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APPLICATIONS OF SPATIAL DATABASES AND STRUCTURES TO THE STUDY OF MIOCENE DEPOSITS OF BOROD BASIN

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Abstract. In this paper we apply spatial database and structure to render the paleo-relief of the Borod Basin. The boreholes data are stored in a spatial database. For the effective surface rendering we use local-Shepard interpolation with variable radius, based on a spatial grid and Delaunay triangulation. The generated pictures are more realistic compared to picture generated by mean of other methods.

1. Local Shepard Interpolation

The classical Shepard operator (see [19]) defined by

$$(S_{n,\mu}f)(x) = \sum_{k=0}^{n} w_k(x) f(x_k) \tag{1}$$

$$w_k(x) = \frac{|x - x_k|^{-\mu}}{\sum\limits_{k=0}^{n} |x - x_k|^{-\mu}}, \tag{2}$$

where $|.|$ denotes the Euclidean norm in $\mathbb{R}^s$, and $X = \{x_0, x_1, \ldots, x_n\} \subset \mathbb{R}^s$ is a set of $n+1$ pairwise distinct points, requires a large amount of computation. The volume of computation can be reduced replacing the weight functions given by (2) with the so called Franke-Little weights:

$$\bar{w}_k(x) = \frac{\dfrac{(R - |x - x_k|)_+^{\mu}}{R^{\mu}|x - x_k|^{\mu}}}{\sum\limits_{i=0}^{n} \dfrac{(R - |x - x_i|)_+^{\mu}}{R^{\mu}|x - x_i|^{\mu}}} \tag{3}$$

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(see [14, 12, 13]). In (3), R is a given positive real constant, and the + subscript denotes the positive part. Thus we obtain the *local Shepard-operator*:

$$\left(\bar{S}^L_{n,\mu} f\right)(x) = \sum_{k=0}^{n} \bar{w}_k(x) f(x_k). \tag{4}$$

This operator reproduces the values of f in x_k and has the degree of exactness equal to zero, that is reproduces the constant.

In order to increase the degree of exactness one tries to replace the values $f(x_k)$ with the values of an interpolation operator: Taylor [9, 11, 8, 3, 10, 6], Lagrange [4, 5, 6], Hermite [1, 5, 6], Birkhoff [2, 5, 6], least square approximation [18, 16, 17, 21] and even spline[6]. The operators obtained in this way are called *combined Shepard operators.*

In this paper we are interested in simple local Shepard operator, given by (4).

2. Spatial data structures

In order to compute the various local Shepard-type interpolants we are interested to report efficiently the point located into the ball $B(x, R)$. The naive approach (computing $d_k = |x - x_k|$ and checking $d_k < R$) needs a time $O(n)$ for each point x. Computational geometry techniques and data structures allow us to perform this task in polylogarithmic time.

Let $P := \{p_1, \dots, p_n\}$ be a set of point from $\mathbb{R}^s$ and Reg a region from the same space. A s-dimensional *range searching problem* asks for the points from P lying inside the query region Reg. If the region is a hyperparallelopiped, i.e. $Reg = [x_1, x'_1] \times \cdots \times [x_s, x'_s]$, then we have an *orthogonal range-searching problem.* If Reg is a ball from $\mathbb{R}^s$, we have a *circular range searching problem.* Our approach is to solve a simpler orthogonal range searching problem instead the circular range searching (since this approach eliminates a large number of points) and then to check the reported points.

One of the most used data structure for orthogonal range query is the *range tree*[7]. A solution based on range tree is given in [20].

Another solution is inspired from a paper of Renka[18] and presented extensively in [21]. The smallest bounding box containing the interpolation nodes $\prod_{k=1}^{s} \left[x^k_{\min}, x^k_{\max}\right]$ is partitioned into an uniform grid of cells, having NR cells on each dimension. Each cell points to the list of point indices contained in that cell. Such an example for the 2D case is given in Figure 1. The algorithm 1 describes the creation of the data structure. If the second argument NR is not provided, we can initialize it with a default value; Renka suggests in [16]

$$NR = \lfloor (N/3)^{1/\dim} \rfloor.$$

The orthogonal range searching is easy to implement using this data structure (the algorithm 2): first the cell which must be scanned are determined (i.e. the cell

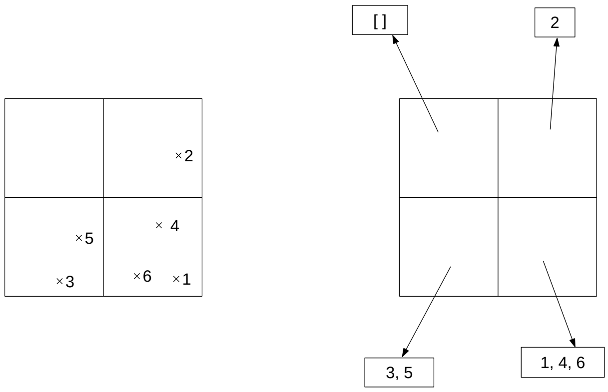

FIGURE 1. A 2D grid of cell and its representation

which intersects the searching domain), and then the list of points corresponding to that cell are concatenated. The points from the outer cells which lie outside the searching range must be eliminated.

Algorithm 1 Creating the cell grid

Input: the set of N points P, the number of cells, NR (optional);
Output: a grid of cell LCELL, each containing the list of points in the cell
set all cells to **nil**;
{compute the cell sizes}
$dc_1 := \min(NR, \lfloor x^1_{\max} - x^1_{\min} \rfloor + 1)$;
$\vdots$
$dc_s := \min(NR, \lfloor x^s_{\max} - x^s_{\min} \rfloor + 1)$;
for $K := N$ **downto** 1 **do**
  {find the cell}
  $i_1 := \min(NR, \lfloor x^k_1 - x^1_{\min} \rfloor + 1)$;
  $\vdots$
  $i_s := \min(NR, \lfloor x^k_s - x^s_{\min} \rfloor + 1)$;
  add K to the list $LCELL(i_1, \ldots, i_s)$;
end for

Now we are able to compute the local Shepard interpolant on a set of points X:

- build the spatial data structure;
- for each point x in X

– perform the orthogonal range searching into the hypercube centered in x and with the radius R
– apply formulas (3) and (4).

Algorithm 2 The orthogonal range searching

$PTLIST$:= **nil**;
{determine the outer cells, i. e. the scan limits}
$imin_1 := \max(1, \lfloor (liminf_1 - x^1_{\min})/dc_1 \rfloor + 1);$
$imax_1 := \min(NR, \lfloor (limsup_1 - x^1_{\min})/dc_1 \rfloor + 1)$
$\vdots$
$imin_s := \max(1, \lfloor (liminf_s - x^s_{\min})/dc_s \rfloor + 1);$
$imax_s := \min(NR, \lfloor (limsup_s - x^s_{\min})/dc_s \rfloor + 1)$
for $i_1 := imin_1$ **to** $imax_1$ **do**
  $\vdots$
  for $i_s := imin_s$ **to** $imax_s$ **do**
    $JL := LCELL(i_1, \dots, i_s);$
    if the cell $(i_1, \dots, i_s)$ is peripheral **then**
      remove the points which lay cell outside the searching range from JL;
    end if
    concatenate $PTLIST$ and JL
  end for
  $\vdots$
end for

This approach has a drawback: the accuracy tends to decrease into the areas where the interpolation nodes are sparse. We can avoid this situation, allowing the radius R to vary with k: the radii are chosen such that the ball $B(x, R)$ contains at least N_w nodes. Thus, instead of an orthogonal range searching we perform a N_w-th nearest neighbor search of x_j and x, respectively. This can be done scanning the grid in a circular fashion starting with the cell containing x. In order to facilitate the scanning we can associate a Boolean indicator to each cell, which is true when the cell was already scanned.

3. The geological data

Borod Depression is located in the western part of the Apuseni Mountains, being bordered by Plopisului Mountains in the north and by Padurea Craiului in the south.

This depressionary area was formed about 12-16 million years ago, during the Badenian, along a fault located on the northern border, in contact with the Plopis

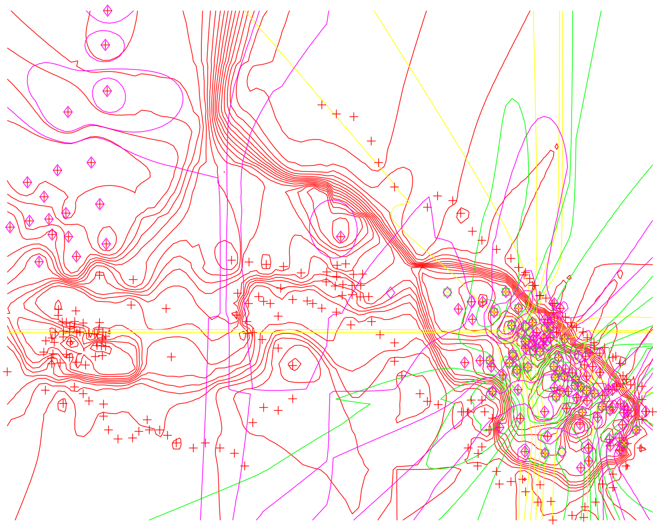

FIGURE 2. Geological boreholes

Mountains. Subsequently it represented a sedimentary basin where a thick succession of sediments (200-1000m) was accumulated.

The Neogene deposits representing the filling of the basin were assigned to three different formations: Borod Formation (Badenian), Cornitel Formation (Sarmatian), and Beznea Formation (Pannonian) [15]. Each formation (unit) consists of banks of rocks with a variable thickness, separated according to well-established criteria.

This area was investigated during the last three decades by using geological drilling (Figure 2. Geological studies based on borehole data imply the drawing of:

- longitudinal and transversal geological profiles
- Isogram maps (isobath and isopachyte)
- 3-D modelling of the paleorelief at various levels (basement and top of the formations);

For each borehole the database contains the following information:

- The borehole ID;
- The abscissa and the ordinate of the borehole;
- From one to four z-coordinates representing the borehole depth, corresponding to Basement, Badenian, Sarmatian, and Pannonian age.

The graphical representations offer a suggestive image on the specific features of the basin formation and on its evolution in time. In the previous decades the graphs – except for the 3-D – were performed manually. This stage was partly overcome due to attempts to electronic processing of data, including 3-D modelling. It is worth to mention that on an international level, computer graphics is currently a common tool in geological sciences.

Our previous trials to draw 3-D block diagrams by using other types of operators were not satisfactory. On the contrary, the method presented in the paper clearly evidences the fractured areas (faults), the space arrangement of the geological blocks, and the relationships among the various formations. The modelling of the paleoenvironment at the basement level evidences the fault along the northern border that shaped the basin formation. In the same time, the sets of faults that shaped the pre-Neogene deposits (older than 65 million years) are also noticeable. The graphs obtained for the top of the formations (Borod-yellow, Cornitel-green, and Beznea-magenta) suggest the presence of some faults that have influenced also the geological structure of these deposits. Some of these faults represent older, basement-related ones that were subsequently reactivated; others are younger and were probably generated by petrographical discontinuities in the deposits that form the cover (Figures 3, 4).

A "scaled" reconstruction of a sedimentary basin at different stages of its evolution is an extremely useful tool in the geological research. 3-D block diagrams represent the most suggestive image of a basin at various formation stages if the method of interpolation of data between different drill locations used was based on the suitable operators.

The variable radius (range) local Shepard operator based on a rectangular network (grids) as well as the Delaunay triangulation definitely provide a more suggestive image on the spatial relationship between geological blocks.

The graph already presented are build on a rectangular grid. A more useful and realistic manner is to generate surfaces over a polygonal convex area. This can be achieved using a triangular grid the Delaunay triangulation (see [7]). The idea is to consider a polygonal area and a sufficiently large number of points in this area; we compute the function values on these points and then the Delaunay triangulation for this set of points. The membership of a point to a region is easily performed with this data structure. Finally, we render the surface on the triangular grid computed in this way.

Figures 5 and 6 give such representations for the surfaces given in Figures 3 and 4, respectively.

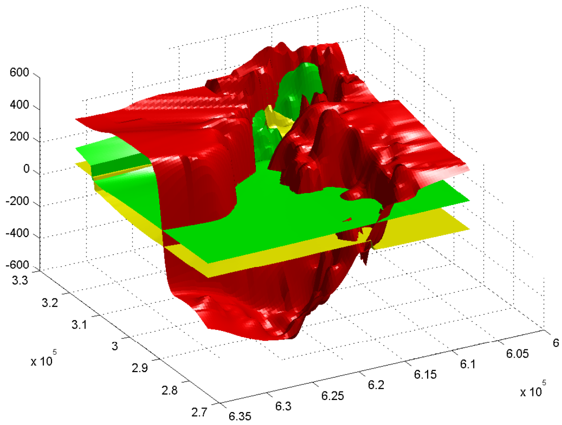

FIGURE 3. Representation: Basement – red, Badenian (top of Borod Formation) – yellow, Sarmatian (top of Cornițel Formation) – green

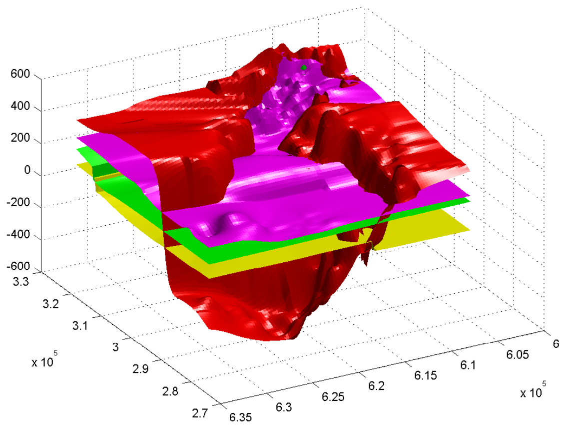

FIGURE 4. Representation: Basement – red, Badenian (top of Borod Formation)– yellow, Sarmatian (top of Cornițel Formation) – green, Pannonian (top of Beznea Formation) – magenta

4. Conclusions

The variable radius Shepard interpolation is a feasible approach for scattered data interpolation. Using spatial data structures leads us to efficient algorithms

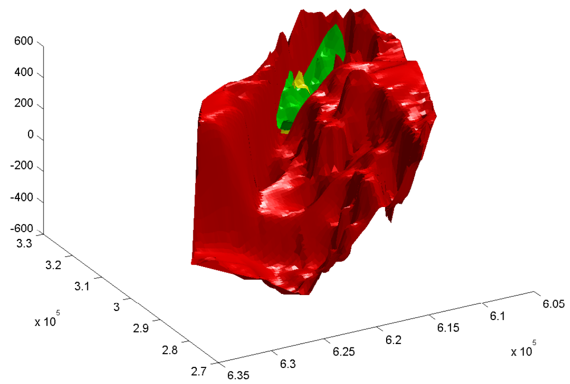

FIGURE 5. Representation of the surfaces from Figure 3 using Delaunay triangulation

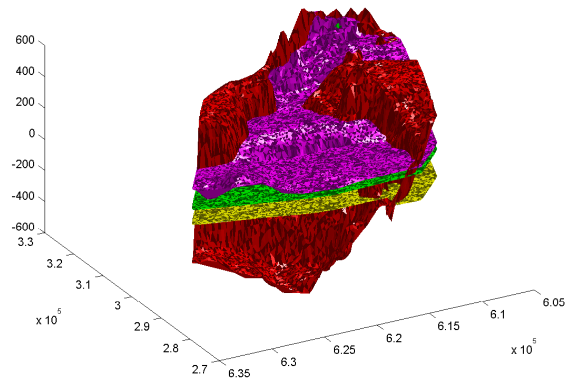

FIGURE 6. Representation of the surfaces from Figure 4 using Delaunay triangulation

for computing such interpolation operators and rendering the corresponding operators.

The method is also suitable from application point of view. It provides a more suggestive image on the spatial relationship between geological blocks and reveals some faults which other previous trials cannot reveal.

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