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# AN APPROXIMATE ALGORITHM TO ESTIMATE PLAUSIBLE LOCATIONS OF UNDISCOVERED HYDROCARBON ACCUMULATIONS IN SPARSELY DRILLED AREAS

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Abstract. This paper uses concepts and principles pertaining to a natural geometrical data structure (the Voronoi diagram) in a theoretical attempt to estimate the sites and perimeters to really succeed in an exploration for undiscovered new hydrocarbon accumulations in an oil basin/system. The proposed algorithm can be applied to oil, methane gas or oil and gas combined reserves in a natural area of hydrocarbon accumulation, characterized by the same hydrocarbon source.

## 1. Introduction

The starting point of this mathematical experiment was a report [7], published by the Petroconsultants Group in 1993, on a new method for estimating undiscovered petroleum potential with applications to the giant oil fields of the world, such as: Arabo-Iranian basin, Campos basin, Gippsland basin, Kutei, South Sumatra, Niger delta, Timan-Pechora, North Sea grabens, Transylvania basin, as well as other petroleum systems. Their estimation is based on the best fit with fractal parabolas of oil field size distributions. Meanwhile, new methodologies meant to estimate the amount of undiscovered hydrocarbon reserves were announced in several reports ([11], [5]), whose results haven't been published so far.

However, a more "delicate" and obviously more difficult problem can be posed:

> "Is it possible to make forecasts/estimates, with a certain degree of plausibility, on the locations (sites, perimeters, extents, zones) of the "presumably existing" hydrocarbon accumulatios, not yet discovered in a sparsely drilled oil system?"

Using our knowledge of Voronoi diagrams ([3], [10]) which we had previously applied to some natural geological data structures (e.g., mineral deposits) we arrived at a first approach of the above problem. The most important question was:

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how to formulate, in mathematical terms, a location principle to find the "most plausible" sites and corresponding extents of new oil fields in a drilled area?

## 2. Voronoi diagrams: general concepts

Let $X$ be a non-empty arbitrary set. A function $d : X \times X \to \mathbb{R}$ is said to be a *distance* or a *metric* on $X$ if it satisfies the following conditions:

$$d(x,y) = 0 \iff x = y;$$

$$d(x,y) = d(y,x);$$

$$d(x,y) \leq d(x,z) + d(z,y) \quad \forall x,y,z \in X.$$

The pair $(X,d)$ is called a *metric space.*

A simple example is the real plane $\mathbb{R}^2$ with the metric defined by:

$$d(x,y) = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2} \quad \forall x = (x_1,x_2), y = (y_1,y_2) \in \mathbb{R}^2.$$

This metric is called the *Euclidean* metric on $\mathbb{R}^2$. Let $(X,d)$ be a metric space. A subset $Y$ of $X$ is said to be *bounded* (with respect to the metric) if

$$\sup\{d(x,y) | x,y \in Y\} \ < \ \infty.$$

Let $Y$ be a non-empty subset of $X$ and $x \in X$. The real number:

$$d(Y,x) = \inf\{d(y,x) | y \in Y\}$$

is called distance from $x$ to $Y$. Let $M(X)$ be the set of all non-empty, bounded subsets of $(X,d)$ and $M'(X)$ the set of all non-empty and closed subsets of $(X,d)$. If $Y, Z \in M(X)$, then the real number:

$$e(Y,Z) = \sup\{d(Y,z) | z \in Z\}$$

is called *gauge* or *excess* of $Y$ from $Z$.

If $(X,d)$ is a metric space, then the function $\rho : M'(X) \times M'(X) \to \mathbb{R}$ defined by:

$$\rho(Y,Z) = \max\{e(Y,Z), e(Z,Y)\} \quad \forall Y,Z \in M'(X)$$

is a metric on $M'(X)$ [8]. This metric is called the *Pompeiu-Hausdorff* metric.

Suppose now that $S = \{C_1, \ldots, C_k\}$ is a finite set of distinct points in $\mathbb{R}^2$ and $f : \mathbb{R}^2 \times S \to [0,\infty[$ is a given function called the *influence* or *authorithy* function.

If $i \in \{1, \ldots, k\}$, then the subset of $\mathbb{R}^2$ defined by:

$$reg(C_i) = \{x \in \mathbb{R}^2 | f(x,C_i) \leq f(x,C_j), \forall j \in \{1,\ldots,k\} \setminus \{i\}\}$$

is said to be the *influence region* of $C_i$.

We call the set $\{reg(C_1), \ldots, reg(C_k)\}$ the *Voronoi diagram* generated by $S$ with influence function $f$. In fact, the Voronoi diagram is a covering of the real plane by a set of regions associated with members of the point set $S$ and an influence function $f$.

The sets $reg(C_i), i = \{1, \ldots, k\}$ are sometimes called *faces* of the Voronoi diagram. The intersection of two faces gives a *Voronoi edge* and the intersection of two edges is called a *Voronoi vertex*.

We'll denote by $Vor(S, f)$ the set consisting of all points of the edges of a Voronoi diagram generated by $S$ and an influence function $f$.

If $i, j \in \{1, \ldots, k\}, i \neq j$, then the subset of $\mathbb{R}^2$ defined by:

$$sep(C_i, C_j) = \{x \in \mathbb{R}^2 \,|\, f(x, C_i) = f(x, C_j)\}$$

is called the *separation curve* of $C_i$ and $C_j$, and the set:

$$dom(C_i, C_j) = \{x \in \mathbb{R}^2 \,|\, f(x, C_i) \leq f(x, C_j)\}$$

defines the *dominance region* of $C_i$ over $C_j$.

The following relations hold true:

$$dom(C_j, C_i) = [\mathbb{R}^2 \setminus dom(C_i, C_j)] \cup sep(C_i, C_j),$$

$$reg(C_i) = \cap\{dom(C_i, C_j) | j \in \{1, \ldots, k\} \setminus \{i\}\}.$$

If the influence function $f$ is the Euclidean metric $d$ of $\mathbb{R}^2$, then the planar (ordinary) Voronoi diagram is obtained. In this case, $sep(C_i, C_j)$ is the perpendicular bisector $m_{ij}$ between $C_i$ and $C_j$, and $dom(C_i, C_j)$ is the half plane defined by $m_{ij}$, containing $C_i$. Therefore, being the intersection of $k-1$ half planes, $reg(C_i)$ is a convex set.

When $f = d$, we call the region $reg(Ci)$ the (ordinary) *Voronoi polygon* associated with $C_i$, or the Voronoi polygon of $C_i$ denoted $V(C_i)$. Since a Voronoi polygon is a closed set, it contains its boundary denoted by $\partial V(C_i)$. The term polygon is used to denote the union of the boundary and of the interior. The boundary of a Voronoi polygon may consist of line segments, half lines or infinite lines, which we call *Voronoi edges*. Alternatively, we may define a Voronoi edge as a line segment, a half line or an infinite line shared by two Voronoi polygons.

If $V(C_i) \cap V(C_j) \neq \emptyset$, then the set $V(C_i) \cap V(C_j)$ gives a Voronoi edge which may degenerate into a point. If $V(C_i) \cap V(C_j)$ is neither empty nor a point, we say that the Voronoi polygons $V(C_i)$ and $V(C_j)$ are *adjacent*.

For the sake of simplicity, if $f = d$, instead of $Vor(S, f)$ we write $Vor(S) = \partial V(C_i) \cup \ldots \cup \partial V(C_k)$.

Now let $A$ be a closed subset of $\mathbb{R}^2$ and $\mathfrak{T} = \{T_1, \ldots, T_k\}$, where each $T_i, i \in \{1, \ldots, k\}$ is a closed subset of $A$. If the elements of the set $\mathfrak{T}$ satisfy $[T_i \setminus \partial T_i] \cap [T_j \setminus \partial T_j] = \emptyset, \forall i, j \in \{1, \ldots, k\}, i \neq j$, then we call the set $\mathfrak{T}$ a *pretessellation* of $A$. A pretessellation $\mathfrak{T}$, where all $T_i, i \in \{1, \ldots, k\}$ are convex sets is called a *convex pretessellation*.

A pretesselation $\mathfrak{T} = \{T_1, \ldots, T_k\}$ with $A = \cup\{T_i | i = 1, \ldots, k\}$ becomes a *tesselation*. A planar Voronoi diagram is a tessellation which consists of convex polygons with three or more vertices. A planar tessellation in which any $T_i$ in $\mathfrak{T}$

is a triangle $\forall i \in \{1, \ldots, k\}$ is called a *triangulation* of $A$. Two vertices sharing an edge in a triangulation are called adjacent.

Given a planar Voronoi diagram where generators are not colinear and their number is three or more, but finite, we join all pairs of generators whose Voronoi polygons share a common Voronoi edge, thus obtaining a new tessellation. If the new tessellation consists only of triangles, we call it a *Delaunay triangulation*; otherwise, we call it a *Delaunay pretriangulation*. In the case of the Delaunay pretriangulation, we partition the non-triangular polygons into triangles by non-intersecting line segments joining the vertices. As a result, the Delaunay pretriangulation becomes a Delaunay triangulation.

## 3. Locational mathematical model

Let's consider an oil system (or a geological-tectonical region) externally delimited, on a geographical map, by the boundary of a simple polygon $A$. Suppose that in this oil system $k$ oil fields have been discovered.

Let the points $C_1, \ldots, C_k$ be the centers/sites/domes and let the simple polygons $\wp_1, \ldots, \wp_k, C_i \in \wp_i \subset A, i \in \{1, \ldots, k\}$ be the extents/contours of these fields, being situated on the same map. Moreover, $\wp_1 \cap \ldots \cap \wp_k = \emptyset$.

We consider the set $S = \{C_1, \ldots, C_k\}$. In addition, let's denote $B_i = \partial \wp_i, i \in \{1, \ldots, k\}$ and $P = \cup\{\wp_i | i = 1, \ldots, k\}$.

Now, we formulate the following question: where, in this region $A$, can the centers of a given number of new, posible oil fields be most plausibly located ?

In order to get an answer it is important to restate and formalize the above verbal problem more precisely, in mathematical terms. With that end in view, let $d$ be the Euclidean metric on $\mathbb{R}^2$, $M'$ the set of all non-empty closed subsets of $\mathbb{R}^2$ and $\rho$ the Pompeiu-Hausdorff metric on $M'$.

In order to mathematically formalize the locational problem, we must adopt an essential assumption:

**Assumption.**

$$B_1 \cap \ldots \cap B_k = Vor(S) \cap A$$

If the above assumption is correct, then we believe the most plausible location of the centers of $m$ undiscovered oil fields in the oil system $A$ leads to the following optimization problem:

**Location problem.** Find $m$ points $C_{k+1}, \ldots, C_{k+m}$ in $A \setminus P$ such that:

$$\rho(Vor(S \cup \{C_{k+1}, \ldots, C_{k+m}\}), B_1 \cup \ldots \cup B_k) =$$

$$\min\{\rho(Vor(S \cup S'), B_1 \cup \ldots \cup B_k) | S' \in \Sigma\}$$

where:

$$\Sigma = \{S' \subseteq A \setminus P | card(S' \setminus S) = m\}.$$

**Remarks.**

(1) In fact, the above location problem is to determine within a simple polygon $A$ the locations of a given number $(m)$ of points, outside of a pretessellation of $A$ (in $A\setminus P$), so that the Pompeiu-Hausdorff distance between two Voronoi diagrams having some common generators is minimized. The distance can be defined as the sum of Pompeiu-Hausdorff distances between the pairs of Voronoi polygons [2] with common generators.

(2) A couple of location problems are similar, although much easier than ours: the recognition of a Dirichlet (Voronoi) tessellation [1], [12] and the geographical optimization problem from [6]. These problems start from a convex tessellation. By contrast, we start from a more general, non convex pretesselation, denoted in the following by $(A, S, P)$.

## 4. The approximate algorithm

Being fully aware of the difficulty of the above location problem, we have tried to find only an approximate solution. Our approximate algorithm is of an incremental type [9] and uses some remarks on distortions of a Voronoi diagram when one point moves [4].

Let $S = \{C_1, \ldots, C_k\}$ be the set of sites/centers of discovered oil fields and an arbitrary point $C_0 \in A \setminus P$. In the following, we denote by $V(i), i = 1, \ldots, k$, the Voronoi polygon of $C_i$ in the Voronoi diagram generated by $S$ and by $V_0(i)$ the Voronoi polygon in the Voronoi diagram generated by $S_0 = S \cup \{C_0\}$. Let $\mathfrak{T}_0$ be the Delaunay triangulation of the set $S_0$.

For $C_i \in S$, we have $V(i) = V_0(i)$, if and only if $C_i$ and $C_0$ are not adjacent vertices in $\mathfrak{T}_0$ [4]. Moreover, if $C_i$ and $C_0$ are adjacent and $\wp_i \subset V(i)$, it doesn't follow that $\wp_i \subset V_0(i)$.

We say that the "center" $C_0$ is admissible in pretessellation $(A, S, P)$ in respect to $\mathfrak{T}_0$, if for every $C_i \in S$, such that $C_i$ and $C_0$ are adjacent vertices in $\mathfrak{T}_0$, then $\wp_i \subset V_0(i)$.

If $C_p, C_q \in A \setminus S, C_p \neq C_q$, let us denote by $V_p(i)$, respectively by $V_q(i)$, the Voronoi polygon of $C_i \in S$ in $Vor(S_p)$, respectively $Vor(S_q)$, where $S_p = S \cup C_p$ and $S_q = S \cup C_q$. Let $\mathfrak{T}_p$, respectively $\mathfrak{T}_q$ be the Delaunay triangulations of $S_p$, respectively $S_q$. Moreover, let $\mathring{A}(p)$ be the set of points in $S$ which are adjacent with $C_p$ in $\mathfrak{T}_p$, and $\mathring{A}(q)$ the points in $S$ adjacent with $C_q$ in $\mathfrak{T}_q$.

Let $C_p$ and $C_q$ be two admissible centers in $(A, S, P)$ corresponding to $\mathfrak{T}_p$ and $\mathfrak{T}_q$, respectively. We say that $C_p$ is preferred to $C_q$ if

$$\pi_p \;=\; \sum_{C_i \in \mathring{A}(p)} \rho(V_p(i), V_p(p)) \quad \leq \quad \sum_{C_i \in \mathring{A}(q)} \rho(V_q(i), V_q(q)) \;=\; \pi_q,$$

where $\rho$ is the Pompeiu-Hausdorff distance.

The number $\pi_p$ evaluates the distortion effect of the point $C_p$ on the Voronoi diagram generated by $S$. At the same time, $\pi_p$ represents a measure of plausibility. The smaller $\pi_p$, the more plausible $C_p$.

Let $G(p,p)$ be a uniform rectangular grid with sides parallel to the coordinate axes, which contains $A$, and the number $p$ of horizontal and vertical grid lines an even integer.

**The algorithm. Step 1.** Let $p$ be the smallest positive even integer such that $m < p^2$ and let $G(p,p)$ be the minimal uniform rectangular grid covering $A$.

Let $n := 0$ and $W^0 := S$.

**Step 2.** Scan the grid $G(p,p)$ rectangle-by-rectangle, in a spiral order, starting from the central rectangle of the grid. For each rectangle $D_{q+1}$ execute the following operations:

**a.** Let $C_0 :=$ the center of $D_{q+1}$;

**b.** Construct the Delaunay triangulation $\mathfrak{T}_*$ of the set $W^* = W^q \cup \{C_0\}$. We distinguish the cases:

*Case I.* If there exists a point $C_i \in S$ which is adjacent to $C_0$ in $\mathfrak{T}_*$ and $C_0 \in \wp_i$, take the next rectangle.

*Case II.* If $C_0 \notin P$, choose the most preferred point $C^*$ between $C_0$ and each of the four rectangle corners which are not in $P$. Let $W^{q+1} := W^q \cup \{C^*\}$ and $n := n+1$. If $n = m$ **stop**, else go to **Step 1** with $p := 2p$.

**Remarks.**

(1) In fact, this algorithm locates a $m$-points planar configuration in a pretessellation, each of the $m$ points having only a local plausibility. This configuration can be a starting point pattern for further, more subtle, improved algorithms.

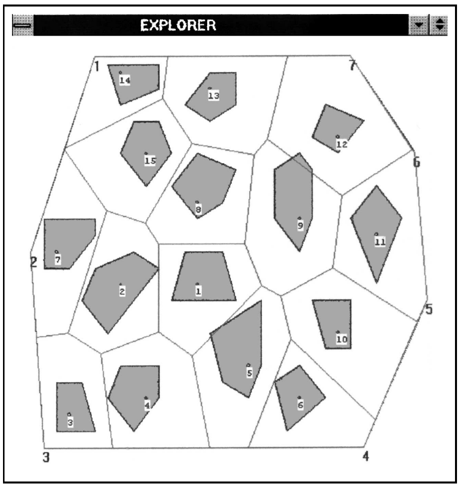
(2) Mathematically, it is easier to insert "new" admissible oil fields closer to the boundary of $A$, but we have preferred a more "central" configuration for geological reasons.

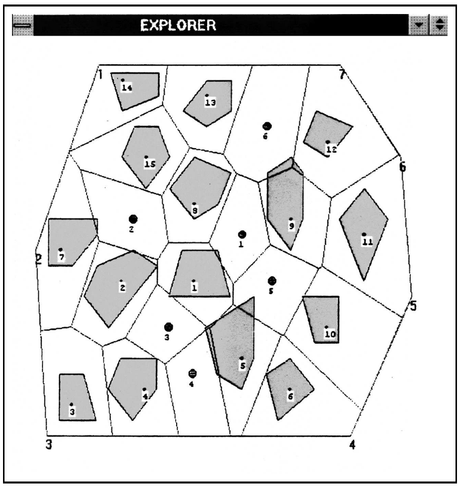
(3) The algorithm can be relativized to a subzone of $A$, called zone of geological interest.

A software package named *EXPLORER* has been developed and tested on both non- and real data. *EXPLORER* enables users to:

- visualize a basin in study with all its fields,
- visualize a particular field and its contour,
- visualize a field and its adjacent neighbours,
- visualize the Voronoi diagram of a basin,
- locate a given number of new plausible oil fields and their possible extents, and
- print the founded pattern of "new" and old fields.

In the following figures we present an example of *EXPLORER* outputs for a fictional basin with 15 active oil fields, their contours or extents (gray polygons) and the Voronoi diagram of these fields (left); the forecasted sites of 6 possible new fields (circled dots) and their plausible (in decreasing rank order, #1 having the highest degree of plausibility) extents as their Voronoi polygons (right).




## 5. An experiment for Transylvania Basin

The *EXPLORER* software application allows the user to "discover" several "new" oil fields as well, in a real oil basin, **Transylvania**, with different plausibilities (depending on the number of scanned rectangles).

We can comunicate, to whom may be interested, two tested results:

(1) Using the information regarding the 23 active fields discovered in the Transylvania basin during 1906-1965, the algorithm proposed 20 "new" locations of methane gas fields. We were surprised to find out that 15 out of these fields were "confirmed" (their extents having a non-empty intersection with at least one contour of an actually discovered field) during 1966-1985 (out of the 29 new fields actually pointed out during this period). Furthermore, 4 more fields were confirmed during 1986-1996 (out of 52 new actually discovered fields). Therefore, 19 out of the 20 sites proposed by algorithm have been confirmed.

(2) Forecasting again 20 possible locations of "new" fields by means of the methane gas field pattern existing in 1985 (i.e., 52 active fields), 17 fields were confirmed during 1986-1996.

## 6. Conclusions

We are aware that the development of a new method/technology to mathematically forecast the sites and/or extents of new oil fields in an oil system needs a strong collaboration between oil geologists, mathematicians and computer engineers. The above algorithm is just a first step toward a new technology. Algorithm's forecasts zones of possible hydrocarbon accumulations require confirmations by geological parameters. But these forecast perimeters, we believe, are the most plausible locations for the new possible oil fields in an oil basin.

By superposing quantitative geological parameter (e.g., permeability, porosity, pressure, &c.) maps, on this prognosticated locations the exploration expenses and time can be drastically diminished. As the tested results on a real oil basin indicate, we are optimistic and forsee a successful completion (new natural geometrical data structures generated by an influence function $f \neq d$; new location principles; new improved algorithms) of this promising research.

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## References

[1] P.F. Ash, E.D. Bolker, *Recognizing Dirichlet tessellations*, Geometriae Dedicata, 19 (1985), pp. 175-206.

[2] M.J. Attallah, *A linear time algorithm for the Hausdorff distance between convex polygons*, Inf. Proc. Let., 17 (1983), pp. 207-209.

[3] F. Aurenhammer, *Voronoi Diagrams - A Survey of a Fundamental Geometric Data Structure*, ACM Comput. Surveys, 23 (1991), pp. 345-405.

[4] L.-M. Cruz Orive, *Distortion of certain Voronoi tessellations when one particle moves*, J. Appl. Prob., 16 (1979), pp. 95-103.

[5] C.C. Barton, G.L. Troussov, *Fractal Methodology for Petroleum Resource Assessment and Fra-A Computer Program That Calculates the Volume and Number of Undiscovered Hydrocarbon*, U.S. Geological Survey; announced in 1997.

[6] M. Iri, K. Murota, T. Ohya, *A Fast Voronoi Diagram Algorithm with Applications to Geographical Optimization*, in Lecture Notes in Control and Information Sciences 59 , Springer Verlag, Berlin, 1984, pp. 273-288.

[7] J. Laherrere, A. Perrodon, G. Demaison, *Undiscovered Petroleum Potential; A new approach based on distribution of ultimate resources*, Petroconsultants S.A., Geneva-London-Houston-St.Leonards-Singapore-Victoria Park, 1994.

[8] I. Muntean, *Analiză funcţională*, Universitatea "Babeş-Bolyai", Cluj-Napoca, 1993.

[9] T. Ohya, M. Iri, K. Murota, *Improvements of the Incremental Method for the Voronoi Diagram with Computational Comparison of Various Algorithms*, J. Operations Res. Soc. Japan, 27 (1984), pp. 306-336.

[10] A. Okabe, B. Boots, K. Sugihara, *Spatial Tessellations; Concepts and Applications of Voronoi Diagrams*, J.Wiley & Sons Ltd, London, 1992.

[11] R.G. Stanley, *The "Checkerboard Method": A New Way to Estimate the Numbers of Undiscovered Hydrocarbon Accumulations in Sparsely Drilled Areas*, U.S. Geological Survey; announced in 1995; not yet published.

[12] A. Suzuki, M. Iri, *Approximation of a tessellation of the plane by a Voronoi diagram*, J. Operations Res. Soc. Japan, 29 (1986), pp. 69-96.

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