from elsewhere at a lower rate, makes this contract similar to an *option* on a forward *interest rate swap* or *swaption* of the *European type*. Remember that the holder of a *swaption* has the right to choose between the market rate and the contractual interest rate at expiry[25]. Therefore this is an option on an amortizing swap.

Definition 3. *An amortizing swap is usually an interest rate swap in which the notional principal for the interest payments declines during the life of the swap.*

The notional principal amount in this case is the building loan in its amortizing phase. In the same vein, the expiry date of the option is the disbursement date of the loan contract. And there is an option on the contractual loan rate at the disbursement date.

6. Outlook

This author intends, in his future research, to set up a mathematical pricing model for forward amortization swaps as discussed above for loan-bank contracts.

References

[1] F. Black, *The Pricing of Commodity Contracts*, in Journal of Financial Economics,3, 1976, 167-179.

[2] F. Black, E. Derman and W. Toy, *A One-Factor Model of Interest Rates and its Application to Treasury Bond Options*, Financial Analysts Journal, Jan-Feb, 1990, 33-39.

[3] F. Black, M. Scholes, *The Pricing of Options and Corporate Liabilities*, in Journal of Political Economics,81, 1973, 637-654.

[4] B. Biermann, *Die Mathematik von Zinsinstumenten*, Oldenbourg Verlag, 1999, Mnchen.

[5] R.R.Jr. Bliss, E.I. Ronn, *Arbitrage-Based Estimation of Nonstationary Shifts in the Term Structure of Interest Rates*, Journal of Finance, 44, 1989 London, 591-610.

[6] M. Bs, *Optionsbewertung und Kapitalmarkt*, Verlag Josef Eul, Bergisch Gladbach/Kln, 1990.

[7] E. Crow, K.E. S Shimizu, *Lognormal Distributions, Theory and Applications*, Marcel Dekker Inc., 1988.

[8] F. Fabozzi, *Bond Markets, Analysis and Strategies*, Prentice-Hall, New Jersey, 1989.

[9] B. Ganter, R. Wille, *Formale Begriffsanalyse: Mathematische Grundlagen*, Springer, Berlin, Heidelberg, 1996.

[10] F. Heitmann, *Bewertung von Zinsfutures*, Diplomarbeit am Instutut fr Entscheidungstheorie und Unternehmensforschung, Universitt Karlsruhe(TH), Januar (1992), Fritz Knapp Verlag, Frankfurt am Main 1992.

[25]The expiry date in this case would be the the date of disbursement.

[11] T.S.Y. Ho, S.B. Lee, *Term Structure Movements and Pricing Interest Rate Contingent Claims*, Journal of Finance, 41, 1986, 1011-1029.

[12] J. Hull, *Options,Futures and Other Derivative Securities*, Englewood Cliffs, New Jersey:Prentice Hall 1989.

[13] J. Hull, A. White, *On Derivatives*, Risk Publications, London, 1996.

[14] B. Knab, R. Schrader, I. Weber, K. Weinbrecht, B. Wichern *Mesoskopisches Simulationsmodell zur Kollektivfortschreibung*, Center for Applied Computer Science, Report 97.295, 1997.

[15] R.V. Kohn, *Derivative Securities - Section 11*, `http://www.math.nyu.edu/faculty/kohn/derivative.securities/section11.pdf`

[16] H. Laux, *Entwicklung der Bauspartechnischen Kennzahlen bei den privaten und den oeffentlich-rechtlichen Bausparkassen bis 1989*, in: Blaetter der Deustchen Gesellschaft fuer Versicherungsmathematik e.V., XVI.1(1): Pages 37 - 60, April 1991

[17] H. Laux, *Verlauf der Bauspartechnischen Kennzahlen in an- und auslaufenden Tarifbestaenden des Bausparens*, in: Blaetter der Deustchen Gesellschaft fuer Versicherungsmathematik e.V., XVI.3(3): Pages 365 - 371, April 1992

[18] W. Lehmann, *Die Bausparkassen*, Fritz Knapp Verlag, Frankfurt am Main, 1965.

[19] B. Luderer, D. Akume *Einige Aspekte der Bewertung von Swaptions*, Technische Universitaet Chemnitz, Fakultaet fuer Mathematik, Preprint 2001-15.

[20] B. Luderer, O. Zuschanke, *Ein einheitlicher Zugang zum Pricing von Swaps*, Preprint 2000-14, TU Chemnitz.

[21] R.F.M.L. Obermann, *Zinsrisikopotential-Kennziffer zur Quantifizierung des Zinsrisikos von Zinsswaps, -Futures und -Optionen*, Fritz Knapp, Frankfurt am Main, 1990.

[22] Z.E. Prisman, *Pricing Derivative Securities*, Academic Press, 2000.

[23] R. Rauleder, *Bewertung, Anwendungsmglichkeiten und Hedginsstrategien von Swaptions*, Fritz Knapp Verlag, Frankfurt am Main, 1994.

[24] P. Reiner, *Zur analytischen Bewertung von Zinsoptionen*, Verlag Peter Lang GmbH, Frankfurt/Main, 1991.

[25] I.M. Vannahme, *Clusteralgorithmen zur mathematischen Simulation von Bausparkollektiven*, doctoral thesis, University of Cologne, Cologne, 1996.

[26] G.-W. Weber, *Mathematische Optimierung in Finanzwirtschaft und Risikomanagement - diskrete, stetige und stochastische Optimierung bei Lebensversicherungen, Bausparvertrgen und Portfolios*, lecture held at Chemnitz University of Technology, summer semester 2001.

[27] P. Wilmott, S. Howison, J. Dewynne, *The Mathematics of Financial Derivatives*, Cambridge, 1995.

[28] P. Wilmott, *Week 9: The Black-Scholes Solution And The "Greeks"* , `http://www.ph.qmw.ac.uk/~oleg/Week9.pdf`

Computer Science Department, University of Buea, P.O. Box 63, Buea, Cameroon
E-mail address: `daniel.akume@minesup.gov.cm`