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THE MV-ALGEBRA STRUCTURE OF RGB MODEL

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Abstract. The aim of this paper is to explore the MV-algebra structure of RGB colour space (see [5]). We start with the construction of the MV-algebra structure of one component of the RGB model (this component represents one colour of the three that gives us the colour of a pixel on the screen). Then we define an MV-algebra structure on RGB model. Using Chang's Subdirect Representation Theorem we prove that RGB model is a subdirect product of MV-algebras of one component.

1. Introduction

Fuzzy sets are well known for their applications to image processing. It is also well known that the fuzzy sets have an MV-algebra structure (see [4]). We intend to develop an similar structure on RGB model (see[5]). This will allows us to use MV-algebras in image processing.

First we recall some definitions and properties of MV-algebras (see e.g. [2], [3]) that will be used later.

Definition 1.1. An MV-algebra is an algebra $\langle A, \oplus, \neg, 0_A \rangle$ with a binary operation $\oplus$, a unary operation $\neg$ and a constant 0_A satisfying the following equations:

(MVi) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$;
(MVii) $x \oplus y = y \oplus x$;
(MViii) $x \oplus 0_A = x$;
(MViv) $\neg\neg x = x$;
(MVv) $x \oplus \neg 0_A = \neg 0_A$;
(MVvi) $\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x$.

Remark 1.2. In particular, axioms (MV1)-(MV3) state that $\langle A, \oplus, 0_A \rangle$ is a commutative monoid. As usually, we denote an MV-algebra $\langle A, \oplus, \neg, 0_A \rangle$ by its universe A.

Remark 1.3. The constant 1_A and the operations $\odot$ and $\ominus$ are defined on each MV-algebra A as follows:

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i) $1_A =_{def} \neg 0_A$;
ii) $x \odot y =_{def} \neg(\neg x \oplus \neg y)$;
iii) $x \ominus y =_{def} x \odot \neg y$.

The following identities are immediate consequences of (MV4):

(MV7) $\neg 1_A = 0_A$;
(MV8) $x \oplus y = \neg(\neg x \odot \neg y)$.

Axioms (MV5) and (MV6) can now be written as:

(MV5') $x \oplus 1_A = 1_A$;
(MV6') $(x \ominus y) \oplus y = (y \ominus x) \oplus x$.

Setting $y = \neg 0_A$ in MV6) we obtain:

(MV9) $x \oplus \neg x = 1_A$.

Following common usage, we consider that $\neg$ operation is more binding than any other operation. Also we consider that $\odot$ operation is more binding than $\oplus$ operation and $\ominus$ operation.

Definition 1.4. Let A be an MV-algebra and $x, y \in A$. We say that $x \leq y$ if and only if x and y satisfy one of the bellow equivalent conditions:

i) $\neg x \oplus y = 1_A$;
ii) $x \odot \neg y = 0_A$;
iii) $y = x \oplus (y \ominus x)$;
iv) there is an element $z \in A$ such that $x \oplus z = y$.

Remark 1.5. It follows that $\leq$ is a partial order, called the natural order of A.

Definition 1.6. An MV-algebra whose natural order is total is called an MV-chain.

Lemma 1.7. *In every MV-algebra A the natural order $\leq$ has the following properties:*

i) $x \leq y$ if and only if $\neg y \leq \neg x$;
ii) if $x \leq y$ then for each $z \in A$, $x \oplus z \leq y \oplus z$ and $x \odot z \leq y \odot z$;
iii) $x \odot y \leq z$ if and only if $x \leq \neg y \oplus z$.

Proposition 1.8. *On each MV-algebra A the natural order determines a lattice structure. Specifically, the join $x \vee y$ and the meet $x \wedge y$ of the elements x and y are given by*

i) $x \vee y = (x \ominus y) \oplus y$;
ii) $x \wedge y = x \odot (\neg x \oplus y)$.

Definition 1.9. The distance function $d : A \times A \to A$ is defined by

$$d(x, y) =_{def} (x \ominus y) \oplus (y \ominus x).$$

Proposition 1.10. *In every MV-algebra A we have:*

i) $d(x, y) = 0_A$ *if and only if* $x = y$*;*
ii) $d(x, y) = d(y, x)$*;*
iii) $d(x, z) \leq d(x, y) \oplus d(y, z)$*;*
iv) $d(x, y) = d(\neg x, \neg y)$*;*
v) $d(x \oplus s, y \oplus t) \leq d(x, y) \oplus d(s, t)$.

2. Construction of MV-algebra structure of one component of RGB model.

RGB model represents one of the most used models to determin a pixel's colour on the screen. The number of colours that can be displayed is directly influenced by the number of bits on which the colours are stored in the computers memory. Also it is influenced by the properties of the screen.

RGB (see [5]) is defined as the set of triplets $(red, green, blue)$ or (r, g, b). The numbers form triplets represent how much red, green, respectively blue contains the pixel's colour.

We consider the number that represents one component of the triplet, stored on t bits. Therefore as the set of possible values for one colour component of RGB model we consider the set

$$C =_{def} \left\{ x \in \mathbb{R} \mid 0 \leq x \leq 2^{t-1} \right\}.$$

We introduce a binary operation $\oplus$, a unary operation $\neg$ and a constant 0_C as follows (when $x, y \in C$):

$$x \oplus y =_{def} \min(2^{t-1}, x + y); \tag{2.1}$$

$$\neg x =_{def} 2^{t-1} - x; \tag{2.2}$$

and 0_C is represented by the real number 0.

Lemma 2.1. *The quadruple* $\langle C, \oplus, \neg, 0 \rangle$ *is an MV-algebra.*

Proof. For proving that $\langle C, \oplus, \neg, 0 \rangle$ is an MV-algebra we have to show that the axioms (MV1)-(MV6) are satisfied.

(MV1): From equation (2.1) we have

$$x \oplus (y \oplus z) = x \oplus \min(2^{t-1}, y + z)$$
$$= \min(2^{t-1}, x + \min(2^{t-1}, y + z)) = \min(2^{t-1}, x + y + z)$$
$$= \min(2^{t-1}, \min(2^{t-1}, x + y) + z) = \min(2^{t-1}, x + y) \oplus z = (x \oplus y) \oplus z.$$

(MV2): Also by equation (2.1) we have

$$x \oplus y = \min(2^{t-1}, x + y) = \min(2^{t-1}, y + x) = y \oplus x.$$

(MV3): Using equation (2.1) we obtain

$$x \oplus 0 = \min(2^{t-1}, x+0) = \min(2^{t-1}, x) = x.$$

(MV4):From equation (2.2) we obtain

$$\neg\neg x = \neg(2^{t-1} - x) = 2^{t-1} - (2^{t-1} - x) = x.$$

(MV5): By equations (2.1) and (2.2) it is easy to see that

$$x \oplus \neg 0 = x \oplus (2^{t-1} - 0) = x \oplus 2^{t-1} = \min(2^{t-1}, x + 2^{t-1})$$
$$= 2^{t-1} = 2^{t-1} - 0 = \neg 0.$$

(MV6): For proving (MV6) we will transform each side of equation to the same expression.

Using equation (2.2) the left side of axiom (MV6) is

$$\neg(\neg x \oplus y) \oplus y = \neg((2^{t-1} - x) \oplus y) \oplus y.$$

Then applying equation (2.1) we obtain

$$\neg(\neg x \oplus y) \oplus y = \neg \min(2^{t-1}, 2^{t-1} - x + y) \oplus y.$$

Applying now several times the equations (2.1) and (2.2) we have

$$\neg(\neg x \oplus y) \oplus y = (2^{t-1} - \min(2^{t-1}, 2^{t-1} - x + y)) \oplus y = \max(0, x - y) \oplus y$$
$$= \min(2^{t-1}, \max(0, x - y) + y) = \min(2^{t-1}, \max(y, x)).$$

Since both x and y are less or equals then 2^{t-1}we have

$$\max(y, x) \leq 2^{t-1}$$

and then we obtain

$$\neg(\neg x \oplus y) \oplus y = \max(y, x).$$

Using now the commutativity of max function we obtain

$$\neg(\neg x \oplus y) \oplus y = \max(x, y) \qquad (1).$$

Using equation (2.2) the right side of axiom (MV6) is

$$\neg(\neg y \oplus x) \oplus x = \neg((2^{t-1} - y) \oplus x) \oplus x.$$

Then applying equation (2.1) we obtain

$$\neg(\neg y \oplus x) \oplus x = \neg \min(2^{t-1}, 2^{t-1} - y + x) \oplus x.$$

Applying now several times the equations (2.1) and (2.2) we have

$$\neg(\neg y \oplus x) \oplus x = (2^{t-1} - \min(2^{t-1}, 2^{t-1} - y + x)) \oplus x = \max(0, y - x) \oplus x$$
$$= \min(2^{t-1}, \max(0, y - x) + x) = \min(2^{t-1}, \max(x, y)).$$

Since both x and y are less or equals then 2^{t-1} we have

$$\max(x, y) \leq 2^{t-1}$$

and then we obtain

$$\neg(\neg y \oplus x) \oplus x = \max(x, y) \qquad (2).$$

From the equations (1) and (2) we obtain

$$\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x.$$

□

The constant 1_C and the operations $\odot$ and $\ominus$ are defined on MV-algebra C as follows:

i) $1_C =_{def} \neg 0$.

From equation (2.2) we obtain

$$1_C = 2^{t-1}. \tag{2.3}$$

ii) $x \odot y =_{def} \neg(\neg x \oplus \neg y)$.

Also by equation (2.2) we obtain

$$x \odot y = 2^{t-1} - ((2^{t-1} - x) \oplus (2^{t-1} - y)).$$

From equation (2.1) we have

$$x \odot y = 2^{t-1} - \min(2^{t-1}, 2^{t-1} - x + 2^{t-1} - y).$$

After calculations we obtain

$$x \odot y = \max(0, x + y - 2^{t-1}).$$

From equation (2.3) we have

$$x \odot y = \max(0, x + y - 1_C). \tag{2.4}$$

iii) $x \ominus y =_{def} x \odot \neg y$.

From equation (2.2) we obtain

$$x \ominus y = x \odot (2^{t-1} - y).$$

By the equations (2.3) and (2.4) we have

$$x \ominus y = \max(0, x - y). \tag{2.5}$$

Remark 2.2. We can introduce an order relation $\leq$ on $\langle C, \oplus, \neg, 0\rangle$ in the same way like in the general case (see Definition 1.4).

It is easy to see that the natural order on $\langle C, \oplus, \neg, 0\rangle$ is a total order and thus the MV-algebra C is an MV-chain. Since $C \subseteq \mathbb{R}$, it is obvious that C is a complete latice.

The order relation on MV-algebra C is induced by the order relation of real numbers.

The distance function $d : C \times C \to C$ is defined by:

$$d(x, y) =_{def} (x \ominus y) \oplus (y \ominus x)$$

Using equation (2.5) we obtain

$$d(x, y) = \min(1_C, \max(0, x - y) + \max(0, y - x)) = \min(1_C, |x - y|).$$

Since both x and y are less or equals then 1_C we have

$$|x - y| \leq 1_C$$

and then we obtain

$$d(x, y) = |x - y| \,. \tag{2.6}$$

Remark 2.3. The distance function is defined following the general case (See Definition 1.9). It is obvious that all the properties of the distance are fullfiled (see Proposition 1.10).

Observe also that we obtain the classical Euclidean distance.

3. Construction of MV-Algebra structure of RGB model

In the previous section we have considered the set C as the set of possible values for one colour component of RGB model (see [5]). We also have proved that $\langle C, \oplus, \neg, 0\rangle$ is an MV-algebra.

In this section we will introduce on RGB model an MV-algebra structure. Let consider the set:

$$RGB =_{def} \{(c_1, c_2, c_3) \mid c_i \in C,\ i \in \{1, 2, 3\}\} \,.$$

In other words RGB is the direct product of family $\{C_i\}_{i\in\{1,2,3\}}$, where

$$C_i = C \text{ for all } i \in \{1, 2, 3\} \,.$$

On RGB set we introduce the operations $\oplus$, $\neg$ and the constant 0 as follows:

$$(c_1, c_2, c_3) \oplus (d_1, d_2, d_3) =_{def} (c_1 \oplus d_1, c_2 \oplus d_2, c_3 \oplus d_3) \tag{3.1}$$

for each (c_1, c_2, c_3) and (d_1, d_2, d_3) from RGB;

$$\neg(c_1, c_2, c_3) =_{def} (\neg c_1, \neg c_2, \neg c_3) \tag{3.2}$$

for each (c_1, c_2, c_3) from RGB;

$$0_{RGB} =_{def} (0, 0, 0) \tag{3.3}$$

where 0 is the constant 0 on MV-algebra C.

Theorem 3.1. *The quadruple $\langle RGB, \oplus, \neg, 0_{RGB}\rangle$ is an MV-algebra.*

Proof. For proving that $\langle RGB, \oplus, \neg, 0_{RGB}\rangle$ is an MV-algebra we have to show that the axioms (MV1)-(MV6) are satisfied.

(MV1): From equation (3.1) we have

$$\begin{aligned}(a_1, a_2, a_3) \oplus ((b_1, b_2, b_3) \oplus (c_1, c_2, c_3))\\ = (a_1, a_2, a_3) \oplus (b_1 \oplus c_1, b_2 \oplus c_2, b_3 \oplus c_3)\\ = (a_1 \oplus (b_1 \oplus c_1), a_2 \oplus (b_2 \oplus c_2), a_3 \oplus (b_3 \oplus c_3)).\end{aligned}$$

By Lemma 2.1, the associative law holds in the MV-algebra C

$$a_i \oplus (b_i \oplus c_i) = (a_i \oplus b_i) \oplus c_i \text{ for } i = 1, 2, 3.$$

From this law we have

$$\begin{aligned}(a_1, a_2, a_3) \oplus ((b_1, b_2, b_3) \oplus (c_1, c_2, c_3)) =\\ = ((a_1 \oplus b_1) \oplus c_1, (a_2 \oplus b_2) \oplus c_2, (a_3 \oplus b_3) \oplus c_3).\end{aligned}$$

From this equality and from equation (3.1) we obtain

$$\begin{aligned}(a_1, a_2, a_3) \oplus ((b_1, b_2, b_3) \oplus (c_1, c_2, c_3))\\ = (a_1 \oplus b_1, a_2 \oplus b_2, a_3 \oplus b_3) \oplus (c_1, c_2, c_3)\\ = ((a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (c_1, c_2, c_3).\end{aligned}$$

(MV2): From equation (3.1) we have

$$(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 \oplus b_1, a_2 \oplus b_2, a_3 \oplus b_3).$$

Using the commutativity of MV-algebra C we obtain

$$(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (b_1 \oplus a_1, b_2 \oplus a_2, b_3 \oplus a_3).$$

From equation (3.1) we have

$$(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (b_1, b_2, b_3) \oplus (a_1, a_2, a_3).$$

(MV3): From the equations (3.1) and (3.3) it is easy to see that

$$(a_1, a_2, a_3) \oplus (0, 0, 0) = (a_1 \oplus 0, a_2 \oplus 0, a_3 \oplus 0) = (a_1, a_2, a_3)$$

since 0 is the neutral element of MV-algebra C.

(MV4): Applying several times the equation (3.2) we obtain

$$\begin{aligned}\neg\neg (a_1, a_2, a_3) = \neg (\neg a_1, \neg a_2, \neg a_3)\\ = (\neg\neg a_1, \neg\neg a_2, \neg\neg a_3) = (a_1, a_2, a_3),\end{aligned}$$

since $\neg\neg a = a$ in MV-algebra C.

(MV5): From equation (3.2) we have

$$(a_1, a_2, a_3) \oplus \neg (0, 0, 0) = (a_1, a_2, a_3) \oplus (\neg 0, \neg 0, \neg 0).$$

Using this equality and from equation (3.1) we obtain

$$(a_1, a_2, a_3) \oplus \neg (0,0,0) = (a_1 \oplus \neg 0, a_2 \oplus \neg 0, a_3 \oplus \neg 0)$$
$$= (\neg 0, \neg 0, \neg 0) = \neg (0,0,0),$$

since $a \oplus \neg 0 = \neg 0$, in MV-algebra C.

(MV6): From equation (3.2) we have

$$\neg (\neg (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3)$$
$$= \neg ((\neg a_1, \neg a_2, \neg a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3).$$

From equation (3.1) we obtain

$$\neg (\neg (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3)$$
$$= \neg (\neg a_1 \oplus b_1, \neg a_2 \oplus b_2, \neg a_3 \oplus b_3) \oplus (b_1, b_2, b_3).$$

Applying successively equations (3.2) and (3.1) we obtain

$$\neg (\neg (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3)$$
$$= (\neg (\neg a_1 \oplus b_1) \oplus b_1, \neg (\neg a_2 \oplus b_2) \oplus b_2, \neg (\neg a_1 \oplus b_3) \oplus b_3).$$

By (MV6) of MV-algebra C applied for each component we have

$$\neg (\neg (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3)$$
$$= (\neg (\neg b_1 \oplus a_1) \oplus a_1, \neg (\neg b_2 \oplus a_2) \oplus a_2, \neg (\neg b_3 \oplus a_3) \oplus a_3).$$

Applying now successively equation (3.1) we obtain

$$\neg (\neg (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3)$$
$$= (\neg (\neg b_1 \oplus a_1), \neg (\neg b_2 \oplus a_2), \neg (\neg b_3 \oplus a_3)) \oplus (a_1, a_2, a_3).$$

By equation (3.2) we have

$$\neg (\neg (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3)$$
$$= \neg (\neg b_1 \oplus a_1, \neg b_2 \oplus a_2, \neg b_3 \oplus a_3) \oplus (a_1, a_2, a_3).$$

Applying equations (3.1) and (3.2) we obtain

$$\neg (\neg (a_1, a_2, a_3) \oplus (b_1, b_2, b_3)) \oplus (b_1, b_2, b_3)$$
$$= \neg ((\neg b_1, \neg b_2, \neg b_3) \oplus (a_1, a_2, a_3)) \oplus (a_1, a_2, a_3)$$
$$= \neg (\neg (b_1, b_2, b_3) \oplus (a_1, a_2, a_3)) \oplus (a_1, a_2, a_3).$$

□

The constant 1_{RGB}, and the operations $\odot$ and $\ominus$ are defined as follows:

i) $1_{RGB} =_{def} \neg (0,0,0)$.

From equations (2.2) and (3.2) we obtain

$$1_{RGB} = (\neg 0, \neg 0, \neg 0) = \left(2^{t-1}, 2^{t-1}, 2^{t-1}\right).$$

By equation (2.3) we obtain

$$1_{RGB} = (1_C, 1_C, 1_C)$$

ii) $(a_1, a_2, a_3) \odot (b_1, b_2, b_3) =_{def} \neg(\neg\,(a_1, a_2, a_3) \oplus \neg\,(b_1, b_2, b_3))$.
Applying equation (3.2) we have:

$$(a_1, a_2, a_3) \odot (b_1, b_2, b_3) = \neg\left((\neg a_1, \neg a_2, \neg a_3) \oplus (\neg b_1, \neg b_2, \neg b_3)\right).$$

By equation (3.1) we obtain

$$(a_1, a_2, a_3) \odot (b_1, b_2, b_3) = \neg\left(\neg a_1 \oplus \neg b_1, \neg a_2 \oplus \neg b_2, \neg a_3 \oplus \neg b_3\right).$$

Applying equation (3.2) we have

$$(a_1, a_2, a_3) \odot (b_1, b_2, b_3) = \left(\neg\left(\neg a_1 \oplus \neg b_1\right), \neg\left(\neg a_2 \oplus \neg b_2\right), \neg\left(\neg a_3 \oplus \neg b_3\right)\right)$$

By definition of $\odot$ on MV-algebra C we obtain:

$$(a_1, a_2, a_3) \odot (b_1, b_2, b_3) = (a_1 \odot b_1, a_2 \odot b_2, a_3 \odot b_3) \tag{3.4}$$

iii) $(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) =_{def} (a_1, a_2, a_3) \odot \neg\,(b_1, b_2, b_3)$.
By equation (3.1) and 3.4 we have:

$$(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1, a_2, a_3) \odot (\neg b_1, \neg b_2, \neg b_3).$$

From equation (3.4) we obtain

$$(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 \odot \neg b_1, a_2 \odot \neg b_2, a_3 \odot \neg b_3).$$

By definition of $\ominus$ on MV-algebra C we obtain:

$$(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 \ominus b_1, a_2 \ominus b_2, a_3 \ominus b_3). \tag{3.5}$$

Remark 3.2. We can introduce a partial order relation $\leq$ on $\langle RGB, \oplus, \neg, 0_{RGB}\rangle$ in the same way like in the general case.

It is easy to verify that:

i) $(a_1, a_2, a_3) \leq (b_1, b_2, b_3)$ if and only if $a_i \leq b_i$, for each $i = 1, 2, 3$.
ii) $(a_1, a_2, a_3) \vee (b_1, b_2, b_3) = (a_1 \vee b_1, a_2 \vee b_2, a_3 \vee b_3)$
iii) $(a_1, a_2, a_3) \wedge (b_1, b_2, b_3) = (a_1 \wedge b_1, a_2 \wedge b_2, a_3 \wedge b_3)$.

Proposition 3.3. *RGB has a complete lattice structure, and this implies that RGB is an MV σ-algebra (see* [1]*).*

Remark 3.4. Convergent sequences can be defined on *RGB*, using MV σ-algebra properties of *RGB* (see [1]).

4. Distance on RGB

To use the MV-algebra structure of RGB model for image processing, we have to define a distance function $d : RGB \times RGB \to RGB$ as follows

$$d\left(\left(a_1, a_2, a_3\right), \left(b_1, b_2, b_3\right)\right) =_{def}$$
$$= \left(\left(a_1, a_2, a_3\right) \ominus \left(b_1, b_2, b_3\right)\right) \oplus \left(\left(b_1, b_2, b_3\right) \ominus \left(a_1, a_2, a_3\right)\right).$$

From equation (3.5) we have

$$d\left(\left(a_1, a_2, a_3\right), \left(b_1, b_2, b_3\right)\right)$$
$$= \left(a_1 \ominus b_1, a_2 \ominus b_2, a_3 \ominus b_3\right) \oplus \left(b_1 \ominus a_1, b_2 \ominus a_2, b_3 \ominus a_3\right).$$

Applying equation (3.1) we obtain

$$d\left(\left(a_1, a_2, a_3\right), \left(b_1, b_2, b_3\right)\right)$$
$$= \left(\left(a_1 \ominus b_1\right) \oplus \left(b_1 \ominus a_1\right), \left(a_2 \ominus b_2\right) \oplus \left(b_2 \ominus a_2\right), \left(a_3 \ominus b_3\right) \oplus \left(b_3 \ominus a_3\right)\right).$$

From definition of the distance function on MV-algebra C we obtain:

$$d\left(\left(a_1, a_2, a_3\right), \left(b_1, b_2, b_3\right)\right) = \left(d\left(a_1, b_1\right), d\left(a_2, b_2\right), d\left(a_3, b_3\right)\right).$$

In Section 2 of this paper we have shown that $\langle C, \oplus, \neg, 0\rangle$ is an MV-chain. In Section 3 we have defined RGB set as a direct product of C set. It is obviously that RGB set is also a subdirect product of C set. From Chang's Subdirect Representation Theorem (see [2]) we obtain:

Proposition 4.1. *The MV-algebra $\langle RGB, \oplus, \neg, 0_{RGB}\rangle$ is a subdirect product of the MV-chains $\langle C, \oplus, \neg, 0\rangle$.*

References

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